Relay Placement in Wireless Networks: Minimizing Communication Cost

Milen Nikolov and Zygmunt J. Haas, *Fellow, IEEE*

***Abstract*—Given *n* source nodes and *k* relay nodes, we model the optimal relay topology problem allowing for simultaneous optimization of the relay node locations and traffic through the network, so that the overall number of packet retransmissions is minimized. We argue that state-of-the-art models and algorithms for relay placement in wireless networks do not reflect salient characteristics of the optimal relays topology and lead to suboptimal solutions. We do not constrain the position of relays to a finite set of discrete points, as the latter may not be feasible in practical networks. In this case, we show that just listing a set of feasible sites for the relays is already at least APX-hard. Exploiting convexity in a special case of the network communication cost function, we give an optimal algorithm for the relay placement problem. However, the algorithm is exponential on the number of nodes in the network. We suggest a practical heuristic algorithm for relay placement: RePlace. We compare RePlace numerically to the optimal algorithm and show that RePlace achieves the optimal or almost optimal solutions. We implement RePlace in the full network stack simulator JiST/SWANS. The relay topologies generated by RePlace eliminate overhead communication cost almost entirely.**

***Index Terms*—Relays, wireless networks, reliability, throughput maximization, communication cost, combinatorial optimization.**

I. Introduction

NETWORKS of wireless sensors are used to monitor various physical processes, ranging from measuring soil moisture for precision agriculture (e.g. [1]) to tracking power consumption in buildings (e.g. [2]). In many of these applications, the deployment success depends on network communication efficiency. For instance, a higher number of packet retransmissions leads to drastically reduced network lifetime [3]. Intuitively, growing number of packet (re)transmissions drives network communication costs up, for instance, via growing energy depletion.

An important factor determining the number of packet retransmissions (and correspondingly communication cost) is the network links quality ([3], [4]). Among others, presence of obstacles between nodes; increasing interference as the density of nodes grows; and separation distance between wireless devices may all influence links' quality. The relative impact of each of these factors on network performance depends on the particular network scenario. A main cause of poor link quality in sparsely deployed outdoor sensor networks is the large separation distance between sensing nodes. This induces low SNR and low packet reception rate (PRR) [4]. To improve links' quality and decrease network communication costs, the latter formally defined in section II-B, network designers often rely on the placement of relay nodes. Relay nodes do not introduce new traffic in the network and only re-transmit the packets received from a set of source nodes.

A simple relay network example is shown in fig. 1 along with a sample traffic demand matrix. We consider the relay placement generated by five different algorithms along with the corresponding traffic routed on each link (i.e. edge labels in blue) for each placement. Notice that in all cases the appropriate placement of relay nodes effectively increases links' PRR and decreases communication cost. Even in the simple case of fig. 1, the optimal placement of relay nodes accounts for more than an 80% reduction in communication cost. It is well known [6] that random placement of relay nodes may theoretically have beneficial effects on network capacity as the network scales. However, in general the number of randomly placed relay nodes required to achieve noticeable impact on network performance is rather large. The random deployment of relay nodes is expensive and not practical. In the sample topology of fig. 1, the random placement of relay nodes achieves only about 45% of communication cost reduction.

Aside from the optimal and random placement, fig. 1 illustrates the relay nodes topologies generated by two other algorithms typically utilized in various studies and applications requiring relay nodes deployment. The first algorithm solves the Euclidean Steiner tree problem and the second solves the General Steiner tree problem on the given sample topology. These algorithms however *do not truly solve* the optimal relay placement problem. The inefficiencies of these algorithms' outputs compared to the optimal relay placement stem from somewhat subtle but fundamental difference between the corresponding problem models.

The Euclidean Steiner Tree (EST) consists of locations and links that interconnect the *n* given fixed nodes in the plane. Each connecting link has an associated weight equal to the Euclidean distance between its vertices. The goal is to pick the locations on the plane that will minimize the sum weight of the interconnecting links. The relay nodes are placed at these locations. The problem is NP-complete [7], however good approximations are efficiently found. While such approaches are frequently used in practice (e.g. [4] in the field of sensor networks; and [8] in the field of robotics), they are not necessarily optimal, as fig. 1

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The authors are with the School of Electrical and Computer Engineering, Cornell University, Ithaca, NY 14853 USA (e-mail: mvn22@cornell.edu; haas@ece.cornell.edu).

