Information flow over wireless networks: a deterministic approach

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Overview

- **Point-to-point channel**
  - Information theory provides an abstraction

- **Wireless network**
  - Does information theory give us a similar picture? *Not yet.*
Basic model for wireless channels

- Key features of wireless channel
  - Broadcast
  - Interference
  - High dynamic range of channel variations

- Basic PHY layer model: additive-Gaussian channel model

\[ y = \sum_i h_i x_i + z \]
What is known?

Point to point: \( C = \frac{1}{2} \log(1 + \text{SNR}) \)
(Shannon 1948)

Multiple access
(Ahlswede, Liao 70’s)

Broadcast
(Cover, Bergmans 70’s)
State of the art

- Unfortunately, we don’t know the capacity of most other Gaussian networks

- Many relaying strategies are developed, but
  - Can’t analyze how they perform in multi relay networks
  - Don’t even know how suboptimal these schemes are

- Current approach
  - Stuck in some toy problems
  - Not scalable to larger networks

- How can we make progress?
Our approach

- Develop a simpler model

- We propose a deterministic channel model
  - De-emphasize the background noise
  - Focus on the interaction between users’ signals

- Advantages:
  - Far more analytically tractable
  - Scalable to multi node networks
  - Helps to visualize information flow and get insights
  - Approximates the Gaussian model
Methodology

- Deterministic network
  - Gaussian network
  - Approximate analysis
  - Perturbation
  - Deterministic model
- Finite field
  - Deterministic network
  - Exact analysis
In this talk …

- Apply our methodology to Gaussian relay networks
  - The model to study
    - cooperative communication strategies for next generation of wireless systems (wimax, UMB, ...)
    - communication protocols for wireless Ad-hoc networks

Characterize its capacity within a constant number of bits

Develop a simple, near optimal cooperative relaying strategy
Outline

- Introduce the deterministic channel model
  - point-to-point
  - multiple access
  - broadcast

- Apply it to the relay network
  - determine the capacity of deterministic relay network

- Going back to Gaussian relay network
  - approximate its capacity
  - determine a near optimal cooperative relaying strategy

- Other interesting applications of deterministic model
Point-to-point

Gaussian

If we have

\[ y = \sqrt{\text{SNR}} \cdot x + z \]

\[ C = \frac{1}{2} \log(1 + \text{SNR}) \]

\[ x = 0.b_1b_2b_3b_4b_5 \ldots \]

If \( n = \frac{1}{2} \log_2 \text{SNR} \) we have

\[ \sqrt{\text{SNR}} \cdot x = b_1b_2 \cdots b_n b_{n+1} \cdots \]

Deterministic

Least significant bits are truncated at noise level

\[ C_{\text{det}}(n) = n \]

\[ n \leftrightarrow \left[ \frac{1}{2} \log \text{SNR} \right]^+ \]

\( n \propto \text{SNR} \) on the dB scale

captures channel strength
Multiple access

Gaussian

\[ y = \sqrt{\text{SNR}_1} x_1 + \sqrt{\text{SNR}_2} x_2 + z \]

- Can visualize where signal interaction happens
- Captures channel strength variations in wireless medium
  - Not captured in other simple models
    - packet collision model

Deterministic

\[ y = \text{SNR}_1 x_1 + \text{SNR}_2 x_2 + z \]

- \( n_1 = \log_2 \text{SNR}_1 = 5 \)
- \( n_2 = \log_2 \text{SNR}_2 = 2 \)
- mod 2 addition
Multiple Access (cont.)

Gaussian

\[ \text{Tx}_1 \quad \text{SNR}_1 \quad \text{Rx} \]
\[ \text{Tx}_2 \quad \text{SNR}_2 \]

\[ \log(1 + \text{SNR}_1) \]
\[ n_1 = \log_2 \text{SNR}_1 = 5 \]
\[ \log(1 + \text{SNR}_2) \]
\[ n_2 = \log_2 \text{SNR}_2 = 2 \]

Deterministic

\[ \text{Tx}_1 \]
\[ \text{Tx}_2 \]
\[ \text{Rx} \]

\[ \text{mod 2 addition} \]

\[ n_1 = \log_2 \text{SNR}_1 = 5 \]
\[ n_2 = \log_2 \text{SNR}_2 = 2 \]

To within 1 bit

captures interference
Broadcast

Gaussian

\[ \text{SNR}_1 \xrightarrow{\text{Rx}_1} \text{Rx}_1 \]
\[ \text{SNR}_2 \xrightarrow{\text{Rx}_2} \text{Rx}_2 \]

\[ \log(1 + \text{SNR}_1) \]

\[ \log(1 + \text{SNR}_2) \]

To within 1 bit

captures broadcast

Deterministic

\[ n_1 = \log_2 \text{SNR}_1 = 5 \]

\[ n_2 = \log_2 \text{SNR}_2 = 2 \]

\[ R_1 \]

\[ R_2 \]
Apply the deterministic model to relay networks

- AWGN
  - Gaussian relay
  - Deterministic model
  - Approximate analysis
- Finite field
  - Deterministic relay
  - Exact analysis
  - Perturbation
Deterministic relay network

