On the Scalability and Capacity of Single-User-Detection Based Wireless Networks with Isotropic Antennas

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***Abstract*— We extend the results of [1] on the capacity of single-user-detection based wireless networks, and we determine the implications of our results on the scalability of such networks. In particular, we consider a wireless network of *N* nodes that are equipped with isotropic antennas. The nodes are stationary or moving arbitrarily in a network domain of an arbitrary one, two, or three dimensional shape, as opposed to the two dimensional circular domain in [1]. In this arbitrary dimensional setting, we derive bounds on the per node end-to-end throughput capacity and the maximum number of simultaneous transmissions whose *SINR*s exceed a given threshold. We derive these bounds with both the bounded propagation model in [1] and a large class of bounded propagation models, which we refer to as the general propagation model. Our results show that with the general propagation model, the maximum number of simultaneous transmissions, whose *SINR*s exceed the threshold has an upper bound that does not depend on *N*, and the per node end-to-end throughput capacity is *O*(1/*N*) for a large class of wireless networks. Moreover, we establish several required conditions for scalability. These conditions show that, for any propagation model, to achieve a desired per node end-to-end throughput as *N* grows, it is necessary to keep the average source-to-destination hop count bounded. Also, for the particular propagation model of [1], we show that the size of the network domain must grow with *N* at a rate that depends on the dimension of the network domain and the path loss exponent.**

***Index Terms*— capacity, capacity bounds, isotropic antennas, scalability, throughput, wireless networks.**

I. Introduction

In every wireless network, the per node end-to-end throughput is upper bounded by the per node end-to-end throughput capacity (λ_{e}). In general, λ_{e} may depend on many parameters of the network, such as the number of nodes (N) [1]-[6]. Hence, understanding the dependence of λ_{e} on N and the other network parameters is vital to determine the necessary conditions for the scalability of a wireless network with the number of nodes.

In this paper, we consider the scalability and capacity of single-user-detection based wireless networks that are not supported by a wired infrastructure, and where the nodes are equipped with isotropic antennas [1]-[6]. Our work in this paper builds on the results of [1]. In [1], a new approach is developed to analyze the capacity and scalability of such networks on the plane, through the use of a more general network model than the models used in [3] and [4]. This generality was achieved by deriving bounds on λ_{e} while making no restrictions on the mobility pattern of the nodes, the temporal variation of source-destination associations, the number of simultaneous transmissions and/or receptions that each node can maintain, the routing protocol, and the spatial-temporal transmission scheduling scheme. Furthermore, a bounded propagation model, called the power law decaying propagation model, is used in [1] as opposed to an unbounded propagation model that was used in [3] and [4].

In the network model of [1], the network domain ($\boldsymbol{Q}$) is a disk of area A, and the reception model is signal-to-interference and noise ratio (*SINR*) threshold based, where a transmission at any given time t is considered *successful*, if its *SINR* is greater than or equal to a threshold value $\beta > 0$. The following are the three main results of [1]:

(A) The maximum number of simultaneously successful transmissions ($N_t^{\max}$) has an upper bound that does not depend on N. This upper bound is called the simultaneous transmission capacity of the network domain ($N_t^{\boldsymbol{Q}}$), and it represents the maximum number of simultaneously successful transmissions that can occur within the network domain, no matter what the number of nodes is. Asymptotically[1], $N_t^{\boldsymbol{Q}}$ is $O(G/\beta)$, where G is the processing gain. Also, for a path loss exponent γ, $N_t^{\boldsymbol{Q}}$ is $O(A^{\min\{\gamma/2,1\}})$ if $\gamma \neq 2$, and $N_t^{\boldsymbol{Q}}$ is $O(A/\log(A))$ if $\gamma = 2$. Moreover, $N_t^{\boldsymbol{Q}}$ is $O(\gamma^{dim})$.

(B) If the area A is fixed, then λ_{e} is $O(1/N)$ even when the mobility pattern of the nodes, the spatial-temporal transmission scheduling policy, the temporal variation of transmission powers, the source-destination pairs, and the possibly multi-path routes between them are optimally chosen. This result continues to hold even when the nodes can maintain multiple transmissions and/or receptions simultaneously, or when the communication bandwidth is partitioned into multiple channels. Moreover, λ_{e} is $O(1/\overline{H})$, where $\overline{H}$ is the average number of hops between a source and a destination.

(C) For practical systems, a desired per node end-to-end throughput is not achievable as $N \rightarrow \infty$, unless the following two conditions apply:

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[1]Suppose f and g are non-negative functions of a real variable x. We say f is $O(g)$ (with respect to x) (or, g is an *asymptotic upper bound* on f) if there are $x_0, y_0 > 0$ such that $f \leq y_0 g$ for all $x \geq x_0$. Also, we say f is $\Theta(g)$, if there are $x_1, y_1, y_2 > 0$ such that $y_1 g \leq f \leq y_2 g$ for all $x \geq x_1$. We will also use the fact that f is $\Theta(g)$ with respect to x, if $\lim_{x\rightarrow\infty}(f/g) \in (0, \infty)$.

(C.1) $\overline{H}$ does not grow indefinitely with N, and

(C.2) A grows with N such that, N is $O(A^{\min\{\gamma/2,1\}})$ if $\gamma \neq 2$, and N is $O(A/\log(A))$ if $\gamma = 2$.

In the network model of [3] and [4], A is fixed and it is assumed that the received power at a distance x from a transmitter equals to the product of the transmitted power[2] and $x^{-\gamma}$. The authors of [3] concluded that, if the nodes are immobile, then λ_{e} vanishes as $N \to \infty$. However, the authors of [4] concluded that there exists a two-dimensional mobility pattern of the nodes that allows λ_{e} to be $\Theta(1)$ with respect to N. Also, both [3] and [4] concluded that $N_t^{\max}$ is $\Theta(N)$.

These results of [3] and [4], differ significantly from the corresponding results in (A) and (B) above. The main reason for these differences, is the different propagation model that was used in [1]. In [1], it is observed that the propagation model of [3] and [4] leads to too optimistic evaluation of the received power of each transmission as N grows large. Hence, in [1], a bounded propagation model, called the power law decaying propagation model, is used. This model replaces $x^{-\gamma}$ above with $(1+x)^{-\gamma}$. This replacement provides a more meaningful estimate of the received power for small transmitter-receiver distances, while approximating the conventionally used propagation model at large distances[3]. This model was also suggested in earlier studies on connectivity [7]-[8] to obtain more realistic results.