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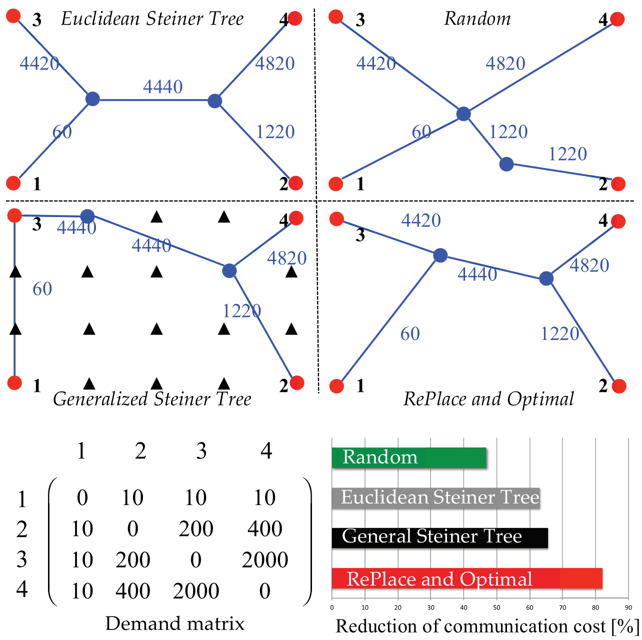

Fig. 1. A network topology of $n = 4$ fixed nodes and a demand matrix $\mathbf{W}$ summarizing the traffic demands for different source destination pairs. There are $k = 2$ relay nodes placed in the network. Edge labels (in blue) indicate the traffic routed on each link in the network (see eq. 7). In this work, we characterize the optimal relay placement problem and discuss inherent inefficiencies of state-of-the-art relay placement algorithms. Even in the simple network topology above, the solutions obtained by the **Euclidean** and **Generalized Steiner** trees algorithms, often suggested for relay placement optimization both in practice and in the technical literature, do not minimize communication cost (defined formally in II-B). In contrast, the **RePlace** algorithm presented in this paper achieves the optimum in this network example.

demonstrates. First, the EST does not account for the traffic loads on links: heavily utilized links may require more relay nodes placed closer to them. For instance, in fig. 1, the traffic between sources 3 and 4 is significantly larger than the rest of the links, shifting the optimal positions of the relay nodes away from the EST. Second, the SNR and respectively the number of packet retransmissions due to poor PRR do not depend linearly on the distance between receiver and transmitter.

These issues seem to be remedied by modeling the relay placement problem as an instance of the General Steiner Tree (GST) model. In the GST problem, the input consists of a graph $G(V, E)$, where each edge $ij \in E$ has an associated cost c_{ij}. Given a set of terminal nodes $N \subseteq V, |N| = n$, the goal is to find a tree ST of minimum cost spanning the vertices in N. Here, the weights of the edges are arbitrary. As part of their influential work [3], Krause et al. utlize the GST model to suggest locations for communication nodes within a network of wireless sensors deployed to maximize the amount of gathered sensing information and minimize communication cost in the network. In [3], V is finite and represents the potential locations of network nodes. Each edge $ij \in E$ in G has weight equal to the expected number of times a packet needs to be retransmitted by i so that the packet is received successfully by j. In this model, the communication cost depends on the locations of the nodes and distance between them but, unlike the case of the EST, not necessarily linearly. The communication nodes are placed at the GST vertices in the tree spanning the set of sensor nodes.

