Information flow over compound wireless relay networks

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Relay networks was formulated in the 1970s, but the complete characterization of the achievable rates of relay networks is open, even for the simplest three node network (single source, single destination, one relay) [1]. This is true even for the special case of Gaussian relay networks, where Gaussian broadcast and multiple access channels model the relay network interactions. However, several interesting coding ideas were developed in [1].

Given this discouraging state of affairs, a natural question to ask is whether we can make positive progress. Our philosophy was to simplify the models and obtain insights with the hope to approximately characterize the general Gaussian relay network capacity. We started with a linear deterministic model which incorporated both the main distinguishing characteristics of wireless channels, i.e., broadcast and multiple access [3]. For such a model we had a complete characterization of the relay network capacity in terms of an intuitively satisfying max-flow min-cut result [3]. This also led to an achievability result for arbitrary deterministic model determining the relay network interactions (broadcast, multiple access) [4]. The analysis for arbitrary deterministic functions needed the notion of typicality and gave insight to an approximate characterization of the wireless relay network capacity presented in [5].

More formally, consider a network represented by a directed relay network $G = (V, E)$ where $V$ is the set of vertices representing the communication nodes in the relay network. The communication problem considered is unicast (or multicast with all destinations requesting the same message). Therefore a special node $S \in V$ is considered the source of the message and a special node $D \in V$ is the intended destination. All other nodes in the network facilitate communication between $S$ and $D$. The received signal $y_j$ at node $j \in V$ and time $t$ is given by

$$y_{jt} = \sum_{i \in \mathcal{N}_j} h_{ij} x_{it}^j + z_j$$

(1)

where each $h_{ij}$ is a complex number representing the channel gain from node $i$ to node $j$, and $\mathcal{N}_j$ is the set of nodes that are neighbors of $j$ in $G$. Furthermore, there is an average power constraint equal to 1 at each transmitter and $z_j$ is Gaussian noise, with unit variance.

The information-theoretic cut-set upper bound [2] is:

$$\mathcal{C} = \max_{p(x_{1|t}, x_{1|t}) \in \Lambda_D} \min_{\Omega \in \Omega_D} I(Y_{1|t}; X_{1|t}|X_{1|t})$$

(2)

where $\Lambda_D = \{ \Omega : S \in \Omega, D \in \Omega^c \}$ is all source-destination cuts (partitions). The main result presented in [5] is summarized below.

**Theorem 1:** Given a Gaussian relay network the capacity $C$ satisfies

$$C - \kappa \leq C \leq \mathcal{C}$$

(3)

Where $\mathcal{C}$ is the cut-set upper bound on the capacity of $G$ as described in equation (2), and $\kappa$ is a constant that could depend on the total number of nodes in $G$ and the topology.

The gap ($\frac{\kappa}{\mathcal{C}}$) does not depend on the channel parameters. Moreover, our achievability strategy based on a “quantize at noise level” and “forward” through a random mapping does not depend on the channel realization. Therefore, the result in Theorem 1 can be extended to compound relay networks where we allow each channel gain $h_{ij}$ to be from a set $\mathcal{H}_{ij}$, and the particular chosen values are unknown to the source node $S$. In this case we can obtain the following result.

**Theorem 2:** Given a compound Gaussian relay network the capacity $C_{cn}$ satisfies

$$C_{cn} - \kappa \leq C_{cn} \leq \mathcal{C}_{cn}$$

(4)

where $\mathcal{C}_{cn} = \max_{p(x_{1|t}, x_{1|t}) \in \Omega_D} \inf_{\Omega \in \Omega_D} \min_{\Omega \in \Omega_D} I(Y_{1|t}; X_{1|t}|X_{1|t})$. Therefore, we can obtain an approximate characterization of the full-duplex compound relay network.

REFERENCES


1Note that this is different from the compress-forward strategy [1].