- Link from node $i$ to node $j$ is described by an integer $n_{ij}$ (channel strength)
Algebraic representation

- Received Signal:

\[
y_{B1} \neq_j (t) = S^{q-\delta} S^{3} x_{A_1}(t) \oplus S^{5-2} x_{A_2}
\]

- \( S = \text{shift}(m_j) \), \( m_j \) is a shift matrix (q x q)
- All operations are in \( F_2 \)
General linear finite-field model

- Channel from i to j is described by an arbitrary channel matrix $G_{ij}$ operating on $F_2^q$

- Received signal:

$$y_j(t) = \sum_{i=1}^{M} G_{ij} x_i(t) \pmod{2}$$

  - Deterministic model: $G_{ij} = S^{q-n_{ij}}$
  - Wireline network also a special case
Cut-set upper bound

For deterministic, linear finite field model

\[ C_{\text{relay}} \leq \overline{C} = \max_{P_{X_1, \ldots, X_M}} \min_{\Omega} I(X_\Omega; Y_{\Omega^c} \mid X_{\Omega^c}) \]

\[ C_{\text{relay}} \leq \overline{C} = \min_{\Omega} \text{rank}(G_{\Omega \rightarrow \Omega^c}) \]
Main result

- Theorem: Cutset bound is achievable,

\[ C_{\text{relay}} = \bar{C} = \min_{\Omega} \text{rank}(G_{\Omega \to \Omega^c}) \]

- In wireline networks, \( \text{rank}(G_{\Omega \to \Omega^c}) \) is just summation of the capacity of links from \( \Omega \) to \( \Omega^c \)

- Our theorem is a generalization of Ford-Fulkerson max-flow min-cut theorem

- Also holds in the multicast scenario
  - Generalization of network coding to achieve the multicast capacity of wireline networks (Ahlswede-Cai-Li-Yeung)
Example: one relay

- How to achieve the capacity?
  - Routing!

- How to achieve the capacity in other networks?

Mathematical expression:

\[ C = \min \left( \max(n_{SD}, n_{SR}), \max(n_{SD}, n_{RD}) \right) \]

\[ = n_{SD} + \min \left( (n_{SR} - n_{SD})^+, (n_{RD} - n_{SD})^+ \right) \]
Multi-stage network (special case)

- Lengths of all paths from S to D are the same
- Major simplification
  - messages do not mix in the network
- Use a network coding strategy (similar to Ahlswede et. al. 2000):
  - S: map each message into a random codeword of length T symbol times
  - Each relay randomly maps the received signal into a transmit codeword
- Min-cut is achieved
General networks

- Consider the time-expanded network with $k$ stages, each $T$ symbol times long
- It is a multi-stage network!
  - Apply the same strategy on (super) messages
- Can achieve $1/k$ of the min-cut of time-expanded network ($\overline{C}_k$)
Key Question: Is $\lim_{k \to \infty} \frac{C_k}{k} = \bar{C}$, min-cut of the original network?

- There are more cuts in the time expanded graph

Yes! (proof based on submodularity of entropy function)

Min-cut is achieved
Back to the Gaussian relay network

- AWGN
  - Gaussian relay
  - Deterministic model
  - Approximate analysis
  - Perturbation

- Finite field
  - Deterministic relay
  - Exact analysis
  - Capacity characterization
  - Optimal communication scheme
Example: one relay

Gaussian

$$h_{SR} \rightarrow R \rightarrow h_{RD} \rightarrow h_{SD} \rightarrow D$$

$$\overline{C} - C \leq C \leq \overline{C}$$

Deterministic

- Gap is at most 1-bit
- On average it is much less than 1-bit

Decode-Forward is optimal

Routing is optimal
Relaying scheme

Deterministic
- S encodes the message over $T$ symbol times
- Each relay randomly maps the received signal into a transmit codeword
- D decodes the message deterministically

Gaussian
- S encodes the message over $T$ symbol times
- Each relay,
  - Quantizes the received signal at noise level
  - Randomly maps it into a Gaussian codeword
- D decodes the message by finding the one that is jointly typical with $y_D$
Properties of the scheme

- Simple
  - Quantize
  - Map to a transmit codeword
- Relays don’t need any channel information
- How does it perform?
Main result

- Theorem: for any Gaussian relay network

\[ \bar{C} - \kappa \leq C \leq \bar{C} \]

- \( \bar{C} \) is the cut-set upper bound on the capacity
- \( \kappa \) is a constant that depends on size of the network, but not the channel gains or SNR’s of the links

- Uniform approximation of the capacity for all channel parameters
- Much stronger than degrees of freedom calculation
Extensions

- We generalize the result to the following scenarios:
  - Multicast to multiple destinations
  - Nodes have multiple antennas
  - Half-duplex constraint
  - Fading (channel variations over time)
Summary

- Complexity of Gaussian model prevents further development in understanding wireless networks

- Development of a linear deterministic model to:
  - Help obtain intuitive engineering insights
  - Help make progress in wireless network information theory

- Future interesting applications of the deterministic model
  - Information theory
  - Wireless communications
  - Networking
The End