The results of [3] are further extended to a spherical network domain in [5] and the authors of [6] concluded that the result of [4], which states that λ_{e} is $\Theta(1)$, continues to hold with a one-dimensional mobility pattern. Finally, the authors of [5] and [6] used the propagation model of [3] and [4], and they concluded that $N_t^{\max}$ is $\Theta(N)$ in their network settings as well.

In light of the results of [1]-[6], the following questions still remain unanswered:

- With the more realistic power law decaying propagation model, what are the capacity limitations of single-user-detection based *arbitrary dimensional* wireless networks, and what are the implications of these limitations on the scalability of such networks?
- Do the main results of [1] and the corresponding results for arbitrary dimensional wireless networks rely strongly on the specific structure of the power law decaying propagation model, or are they indeed valid with a large class of propagation models?

The main contribution of this paper is providing answers to the above questions. We answer these questions by deriving the corresponding bounds on $N_t^{\boldsymbol{Q}}$, $N_t^{\max}$, and λ_{e} for the arbitrary dimensional case, i.e., for a network domain $\boldsymbol{Q}$ that has diameter D and dimension $dim \in \{1,2,3\}$. In addition to the power law decaying propagation model, we introduce the general propagation model, which is the class of all bounded propagation models for which the received power monotonically decreases as the transmitter-receiver separation increases. With the help of the bounds that we derive, we prove that the result (A) above still holds, in the sense that with both propagation models, $N_t^{\max}$ is still upper bounded by the simultaneous transmission capacity of the network domain ($N_t^{\boldsymbol{Q}}$), which does not depend on N. Moreover, we show that $N_t^{\boldsymbol{Q}}$ is $O(G/\beta)$ with both propagation models. Additionally, we find the arbitrary dimensional analogues of the asymptotic upper bounds on $N_t^{\boldsymbol{Q}}$, $N_t^{\max}$, and λ_{e}. In particular, we show that with the power law decaying propagation model, $N_t^{\boldsymbol{Q}}$ is $O(D^{\min\{\gamma,dim\}})$ when $\gamma \neq dim$, and $N_t^{\boldsymbol{Q}}$ is $O(D^{dim}/\log(D))$ when $\gamma = dim$. Also, we show that $N_t^{\boldsymbol{Q}}$ is $O(\gamma^{dim})$. Furthermore, we show that the $O(1/N)$ result in (B) continues to hold with the general propagation model, and the $O(1/\overline{H})$ result in (B) holds for every propagation model.

Finally, regarding the scalability of practical systems, we show that condition (C.1) is required for every propagation model, and the arbitrary dimensional analogue of condition (C.2) is as follows: D grows with N, such that N is $O(D^{\min\{\gamma,dim\}})$ if $\gamma \neq dim$, and N is $O(D^{dim}/\log(D))$ if $\gamma = dim$.

Our results in this paper also extend the results of [5] and [6] in a similar way [1] extended the results of [3] and [4], as we pointed out in the beginning of this section.

The outline of the paper is as follows: In section II we describe the network model and define the quantities that are used in the derivations. In section III, we derive the upper bounds on the simultaneous transmission capacity and the per node end-to-end throughput capacity. Section IV presents the analysis of the upper bounds. In section V, we discuss the implications of the results on scalability. Finally, section VI concludes the paper.

II. Network Model and Definitions

Firstly, we discuss our network model, which is an extension of the network model in [1].

A. Network domain and Nodes **:** All of the N nodes are located within the bounded network domain $\boldsymbol{Q} \subseteq \Re^{dim}$ at all times, where $\Re$ is the real line.[4] The *diameter of the network domain*, D, is defined as $D \triangleq \sup_{u,v\in\boldsymbol{Q}} \|u-v\|$. There are no restrictions on whether or how nodes move.

B. Transmitter and Receiver Model **:** Each node is capable of being a transmitter and/or a receiver at any given time. All transmitters and receivers have isotropic antennas. There are no restrictions on how the transmission power is varied during a transmission. For the time being, we assume that all transmissions take place within the same communication bandwidth, but in section III-B, we will also generalize our results to the case where the communication bandwidth is partitioned into sub-channels of smaller bandwidth. Information from a transmitter can be transmitted to its intended receiver at a rate not exceeding $W_{\max}$ bits/s only when the *SINR* at the receiver is not below $\beta > 0$. The processing gain $G > 0$ represents the factor by which the total received interference power is reduced at each receiver. Each receiver is capable of maintaining at most s simultaneous transmissions intended for itself, given that the *SINR*s of these transmission are greater

[2] In [3] and [4], the *SINR* based threshold model with this propagation model is called *physical model*.

[3] Interested reader can see [1] or [12] for a detailed discussion on this issue and for more detailed discussions on comparisons among the network models of [1], [3] and [4].

[4] If all points of $\boldsymbol{Q}$ lie on a line, then $dim = 1$, else if all points of $\boldsymbol{Q}$ lie on a plane, then $dim = 2$, otherwise $dim = 3$. Also, distance, area, and volume measures have the units [m], [m^2], and [m^3], respectively.

than or equal to β. Also, each node can originate at most s simultaneous transmissions. So, it is possible that a node may attempt transmitting to s other receivers, while it is receiving from s other transmitters. In any of our results, letting s tend to infinity corresponds to relaxation of the above constraints that involve the parameter s (this corresponds to the model in [1]).

We will refer to the transmitter-receiver model explained so far as the *full-duplex* transmission model. Additionally, the *half-duplex* transmission model has one more constraint: no node can transmit and receive at the same time. Thus, the model in [3]-[6], that a node is able to maintain either one transmission or one reception at any given time, corresponds to the half-duplex transmission model with $s = 1$. In general, the reason for the existence of the parameter s is due to the hardware limitations of the transmitters and the receivers. Due to clock frequency, storage, and memory limitations of the transmitter or receiver controller circuitry, the transmitters and receivers cannot process more than a certain number of transmissions or receptions within a given time interval.

When there are multiple sub-channels, implementation of the full-duplex model at a node can take one of three different forms at any time: (i) the node may be transmitting over some sub-channels while it is receiving on some of the other sub-channels, (ii) the node may be transmitting and receiving over the same sub-channels, or (iii) some combination of (i) and (ii). Implementation of (ii) and (iii) may be done by equipping the nodes with multiple antennas. In narrowband systems, (i) is implemented instead of (ii) and (iii), since in (ii) and (iii) a node may directly contribute interference to its own reception, and this may significantly reduce the *SINR* of its own reception. For wideband systems with sufficiently large processing gain, (ii) and (iii) may allow supporting larger transmission rates as opposed to (i). In this paper, the results that we derive for multiple sub-channels are valid for all the three possible implementations of the full-duplex model.