The GST model for placing relay nodes appears sound, however it omits a few important factors affecting the optimal placement of relay nodes. First, the ideal locations of the relay nodes depend on how the routing in the network is constructed: the communication cost on a link $ij \in E$ depends on the amount of traffic flow on ij. The more the packets flowing on link ij are, the greater is the expected number of retransmissions, i.e. communication cost, on link ij. Furthermore, the ideal routing in the network depends on the positions of the relay nodes: a better routing could be achieved if the relay nodes are positioned elsewhere within a set of available locations. This hints that the locations of the relay nodes and the routing should be optimized *simultaneously*. The GST approach to modeling the optimal relay placement does not capture these aspects of the problem. This is corroborated by the example in fig. 1, where Steiner tree based solutions do not match the optimal solution, achieving markedly lower reduction in communication costs even in this very simple network scenario. Furthermore, in practice, the relay nodes may occupy a continuum of points in the plane. In some cases, confining the points to a pre-specified set of discrete locations as per the General Steiner tree model may not be feasible. To obtain a good approximation of the best relay nodes positions in network where n nodes span large area, one may need a large amount of possible locations in V, which increases significantly the complexity of the respective General Steiner tree approximation algorithms.

In this study, we revisit afresh the problem of placing optimally a set of k relay nodes within a network of n fixed nodes, with the goal of minimizing network communication cost. We formulate this task as a novel optimization problem that generalizes previously studied wireless network node placement problems.

- The presented optimization framework allows nodes to be placed at a *continuum of points* on the plane[1]. Furthermore, the relay nodes' *locations and routing traffic patterns* in the network are simultaneously optimized.
- We show that even listing the set of potential feasible sites for the optimal placement of relay nodes is at least APX-hard via equivalence to the clique problem in certain classes of intersection graphs.
- Exploiting convexity in a special case of the network communication cost function, we describe an optimal algorithm solving the relay placement problem and minimizing network overhead retransmissions. The optimal algorithm is exponential on the number of nodes in the network and hence not practical for larger networks.
- We suggest an efficient relay placement and routing heuristic algorithm (**RePlace**). Numerically we show that **RePlace** outputs optimal or very close to the optimal solutions (within 2-3% of the optimal communication cost) in small network instances. **RePlace** is implemented in

[1]We do not consider here spatial restriction on the possible locations of the relay nodes in the plane (e.g. due to physical obstacles such as ponds or rivers), however the framework generalizes to settings, where relay nodes may only be placed within constraints of continuous convex sets of points.

the full network stack simulator JiST/SWANS [9], where practical network effects (e.g. link asymmetry, interference, noise, collisions, etc.) are present. As the number of relay nodes increases **RePlace** eliminates almost entirely overhead communication costs. We compare **RePlace** to alternative state-of-the-art relay placement schemes and show that **RePlace** outperforms them, achieving lower delay and communication cost.

II. Preliminaries and System Model

A. Link Model

Let ij denote the wireless link between two network nodes i and j. We calculate the packet error rate r_{ij} on link ij, assuming the log-distance path-loss model [10]. This wireless signal propagation model has been known to realistically characterize a number of low power network deployment scenarios (e.g. see [13], [4], [11], and [12] among others). For instance, as demonstrated in the network deployments studied in [11], the log-distance path loss model accurately describes the link performance of both outdoor and indoor wireless networks operating over IEEE 802.11b hardware. Assuming i's transmit power is P_t, the received power P_r at j is

$$P_r = P_t - a(d_0) - 10\alpha \log_{10}(d_{ij}/d_0) + \eta(0,\sigma) + \chi(0,\sigma_1) \tag{1}$$

and the SNR at j is correspondingly

$$\gamma(d_{ij}) = P_t - a(d_0) - 10\alpha \log_{10}(d_{ij}/d_0) - \eta(0,\sigma) - \chi(0,\sigma_1) \tag{2}$$

where $a(d_0)$ is the attenuation at reference distance d_0. $\eta(0,\sigma)$ and $\chi(0,\sigma_1)$ are normal random variables modeling the thermal noise power and shadowing respectively. The path loss exponent α varies depending on the network deployment scenario. For instance, setting $\alpha = 4$ and $\alpha = 3$ accurately models respectively indoor and outdoor signal propagation (e.g. [11], [12]).

