Approximate Capacity of the Gaussian Two Pair Bidirectional Relay Network

Salman Avestimehr
Caltech-Cornell

In collaboration with:
Amin Khajehnejad (Caltech), Aydin Sezgin (UC Irvine)
Babak Hassibi (Caltech)

ISIT 2009
Overview

- Two pairs communicate via a relay
- How to relay the information?
- Capacity region?
State of the art

- Two-way relay channel has been studied extensively
  - Decode & Broadcast: (Oechtering et. al.)
  - PHY network coding: (Katti et. al., Narayanan et. al., Hausl et. al., Baik et. al.)
  - Max achievable rate:
    - Equal channel gains: (Narayanan-Wilson-Sprintson)
    \[ R_{AB} = R_{BA} = \log\left(\frac{1}{2} + \text{SNR}\right) \]
    - In general: (Nam-Chung-Lee., Gunduz-Tuncel-Nayak, Avestimehr-Sezgin-Tse)
      constant - bit gap approximation

A <-> R <-> B
Outline

- Extend the results for the two-pair case
- Characterize the capacity of a noiseless (deterministic) version of the problem
- Propose a strategy that achieves within 2 bits/sec/Hz of the capacity of the Gaussian case
Deterministic channel model (A.-Diggavi-Tse)

Receiver gets those bits that arrive above noise level.

Point to point:

- Transmitter (Tx) sends messages.
- Receiver (Rx) receives messages.
- Bits $b_1, b_2, b_3, b_4, b_5$ are transmitted.
- Noise is present.

MAC:

- Transmitters (Tx1, Tx2) send messages.
- Receiver (Rx) receives messages.
- Receiver gets the modulo sum of those bits that arrive at the same signal level.

$\oplus$ represents addition modulo 2.
Single pair

- Optimal scheme (deterministic): Reorder-forward equations

  uplink

  \[ R_{AB} \leq \min(n_1, n_4) = 4, \quad R_{BA} \leq \min(n_2, n_3) = 2 \]

- Near optimal scheme (Gaussian):
  - Relay decodes the strong codeword first
  - Quantizes the remaining message (AST 2008), or decode the lattice point (NCL 2008), then broadcast
Single pair

- Just need to forward enough new equations (random mixing suffices)
- In the Gaussian case CF also achieves within 1-bit of the cut-set (Gunduz et. al.)
Multiple pairs

- Random mixing of equations does not seem to work for multiple pairs
- We need to manage interference and carefully create and forward the equations
- Idea: orthogonalize the pairs over signal levels
- How to allocate the signal levels?
An algorithm for signal level allocation

- Assume we want to achieve \( R = (R_{A1}, R_{B1}, R_{A2}, R_{B2}) \)

1. Pick a pair with non-zero rates (say \( R_{A1} \neq 0, R_{B1} \neq 0 \))
   - Assign the strongest signal level in UL that is connected to both
   - Assign the weakest signal level in DL that is connected to both
   - Update \( R = (R_{A1} - 1, R_{B1} - 1, R_{A2}, R_{B2}) \).

2. Pick a user with non-zero rate (say \( R_{A1} \neq 0 \))
   - Assign the strongest signal level in UL that is connected to \( A_1 \)
   - Assign the weakest signal level in DL that is connected to \( B_1 \)
   - Update \( R = (R_{A1} - 1, R_{B1}, R_{A2}, R_{B2}) \).
Main result (for deterministic networks)

- Theorem (AKSB, ITW 09):

  For deterministic multi-pair bidirectional relay networks
  
  - Capacity region = cut-set bound region
  - It is achieved by an equation forwarding scheme
Structure of the optimal scheme (deterministic)

- To achieve \((R_{A1}, R_{B1}, R_{A2}, R_{B2})\), \(R_{Ai} \geq R_{Bi}\):
  - Create and forwards \(R_{Bi}\) equations for pair \(i\)
  - Forward \(R_{Ai} - R_{Bi}\) bits from \(A_i\) to \(B_i\)
  - Signal level allocation is according to the channel gains

\[
\begin{align*}
n_{A1R} &\geq n_{A2R} \geq n_{B1R} \geq n_{B2R} \\
n_{RB2} &\geq n_{RB1} \geq n_{RA2} \geq n_{RA1}
\end{align*}
\]
Transition to Gaussian channel model