C. Propagation Model **:** At any given time, suppose a transmitter transmits with power P, and x is the Euclidean distance between the transmitter and a given receiver. We assume that the received power is equal to $P \cdot a(x)$, where $a : [0, \infty) \mapsto (0, 1]$ is called the *attenuation function*. In addition to the power law decaying propagation model, where $a(x) = (1 + x)^{-\gamma}$ and $\gamma \geq 0$, we introduce the *general propagation model*, which is defined by the class of attenuation functions with the following properties: $a(0) = 1$, $0 < a(x) \leq 1$ for every $x \geq 0$, and $a(x + y) \leq a(x)$ for every $x, y \geq 0$. Note that the power law decaying propagation model satisfies these properties.

D. Traffic Pattern **:** As in [1], we make no restrictions on the variation of source-destination associations over time and the possibly multi-path routing protocol. Also, as in [1]-[6], intermediate nodes do not jointly encode, transmit, and decode information that comes from different sources.

Next, we define the quantities that we use in our derivations. We denote the *volume* of the network domain by $V_{\boldsymbol{Q}}$, and we define it as the outer measure[5] of $\boldsymbol{Q}$ in $\Re^{dim}$ [9]. When D is allowed to be a variable parameter, we call $\boldsymbol{Q}$ a *regularly scaling* network domain if $\boldsymbol{Q}$ has a nonempty interior for every $D > 0$ and scaling D by a constant results in scaling all linear dimensions of $\boldsymbol{Q}$ by the same constant. Thus, $V_{\boldsymbol{Q}}$ is directly proportional to D^{dim} if $\boldsymbol{Q}$ is regularly scaling.

At any given time t, N_t denotes the total number of successful transmissions. *Simultaneous transmission capacity of the network*, $N_t^{\max}$, is defined as the maximum value of N_t over all the possible placements of the N nodes, selections of transmitters and their intended receivers, and selections of transmission powers. *Simultaneous transmission capacity of the network domain*, $N_t^{\boldsymbol{Q}}$, is defined as the maximum value of N_t over all the number of nodes, the placements of them, selections of transmitters and their intended receivers, and selections of transmission powers. It follows immediately from these definitions that $N_t^{\max} \leq N_t^{\boldsymbol{Q}}$.

$b_i(T)$ denotes the total amount of information (in bits) generated by node i and received by its destinations during the T-second time interval [0,T]. The *end-to-end throughput of node i*, λ_i, is defined as follows: $\lambda_i \triangleq \lim_{T\to\infty} b_i(T)/T$,[6] for every $1 \leq i \leq N$. An end-to-end throughput λ_0 is said to be *achievable by all nodes*, if there exist a mobility pattern, a traffic pattern, a spatial-temporal transmission scheduling policy, and a temporal variation of transmission powers for the network nodes, such that $\lambda_i \geq \lambda_0$, for all $1 \leq i \leq N$. Also, an end-to-end throughput λ_0 is said to be *achievable on average*, if there exist a mobility pattern, a traffic pattern, a spatial-temporal transmission scheduling policy, and a temporal variation of transmission powers for the network nodes, such that $\frac{1}{N}\sum_{i=1}^{N} \lambda_i \geq \lambda_0$. So, if λ_0 is achievable by all nodes, then it is achievable on average. Since the contrapositive statement is also true, we will shortly say that λ_0 is *not achievable* if λ_0 is not achievable on average.

Finally, the *per node end-to-end throughput capacity*, λ_{e}, is defined as the supremum of all end-to-end throughputs that are achievable by all nodes. Similarly, the *per node average end-to-end throughput capacity*, λ_m, is defined as the supremum of all end-to-end throughputs that are achievable on average. It follows immediately from these definitions that $\lambda_m \geq \lambda_{\mathrm{e}}$.

III. Derivation of the Results

A. Upper Bounds on Simultaneous Transmission Capacity

1) Upper bounds that hold with every propagation model: In this subsection, during the proof of Theorem 1, we show that the number of simultaneously successful transmissions that any given node receives cannot exceed 1+G/β. Then, we combine this result with the constraints of the half-duplex and the full-duplex transmission models to establish Theorem 1.

Theorem 1: *For every propagation model and for every time t,*

$$N_t^{\max} \leq N \min\left\{1 + \frac{G}{\beta}, c \cdot s\right\}, \qquad \text{(T1.1)}$$

where $c \triangleq 1$ *if transmissions are full-duplex, and* $c \triangleq 1/2$ *if transmissions are half-duplex.*

[5]The *outer measure* of a set $E \subseteq \Re^{dim}$ is defined as $\inf\{\Sigma_k \mathrm{Vol}(I_k) : I_1, I_2, \ldots$ are closed cubes in $\Re^{dim}$ such that $E \subseteq \cup_k I_k\}$, where $\mathrm{Vol}(I_k)$ denotes the volume of the cube I_k for each k.

[6]Our results also hold with the more general definition $\lambda_i \triangleq \lim_{T\to\infty} \inf_{t>T} b_i(t)/t$, which does not require the existence of the limit. A similar definition was used in [3] and [4].

Proof: At an arbitrary time t, for every $1 \le i \le N$, let $r_i(t)$ be the number of successful transmissions for which node i is the intended receiver, $P_{ki}(t)$ be the received power of the k^{th} successful transmission intended for node i, for every $1 \le k \le r_i(t)$. Also, let $\xi_i(t)$ be the power of thermal noise present in the communication bandwidth at node i, and *SINR*$_{ki}(t)$ be the *SINR* of the k^{th} successful transmission intended for node i. If $1 \le i \le N$ and $r_i(t) > 1$, then for all $1 \le k \le r_i(t)$,

$$\beta \le SINR_{ki}(t) \le \frac{P_{ki}(t)}{\xi_i(t) + \frac{1}{G}\sum_{j=1,j\neq k}^{r_i(t)} P_{ji}(t)}$$

$$\Rightarrow \frac{G}{\beta} \ge \sum_{j=1,j\neq k}^{r_i(t)} \frac{P_{ji}(t)}{P_{ki}(t)} + \frac{G\xi_i(t)}{P_{ki}(t)} \quad (1)$$