Per the physical reception model in Gupta and Kumar's seminal paper [6], the received SNR has to be greater than a minimum threshold ϕ for a successful transmission:

$$\gamma(d_{ij}) \geq \phi \tag{3}$$

At present, we assume that there is no interference from nodes' transmissions. In section VI-B, we study the effect of interference. The bit error rate p_{ij} on link ij is

$$p_{ij} = Q\left\{\left[2\gamma(d_{ij})\right]^{\frac{1}{2}}\right\} \tag{4}$$

if BPSK modulation is utilized. As usual, the $Q(\cdot)$ function here represents the tail probability of the standard Gaussian distribution. The results below are easily extensible for other commonly utilized modulation schemes.

The packet error rate r_{ij} on ij is then given by

$$r_{ij} = 1 - (1 - p_{ij})^b \tag{5}$$

where b is the packet length in bits.

Physical link cost: the cost c_{ij} of link ij is defined as

$$c_{ij} = 1/(1 - r_{ij}) \tag{6}$$

c_{ij} accounts for the number of dropped packets due to low SNR at receiver j in the network. Notice that c_{ij} captures well links' communication cost in terms of expected number of retransmissions [3]. The physical link cost c_{ij} however does not account for the full communication cost on link ij.

B. Network Model

Let V be a set of homogeneous network nodes. We assume that links in the network are symmetric. The network is modeled as an undirected, connected, weighted graph $G(V, E)$. E is the set of wireless links. The weight of edge $ij \in E$ is c_{ij}. The graph G is captured by its weighted adjacency matrix $\mathbf{C} = \left[c_{ij}\right]_{|V|x|V|}$. Notice that the values c_{ij} depend on the distance between the nodes i and j, and hence on the positions of the nodes i and j. Link ij exists iff $d_{ij} \leq R$, where R is defined as follows.

Definition 1: Maximum Transmission Range R: *The maximum transmission range R of node i is the maximum distance away from i at which, a fraction β of the time, the BER is less than* $0.5 - \epsilon, \epsilon > 0$.

The parameters ϵ and β depend on QoS constraints.

Suppose the initial demand matrix $\mathbf{W} = [w_{sd}]_{|V|x|V|}$ is provided. Each entry w_{sd} of $\mathbf{W}$ captures bidirectional traffic demand [packets/sec] between nodes s and d: the sum of the traffic demands from node s to d and from node d to s. A demand pair is denoted $\langle sd \rangle$. For all given pairs $\langle sd \rangle$ we can find a routing path $y_{sd} \subset E$ connecting s with d. This can be done using any variant of Floyd-Warshall's algorithm, for instance. Let Y^{ij} be the set of all $\langle sd \rangle$, such that $ij \in y_{sd}$. The traffic on ij is then

$$q_{ij} = \sum_{\langle sd \rangle \in Y^{ij}} w_{sd} \tag{7}$$

Link ij is utilized if $q_{ij} > 0$.

Network communication cost: Assuming each packet error on ij is an independent event with probability r_{ij}, the expected number of packet retransmissions until q_{ij} packets are successfully received is given by

$$f_{ij} = q_{ij} c_{ij}. \tag{8}$$

The quantity f_{ij} is the communication cost on link ij. The greater the packet error rate r_{ij} on ij, the larger the physical cost c_{ij}. The larger the q_{ij} of link ij, the greater f_{ij} and the average packet delay on link ij. If link ij is not utilized it does not carry traffic: $q_{ij} = f_{ij} = 0$. If $p_{ij} > 0$, then $c_{ij} > 1$ and $f_{ij} > q_{ij}$. Ideally, if $p_{ij} = 0$, then $c_{ij} = 1$ and $f_{ij} = q_{ij}$.

The total *network communication cost* is given by

$$F = \sum_{ij \in E} f_{ij} \tag{9}$$

Minimizing F would improve network performance in terms of network goodput and average packet delay. In the following sections, we study and quantify how the addition of relay nodes, so that F is minimized, impacts these network performance metrics.

III. Communication Cost Minimization

Given a network of n fixed nodes and a set K of new relay nodes, where $k = |K|$, our task is the minimization of overall communication cost in G', where G' is the resulting network graph with vertex set $N \cup K$ and edge set E'. The relay nodes are not sources of traffic and can only offload the traffic from the original n nodes.