- Three challenges
  1. Deal with the additive noise
  2. Decoding the equations
  3. Power leakage from the signals of lower levels to those transmitted at higher levels

- Solutions
  1. Coding
  2. Use lattice codes
  3. Leakage inevitable, however:
     - Treat all lower level interferences as noise
     - Decrease the rates to be able to decode (loosing constant bits)
Relaying scheme: uplink

- Weak users: use a lattice code
- Strong users: use a superposition of a lattice code and a random code

\[ x_{A_1} = \sqrt{\alpha_{A_1}^{(1)}} x_{A_1}^{(1)} + \sqrt{\alpha_{A_1}^{(2)}} x_{A_1}^{(2)} \]

\[ x_{A_2} = \sqrt{\alpha_{A_2}^{(1)}} x_{A_2}^{(1)} + \sqrt{\alpha_{A_2}^{(2)}} x_{A_2}^{(2)} \]

\[ x_{B_1} = \sqrt{\alpha_{B_1}} x_{B_1} \]

\[ x_{B_2} = \sqrt{\alpha_{B_2}} x_{B_2} \]

\[ | h |_{A_1R} \geq | h |_{A_2R} \geq | h |_{B_1R} \geq | h |_{B_2R} \]

Received signal levels at the relay

- \( R_{A_1} - R_{B_1} \) bits of \( A_1 \)
- \( R_{A_2} - R_{B_2} \) bits of \( A_2 \)
- \( R_{B_1} \) eqns of pair 1
- \( R_{B_2} \) eqns of pair 2
Decoding at the relay

- Relay successively decodes the received equations and bits by treating interference at lower levels as noise.

1. Decode $x_{A_1}^{(1)}$: $R_{A_1}^{(1)} \leq \log(1 + \text{SINR}_{A_1^{(1)}})$

2. Decode $x_{A_2}^{(1)}$: $R_{A_2}^{(1)} \leq \log(1 + \text{SINR}_{A_2^{(1)}})$

3. Decode $x_{A_1}^{(2)} + x_{B_1}$: $R_{A_1}^{(2)} = R_{B_1} \leq \log(\text{SINR}_{A_1^{(2)}})$

4. Decode $x_{A_2}^{(2)} + x_{B_2}$: $R_{A_2}^{(2)} = R_{B_2} \leq \log(\text{SINR}_{A_2^{(2)}})$

Received signal levels at the relay:
- $R_{A_1} - R_{B_1}$ bits of $A_1$
- $R_{A_2} - R_{B_2}$ bits of $A_2$
- $R_{B_1}$ eqns of pair 1
- $R_{B_2}$ eqns of pair 2
Relaying scheme: downlink

- Relay uses a superposition of four Gaussian codewords to broadcasts the decoded information

\[\begin{align*}
| h |_{A1R} &\geq | h |_{A2R} \geq | h |_{B1R} \geq | h |_{B2R} \\
| h |_{RB2} &\geq | h |_{RB1} \geq | h |_{RA2} \geq | h |_{RA1}
\end{align*}\]
Gaussian two-pair two-way relay network

- **Theorem:**

If \( R=(R_{A1}, R_{B1}, R_{A2}, R_{B2}) \) is in the cut-set region of the two-pair two-way relay network, then \((R_{A1}^{-2}, R_{B1}^{-2}, R_{A2}^{-2}, R_{B2}^{-2})\) is achievable.
Summary

- Managing interference by orthogonalizing the pairs over signal levels is near optimal.
- Characterized the capacity region of the two pair bidirectional relay network within 2 bits/sec/Hz.

Future directions:
- Generalizing to multiple pairs.
- A distributed algorithm for assigning signal levels.
Questions?