$$\begin{aligned} &\overset{(a)}{\Rightarrow} r_i(t)\frac{G}{\beta} \ge \sum_{k=1}^{r_i(t)} \sum_{j=1,j\neq k}^{r_i(t)} \frac{P_{ji}(t)}{P_{ki}(t)} + \sum_{k=1}^{r_i(t)} \frac{G\xi_i(t)}{P_{ki}(t)} \\ &\quad \overset{(b)}{\ge} \sum_{k=1}^{r_i(t)-1} \sum_{j=k+1}^{r_i(t)} \left(\frac{P_{ji}(t)}{P_{ki}(t)} + \frac{P_{ki}(t)}{P_{ji}(t)}\right) \\ &\quad \overset{(c)}{\ge} \sum_{k=1}^{r_i(t)-1} \sum_{j=k+1}^{r_i(t)} 2 \\ &\quad = r_i(t)(r_i(t)-1), \\ &\Rightarrow r_i(t) \le 1 + \frac{G}{\beta}, \quad \text{if } 1 \le i \le N \text{ and } r_i(t) > 1, \end{aligned} \quad (2)$$

where (a) follows from adding all $r_i(t)$ inequalities in (1) for each value of i, (b) follows from rearranging the order of the double sum and the non-negativity of the single sum, and (c) follows from the fact that $x + \frac{1}{x} \ge 2$ for every $x > 0$. Clearly, (2) also holds if $1 \le i \le N$ and $r_i(t) \le 1$. Since $N_t = \sum_{i=1}^{N} r_i(t)$, this implies that $N_t \le N(1 + G/\beta)$. Also, since each node can maintain at most s simultaneous transmissions intended for itself, $r_i(t) \le s$. Since $N_t = \sum_{i=1}^{N} r_i(t)$, this implies $N_t \le sN$. On the other hand, if the transmissions are half-duplex, then $N_t \le sN/2$. This is so, because in this case, no node can transmit and receive simultaneously and each node can be the transmitter or the receiver of at most s simultaneous transmissions. Together with the inequalities $N_t \le N(1 + G/\beta)$ and $N_t \le sN$, this implies that $N_t \le N \min\{(1 + G/\beta), c \cdot s\}$. While deriving this upper bound on N_t, we made no restrictions on the propagation model, the placement of the N nodes, the choices of the transmitters, their intended receivers and the transmission powers. Hence, this upper bound on N_t is an upper bound on $N_t^{\max}$ for every propagation model. This completes the proof. ■

2) Upper bounds that hold with the general propagation model: In this subsection, we prove Theorem 2. This theorem allows us to show that the main results of [1] are not tied to the particular propagation model in [1], but rather they hold for the general propagation model as well.

Theorem 2: *With the general propagation model, for every time t,*

$$N_t^{\max} \le N_t^{\boldsymbol{Q}} \le 1 + \frac{G}{\beta a(D)}, \quad \text{(T2.1)}$$

Proof: At an arbitrary time t, we index each transmitter-receiver pair that belongs to the same successful transmission with a unique integer from $\{1,2\ldots N_t\}$. Thus, receiver i is the intended receiver of transmitter i, for all $1 \le i \le N_t$. Let $P_j^t(t)$ be the power transmitted by transmitter j. Now, let $\zeta_i(t)$ denote the power of noise at receiver i, and let $d_{ji}(t)$ be the distance between transmitter j and receiver i. Also, let $P_{ji}^r(t)$ be the power received by receiver i from transmitter j. Finally, let *SINR*$_i(t)$ be the *SINR* at receiver i. Then, for all $1 \le i \le N_t$,

$$\begin{aligned} &\beta \le SINR_i(t) \le \frac{P_{ii}^r(t)}{\zeta_i(t) + \frac{1}{G}\sum_{j=1,j\neq i}^{N_t} P_{ji}^r(t)} \\ &\Rightarrow \tfrac{G}{\beta}P_{ii}^r(t) - G\zeta_i(t) \ge \textstyle\sum_{j=1,j\neq i}^{N_t} P_{ji}^r(t) \\ &\qquad = \textstyle\sum_{j=1,j\neq i}^{N_t} P_j^t(t)a(d_{ji}(t)) \\ &\qquad \overset{(d)}{\ge} a(D)\textstyle\sum_{j=1,j\neq i}^{N_t} P_j^t(t) \\ &\Rightarrow \textstyle\sum_{i=1}^{N_t}\sum_{j=1,j\neq i}^{N_t} P_j^t(t) \\ &\qquad \le \tfrac{G}{\beta a(D)}\textstyle\sum_{i=1}^{N_t} P_{ii}^r(t) - \tfrac{G}{a(D)}\sum_{i=1}^{N_t}\zeta_i(t), \end{aligned} \quad (3)$$

where (d) follows from the monotonicity of $a(\cdot)$, and the fact that $d_{ji}(t) \le D$ for every i, j, and t.

Now, let $P_T(t) \triangleq \sum_{i=1}^{N_t} P_i^t(t)$, $P_R(t) \triangleq \sum_{i=1}^{N_t} P_{ii}^r(t)$, and $\zeta_R(t) \triangleq \sum_{i=1}^{N_t} \zeta_i(t)$. So, $P_T(t)$ is the *total transmitted power at time t*, $P_R(t)$ is the *total received power from intended transmitters at time t*, and $\zeta_R(t)$ is the *total received noise power at time t*. Hence, from (3) and the above definitions,

$$\begin{aligned} &(N_t - 1)\,P_T(t) \le \frac{GP_R(t)}{\beta a(D)} - \frac{G\zeta_R(t)}{a(D)} \\ &\Rightarrow N_t \le 1 + \frac{GP_R(t)}{\beta a(D)P_T(t)} - \frac{G\zeta_R(t)}{a(D)P_T(t)} \\ &\Rightarrow N_t \le 1 + \frac{G}{\beta a(D)}, \end{aligned} \quad (4)$$

where (4) follows from $a(\cdot) \le 1$, which implies that $P_R(t) \le P_T(t)$,[7] and the fact that noise and transmission powers are non-negative. By definition, $N_t^{\boldsymbol{Q}}$ is the maximum of N_t over all the number of nodes, their placements, the choices of transmitters, their intended receivers, and the transmission powers. We did not restrict any of these choices while deriving the last inequality in (4). Hence, its right-hand side is an upper bound on $N_t^{\boldsymbol{Q}}$, which is not less than $N_t^{\max}$. This completes the proof. ■

3) Upper bounds that hold with the power law decaying propagation model: In this subsection, we derive an upper bound on $N_t^{\boldsymbol{Q}}$ (hence, $N_t^{\max}$) with the power law decaying propagation model. This upper bound, as well as the upper bound in (T2.1), are due to co-channel interference. As it can be deduced from the definition of $N_t^{\boldsymbol{Q}}$ and Theorems 2 and 3, when all transmissions occur at the same channel, it becomes impossible to satisfy the *SINR* threshold requirements of all transmissions at the same time, if the number of transmissions exceeds a certain finite number.

Theorem 3: *With the power law decaying propagation model, for every time t,*

$$N_t^{\max} \le N_t^{\boldsymbol{Q}} \le U_{\gamma,dim} \triangleq \frac{\left(1+\frac{G}{\beta}\right)d^{dim}}{dim\int_1^{1+d}\frac{(u-1)^{dim-1}}{u^{\gamma}}du}, \quad \text{(T3.1)}$$

[7] Note that this is exactly the inequality, where the bounded behavior of the propagation model plays a key role. If one uses an unbounded propagation model, such as the propagation model in [3] and [4], it is not necessarily true that $P_R(t) \le P_T(t)$. In fact, with the propagation model of [3] and [4], one can make $P_R(t)$ as large as desired, provided the transmitter-receiver pairs are sufficiently close so that $P_R(t) > P_T(t)$. Certainly, such a case is unrealistic due to the law of conservation of energy.

TABLE I

THE VALUE OF THE INTEGRAL IN (T3.1)

	dim=3	dim=2	dim=1
$\gamma \notin \{k\}_{k=1}^{dim}$	$\frac{2}{\upsilon}+\sum_{k=1}^{3} \frac{((-1)^k+I_{\gamma=2})}{(1+d)^{\gamma-k}(\gamma-k)}$	$\frac{1}{\upsilon}-\sum_{k=1}^{2} \frac{(1+d)^{k-\gamma}}{(-1)^k(\gamma-k)}$	$\frac{1-(1+d)^{1-\gamma}}{\upsilon}$
$\gamma = 1$	$\frac{d^2}{2} + \log(1+d) - d$	$d - \log(1+d)$	$\log(1+d)$
$\gamma = 2$	$d+\frac{d}{1+d}-2\log(1+d)$	$\log(1+d) - \frac{d}{1+d}$	
$\gamma = 3$	$\log(1+d) - \frac{d(3d+2)}{2(1+d)^2}$		

where $d \triangleq D/c_{dim}^{1/dim}$, $c_1 \triangleq \frac{1}{2}$, $c_2 \triangleq \frac{2}{3} - \frac{\sqrt{3}}{2\pi}$, $c_3 \triangleq \frac{5}{16}$ *and the integral in the denominator is given by the expressions in Table I.*[8]

Proof outline: By using the same technique as the one used to derive inequality (7) in [1], we find that

$$\frac{GN_t}{\beta} \geq \sum_{i=1}^{N_t} \sum_{j=1,j\neq i}^{N_t} a(l_{ij}(t)), \tag{5}$$

where $l_{ij}(t)$ is the distance between receiver i and receiver j. Next, we use the following lemma:

Lemma 1: (*Interpoint distance sum inequality*) *Let* $B_{dim}(D)$ *be a dim-dimensional ball*[9] *with diameter D. Let* $n \geq 2$ *points be arbitrarily placed in* $B_{dim}(D)$*. Suppose each point is indexed by a distinct integer from* $\{1, 2 \ldots n\}$*. Let* l_{ij} *be the Euclidean distance between point i and point j. For every* $1 \leq i \leq n$*, define the* m^{th} *closest point to point i,* z_{im}*, and the Euclidean distance between point i and the* m^{th} *closest point to point i,* u_{im}*, as follows:*

$$z_{i1} \triangleq \underset{j\in\{1,2\ldots n\}\backslash\{i\}}{\arg\min} \{l_{ij}\}$$

$$z_{im} \triangleq \underset{\substack{j\in\{1,2,\ldots n\},\\ j\notin\{i\}\cup\{z_{ik}\}_{k=1}^{m-1}}}{\arg\min} \{l_{ij}\}, \quad \textit{if} \quad 2 \leq m \leq n-1$$

$$u_{im} \triangleq l_{iz_{im}}, \qquad \textit{if } 1 \leq m \leq n-1.$$

Then, for all $1 \leq m \leq n-1$, $\Sigma_{i=1}^n u_{im}^{dim} \leq md^{dim}$.

Proof outline: The proof of this lemma parallels the proof of Lemma 1 in [1], with the following changes: the two dimensional disks in the proof in [1] are replaced here with dim-dimensional balls, and the overlap ratio f_{im}^S in [1] is computed here for one and three dimensional balls as well. This results in $f_{im}^S = f(y)|_{y=\frac{u_{im}}{D}}$, where for $0 < y \leq 1$, $f(y) = \frac{1}{2}$ for dim=1, and $f(y) = \frac{1}{2} - \frac{3y}{16}$ for dim=3. Hence, $f(y) \geq f(1) = c_{dim}$. From there, the proof follows from inequalities (L1.3) to (L1.9) in [1], while replacing c_2 in [1] with c_{dim}, and using the volume of the balls instead of the area of the disks. □

We note that Lemma 1 is also valid when $B_{dim}(D)$ is replaced with $\boldsymbol{Q}$, because $\boldsymbol{Q}$ is a subset of a dim-dimensional ball with diameter D. If we set $n = N_t$ and the location of points as the location of the receivers at time t, then $u_{im}(t)$ becomes the distance between receiver i and the m^{th} closest receiver to receiver i at time t. Thus, Lemma 1 implies that $md^{dim} \geq \Sigma_{i=1}^{N_t} u_{im}^{dim}(t)$. Now, dividing both sides by N_t and taking the dim^{th} root of both sides, we find that for every $1 \leq m \leq N_t - 1$,

$$d\big(\tfrac{m}{N_t}\big)^{1/dim} \geq \Big(\sum_{i=1}^{N_t} \tfrac{u_{im}^{dim}(t)}{N_t}\Big)^{1/dim} \overset{(a)}{\geq} \tfrac{1}{N_t}\sum_{i=1}^{N_t} u_{im}(t), \tag{6}$$

where (a) follows from Jensen's Inequality [10]. Next, from (5), we obtain that

$$\begin{aligned}
\tfrac{GN_t}{\beta} &\geq \sum_{i=1}^{N_t} \sum_{j=1,j\neq i}^{N_t} a(l_{ij}(t)) = \sum_{m=1}^{N_t-1} \sum_{i=1}^{N_t} a(u_{im}(t)) \\
&= N_t \sum_{m=1}^{N_t-1} \Big(\sum_{i=1}^{N_t} \tfrac{(1+u_{im}(t))^{-\gamma}}{N_t}\Big) \\
&\overset{(b)}{\geq} N_t \sum_{m=1}^{N_t-1} \Big(1 + \tfrac{1}{N_t}\Sigma_{i=1}^{N_t} u_{im}(t)\Big)^{-\gamma} \\
&\overset{(c)}{\geq} N_t \sum_{m=1}^{N_t-1} \Big(1+d\big(\tfrac{m}{N_t}\big)^{1/dim}\Big)^{-\gamma},
\end{aligned} \tag{7}$$

where (b) follows from Jensen's Inequality and (c) follows from (6). Now, (7) implies that

$$\begin{aligned}
\tfrac{G}{\beta} &\geq \sum_{m=1}^{N_t-1} \Big(1 + d\big(\tfrac{m}{N_t}\big)^{1/dim}\Big)^{-\gamma} \\
&\overset{(d)}{\geq} \int_1^{N_t} \Big(1 + d\big(\tfrac{x}{N_t}\big)^{1/dim}\Big)^{-\gamma} dx \\
&\overset{(e)}{=} dim \tfrac{N_t}{d^{dim}} \int_{1+d/N_t^{1/dim}}^{1+d} \tfrac{(u-1)^{dim-1}}{u^\gamma} du,
\end{aligned} \tag{8}$$

where (d) is due to the fact that if a and b are integers such that $b \geq a$, and $f(x)$ is a continuous and non-increasing function of x over $[a, b+1]$, then $\sum_{m=a}^b f(m) \geq \int_a^{b+1} f(x)dx$, and (e) follows from changing the variable of integration by defining u=1+$d(x/N_t)^{1/dim}$. Denoting the integral in (8) by I, we write:

$$\begin{aligned}
I &= \int_1^{1+d} \tfrac{(u-1)^{dim-1}}{u^\gamma} du - \int_1^{1+d/N_t^{1/dim}} \tfrac{(u-1)^{dim-1}}{u^\gamma} du \\
&\overset{(f)}{\geq} \int_1^{1+d} \tfrac{(u-1)^{dim-1}}{u^\gamma} du - \int_1^{1+d/N_t^{1/dim}} (u-1)^{dim-1} du \\
&= \int_1^{1+d} \tfrac{(u-1)^{dim-1}}{u^\gamma} du - \big(dim \tfrac{N_t}{d^{dim}}\big)^{-1},
\end{aligned} \tag{9}$$

where (f) is due to the inequality $\int_E f_1 dx \leq \int_E f_2 dx$ when f_1 and f_2 are continuous functions with $f_1 \leq f_2$ on $E \subseteq \Re$ [11]. Now, (8) and (9) imply that $N_t \leq U_{\gamma,dim}$. The rest of the proof follows along the same lines as the proof of Theorem 2 after (4). ■

B. Upper Bounds on Throughput Capacity

The next theorem provides the upper bounds on $\lambda_{\rm e}$ and λ_m that hold with each propagation model and with multiple sub-channels.

Theorem 4: (i) *For every propagation model,*

$$\lambda_{\rm e} \leq \lambda_m \leq \tfrac{W_{\max}}{H} \min\Big\{1 + \tfrac{G}{\beta}, c \cdot s\Big\}. \tag{T4.1}$$

(ii) *With the general propagation model,*

$$\lambda_{\rm e} \leq \lambda_m \leq \tfrac{W_{\max}}{HN}\Big(1 + \tfrac{G}{\beta a(D)}\Big). \tag{T4.2}$$

(iii) *With the power law decaying propagation model,*

$$\lambda_{\rm e} \leq \lambda_m \leq \tfrac{W_{\max} U_{\gamma,dim}}{HN}. \tag{T4.3}$$

Moreover, if each transmission occurs over one of M non-overlapping sub-channels with maximum transmission rates $W_1, W_2 \ldots W_M$*, then* (i), (ii), *and* (iii) *still hold, if* $W_{\max}$ *is replaced with* $\Sigma_{m=1}^M W_m$.

[8] In Table I, $\upsilon \triangleq \Pi_{k=1}^{dim}(\gamma-k)$. Also, $I_{\gamma=2} \triangleq 1$ if γ=2, and $I_{\gamma=2} \triangleq 0$ if $\gamma \neq 2$.

[9] A *dim-dimensional ball* with diameter D is a closed line segment having length D when $dim = 1$, a closed circular disk having diameter D when $dim = 2$, and a closed sphere having diameter D when $dim = 3$.

TABLE II
ASYMPTOTIC UPPER BOUNDS ON $N_t^{\boldsymbol{Q}}$, $N_t^{\max}$, λ_{e}, AND λ_m

	$N_t^{\boldsymbol{Q}}$	$N_t^{\max}$	λ_{e} and λ_m
D	$O(D^{min\{\gamma,dim\}})$, if $\gamma \neq dim$ $O(D^{dim}/\log(D))$, if $\gamma = dim$	$O(1)$	$O(1)$
γ	$O(\gamma^{dim})$	$O(1)$	$O(1)$
G/β	$O(G/\beta)$	$O(1)^*$	$O(1)^*$
s	$O(1)$	$O(1)^*$	$O(1)^*$
N	–	$O(1)$	$O(1/N)$
$\overline{H}$	–	–	$O(1/\overline{H})$
$W_{\max}$	–	–	$O(W_{\max})$
Power law	(T3.1)	(T1.1)	(T4.1)
General	(T2.1)	(T2.1)	(T4.2)
All	–	(T1.1)	(T4.1)

Proof: The result in the following lemma was derived in the proof of Theorem 2 in [1]:[10]

Lemma 2: If U is a time-invariant upper bound on $N_t^{\max}$, then $\lambda_{\mathrm{e}} \leq \lambda_m \leq \frac{W_{\max}U}{\overline{H}N}$.

Replacing U with the time-invariant upper bounds in Theorems 1, 2, and 3 proves (*i*), (*ii*), and (*iii*). The remaining part of the proof follows from the proof of Corollary 2 in [2]. ■

IV. ANALYSIS OF THE RESULTS

In this section, we discuss how the upper bounds in Theorems 1 to 4 depend on various parameters of the network. In Table II,[11] we present the corresponding asymptotic upper bounds on $N_t^{\boldsymbol{Q}}$, $N_t^{\max}$, λ_{e}, and λ_m. These results describe the asymptotic growth rates of $N_t^{\boldsymbol{Q}}$, $N_t^{\max}$, λ_{e}, and λ_m as a given network parameter grows, while the other network parameters remain fixed.[12]

In Table II, the results on $N_t^{\boldsymbol{Q}}$ imply that if the network domain $\boldsymbol{Q}$ is regularly scaling (e.g., $\boldsymbol{Q}$ is a sphere for each value of D), then $N_t^{\boldsymbol{Q}}$ is $O(V_{\boldsymbol{Q}}^{\min\{\gamma/dim,1\}})$ if $\gamma \neq dim$ and $O(V_{\boldsymbol{Q}}/\log(V_{\boldsymbol{Q}}))$ if $\gamma = dim$. Hence, $N_t^{\boldsymbol{Q}}$ grows at most linearly with the volume of $\boldsymbol{Q}$, and this can happen only when $\gamma > dim$.

We note from Table II that the asymptotic upper bound on $N_t^{\boldsymbol{Q}}$ grows indefinitely with D or γ, whereas $N_t^{\max}$, λ_{e}, and λ_m are $O(1)$ with respect to D or γ. This is due to change in the dominant upper bound on $N_t^{\max}$, λ_{e}, and λ_m as D or γ grows. For $N_t^{\max}$, this change is from (T3.1) to (T1.1), and for λ_{e} and λ_m, this change is from (T4.3) to (T4.1). The reason is that beyond some finite values of D or γ, although the network domain provides sufficient space and attenuation to schedule more successful transmissions, the upper bound on the number of simultaneous receptions per node, i.e., $\min\{1+G/\beta, cs\}$, becomes the limiting factor, and it does not allow scheduling more transmissions. The quantity $U_{\gamma,dim}$ in (T3.1) and (T4.3) grows indefinitely with D and γ. Hence, in the region of (D,γ) pairs bounded by the D axis, by the γ axis, and by the set of (D,γ) pairs for which $U_{\gamma,dim}/N = \min\{1+G/\beta, cs\}$, $N_t^{\max}$, λ_{e}, and λ_m are limited by shortage of space and attenuation. Since this region

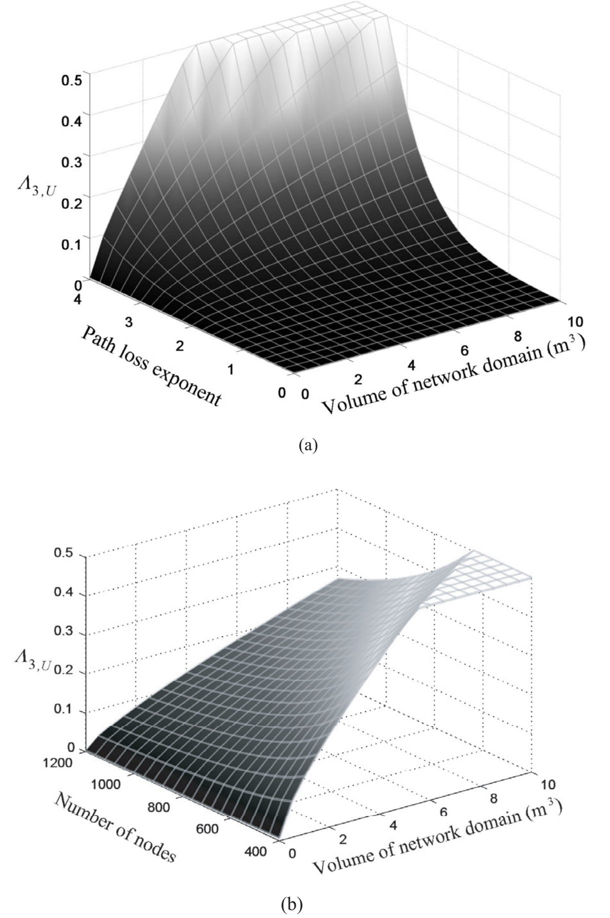

Fig. 1. The upper bound on λ_{e} and λ_m when $\boldsymbol{Q}$ is a sphere (*dim*=3), G=10, and β=20 [dB]. In Fig. 1(*a*), $(N,\overline{H})$=(900,1) and the figure shows the presence of a region of $(V_{\boldsymbol{Q}},\gamma)$ pairs (hence, (D,γ) pairs), where the limitation of λ_{e} and λ_m is due to shortage of space and attenuation. For $(V_{\boldsymbol{Q}},\gamma)$ pairs outside this region, where $\Lambda_{3,U}$=0.5, shortage of inactive pairs of nodes becomes the dominant limitation. In Fig. 1(*b*), $(\gamma,\overline{H})$=(3.2,1) and the figure demonstrates that if $V_{\boldsymbol{Q}}$ is fixed and N is increased, then λ_{e} and λ_m vanish as N grows large. However, if $V_{\boldsymbol{Q}}$ also increases with N, then it is possible to keep the upper bound at a constant level, so that it does not rule out the possibility of achieving a desired per node end-to-end throughput as N grows large.

[10] One can observe from the proof of Theorem 2 in [1] that one particular case where Lemma 2 applies is when it is assumed that time is divided into slots and in each time slot the transmission powers and the path gains stay the same (this was also assumed in [3]-[6]). In that case, "time t" should be interpreted as "time slot t" in Theorems 1, 2, and 3. Another case where Lemma 2 applies is when it is assumed that data can flow in a given transmission only at time instants where the *SINR* of the transmission is greater than or equal to β [1]-[2]. In this case, in the definition of $N_t^{\max}$, "number of simultaneously successful transmissions at time t" should be interpreted as "number of simultaneous intended transmitter-receiver pairs whose links have *SINR* greater than or equal to β at time instant t".

[11] In Table II, the propagation models are gray-shade coded according to the last three cells in the leftmost column. The gray-shade of each cell represents the propagation model for which the result in that cell holds. The last three rows indicate the equation from which the corresponding results in each column is derived. For example, when γ=*dim*, the $O(D^{dim}/\log(D))$ result is obtained by showing that the upper bound on $N_t^{\boldsymbol{Q}}$ in (T3.1) is $\Theta(D^{dim}/\log(D))$, given that the parameters other than D are fixed (because $\lim_{D\to\infty}[U_{\gamma,dim}/(D^{dim}/\log(D))] = (1+G/\beta)/(c_{dim}dim) \in (0,\infty)$ in that case). The results with * improve to $O(\min\{s,G/\beta\})$, if the parameters s and G/β grow together. Also, the $O(\gamma^{dim})$ result assumes $D>0$, while for D=0 the result becomes $O(1)$.

[12] Note that in the derivation of the theorems, we made no restrictions on the interdependence of network parameters. In [1]-[6], while deriving the asymptotic bounds, the approach was to keep other parameters fixed, while a parameter is being increased. Hence, for comparison purposes, only in this section, we adopt a similar approach. In section V, we will also consider practical systems with possible dependencies, such as the dependency between D and N.

expands as N grows, it supports the following claim (which is also noted in [1] when *dim*=2) for arbitrary dimensional networks: *For large N, there is a region of* (D,γ) *pairs where additional space or attenuation provides considerable increases in* $N_t^{\max}$, λ_e, *and* λ_m, *as their behaviors resemble the asymptotic behavior of* $U_{\gamma,dim}$, *and beyond this region their behaviors change into* $\Theta(1)$ *with respect to D and* γ.

In Fig. 1(a) and Fig. 1(b), the above observations are demonstrated for a half-duplex example with s=1. In these figures, the normalized upper bound $\Lambda_{dim,U} \triangleq \min\{U_{\gamma,dim}/N, 1/2\}/\overline{H}$ on λ_e and λ_m is plotted, where $\Lambda_{dim,U}$ follows from normalizing (T4.1) and (T4.3) by $W_{\max}$. Fig. 1(b) suggests increasing $V_{\boldsymbol{Q}}$ with N to prevent the vanishing of $\Lambda_{dim,U}$ as N grows large. The next section elaborates on how this should be done.

V. Implications on Scalability

In this section, using the power law decaying propagation model, we investigate the required conditions to achieve a desired throughput $\lambda_0 > 0$ as $N \to \infty$; i.e., the required conditions for the existence of an integer N_0 such that for all $N \geq N_0$, the throughput λ_0 is achievable by all nodes or on average. Such conditions have been established in [1] for a circular network domain, and here we derive the corresponding conditions for the arbitrary dimensional network domain $\boldsymbol{Q}$. Hence, the conditions that we establish here depend on the dimension of the network domain as well.

Since $\overline{H} \geq 1$, (T4.3) implies that one or more of the quantities $W_{\max}$, D, γ, or $W_{\max}{\cdot}G/\beta$ must grow with N, $\overline{H}N$ must be $O(W_{\max}U_{\gamma,dim})$, and $\overline{H}$ must be $O(W_{\max}\min\{1{+}G/\beta, s\})$ due to (T4.1).

It is pointed out in [1] that in practical systems, arbitrarily large growth of $W_{\max}$, γ, or $W_{\max}{\cdot}G/\beta$ with N is not feasible. Also, s cannot grow arbitrarily large due to hardware limitations. Thus, $\overline{H}$ must be $O(1)$ (hence, $\Theta(1)$, since $\overline{H} \geq 1$). The only remaining parameter, which can compensate for the growth in N, is D. Since, $U_{\gamma,dim}$ is $\Theta(D^{min\{\gamma,dim\}})$ when $\gamma \neq dim$, and $\Theta(D^{dim}/\log(D))$ when $\gamma = dim$, we obtain the following result as a necessary condition for the scalability of practical systems:

Corollary 1: *A desired per node end-to-end throughput* $\lambda_0 > 0$ *is not achievable as* $N \to \infty$, *unless:*

(i) $\overline{H}$ *is* $\Theta(1)$ *with respect to N, and*[13]

(ii) D *grows with* N *such that,* N *is* $O(D^{\min\{\gamma,dim\}})$ *when* $\gamma \neq dim$, *and* $O(D^{dim}/\log(D))$ *when* $\gamma = dim$.

Moreover, if $\boldsymbol{Q}$ *is regularly scaling, then condition* (ii) *is equivalent to*: N *is* $O(V_{\boldsymbol{Q}}^{\min\{\gamma/dim,1\}})$ *when* $\gamma \neq dim$, *and* $O(V_{\boldsymbol{Q}}/\log(V_{\boldsymbol{Q}}))$ *when* $\gamma = dim$. ■

Fig. 2 illustrates Corollary 1 when $dim = 3$. The corollary shows that if N grows fast enough to dominate the curves in the figure (e.g., when the node density is fixed in free space, where $\gamma = 2$), then a desired throughput is not achievable as $N \to \infty$.

[13]Note that condition (i) is a consequence of (T4.1). Hence, condition (i) is required for every propagation model.

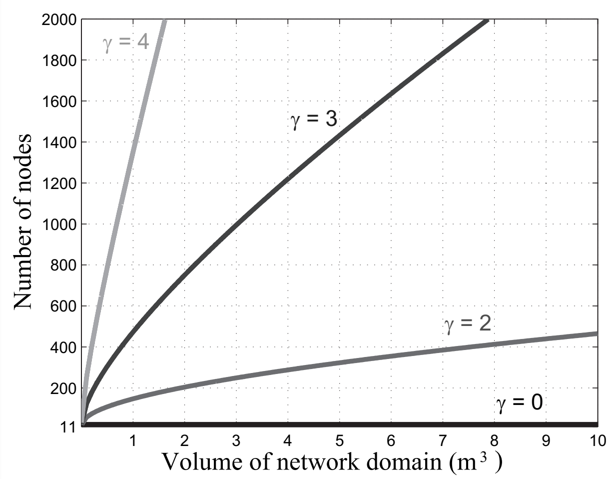

Fig. 2. $\boldsymbol{Q}$ is sphere, G=10, β=20 [dB] and $\overline{H}$=1. The ($V_{\boldsymbol{Q}}$,N) pairs, for which $\Lambda_{dim,U}$=0.1 are plotted. As $\Lambda_{dim,U}$ decreases with N and increases with $V_{\boldsymbol{Q}}$, in the regions above the curves, e.g., (γ,$V_{\boldsymbol{Q}}$,N)=(2,5,400), a normalized throughput of 0.1 is not achievable. Corollary 1 provides the asymptotic behavior of the ($V_{\boldsymbol{Q}}$,N) pairs on the curves; e.g., if γ=2, then N is Θ ($V_{\boldsymbol{Q}}^{2/3}$).

VI. Conclusion

In this paper, we studied the capacity of single-user-detection based arbitrary dimensional wireless networks with isotropic antennas. We extended the results of [1] by deriving upper bounds on the simultaneous transmission capacity and the per node end-to-end throughput capacity, which hold for any one, two or three dimensional network domain. Next, we established the corresponding asymptotic upper bounds on the simultaneous transmission capacity and the per node end-to-end throughput capacity. Moreover, in this arbitrary dimensional setting, we have shown that the main results of [1] hold for a large class of propagation models, in addition to the specific propagation model considered in [1]. Finally, we established several required conditions to achieve a desired per node end-to-end throughput as the number of nodes grows large. These conditions make it necessary to keep the average number of hops between a source and a destination bounded, and to increase the size of the network domain at certain rates that we determined, which depend on the dimension of the network domain and on the attenuation in the medium.

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