From (9), $f_{ij}, ij \in E'$, depends both on the distance between i and j (via the term c_{ij}) and the traffic routed through link ij (via the term q_{ij}). Hence, we need to jointly optimize two sets of variables to minimize communication cost:

(a) *the positions of relay nodes*; and
(b) *the routing* over the links in the relay network. Notice also that
(c) *changing the positions of relay nodes* may affect the optimal routing paths in the network and *changing the routing paths* in the network in turn may affect the optimal positions of the relay nodes.

A. General Relay Placement Problem

Let $\mathbf{Q} = [q_{ij}]_{(n+k)(n+k)}$ denote the network traffic flow matrix, where q_{ij} denotes the traffic on link $ij \in E'$ as defined in (7). The coordinates of the k relay nodes are denoted by $(x_j, y_j), 1 \le j \le k$. We let $\mathbf{v} = [(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)]$, $\mathbf{v} \in \mathbb{R}^{2k}$. $\mathbf{C}(\mathbf{v})$ denotes the weighted adjacency matrix of G'. Its entries depend on $\mathbf{v}$. The general relay placement problem (GRPP) becomes

$$\min_{\mathbf{Q},\mathbf{v}} \{F(\mathbf{Q}, \mathbf{v})\} \equiv \min_{\mathbf{Q},\mathbf{v}} \{\mathbf{Q} : \mathbf{C}(\mathbf{v})\} = \min_{\mathbf{Q},\mathbf{v}} \left\{ \sum_{i=1}^{n+k-1} \sum_{j=i+1}^{n+k} f_{ij} \right\}, \quad \text{where } f_{ij} = q_{ij} c_{ij}, \forall i, j \tag{10}$$

As usual, $\mathbf{A}$:$\mathbf{B}$ denotes the inner product of matrices $\mathbf{A}$ and $\mathbf{B}$; and we have

$$\min_{\mathbf{Q},\mathbf{v}} \left\{ \sum_{i=1}^{n+k-1} \sum_{j=i+1}^{n+k} f_{ij} \right\} = \sum_{i=1}^{n+k-1} \sum_{j=i+1}^{n+k} q_{ij} \left[1 - Q\left(\sqrt{2\gamma\left(d\left(x_i, y_i, x_j, y_j\right)\right)} \right) \right]^{-b}, \quad \text{where } f_{ij} = q_{ij} c_{ij}, \forall i, j \tag{11}$$

$d(x_i, y_i, x_j, y_j)$ denotes the Euclidean distance between any two points i and j.

The candidate solutions to the GRPP (10) are uncountably many since $\mathbf{v}$ may include any point on the plane. However, the different possible routing paths that satisfy (7) and hence the different matrices $\mathbf{Q}$ are finitely many. If we were able to compute the optimal vector $\mathbf{v}$ for each possible input $\mathbf{Q}$, we would have an optimal enumeration algorithm for solving the GRPP (10).

B. Relay Placement With Fixed Routing

Suppose the traffic matrix $\mathbf{Q}$ is provided. Can we find the optimal locations (i.e. $\mathbf{v}$) of the relay nodes and solve

$$\min_{\mathbf{v}} \left\{ F^{\mathbf{Q}}(\mathbf{v}) \right\} = \min_{\mathbf{v}} \left\{ \sum_{i=1}^{n+k-1} \sum_{j=i+1}^{n+k} q_{ij} \left[1 - Q\left(\sqrt{2\gamma\left(d\left(x_i, y_i, x_j, y_j\right)\right)} \right) \right]^{-b} \right\} \quad \text{s.t. } d(x_i, y_i, x_j, y_j) \le R, \forall q_{ij} > 0 \tag{12}$$

where q_{ij}'s are no longer optimization variables but the entries of the given matrix $\mathbf{Q}$? We label this problem the Relay Placement with Fixed Traffic (RPFT).

Note that RPFT's solution must satisfy a set of simple geometrical constraints due to the properties of wireless links. Showing that the function $F^{\mathbf{Q}}(\mathbf{v})$ is convex under these constraints, we are able to provide an optimal solution to the RPFT. This allows us to pose GRPP as a combinatorial problem.

1) Convexity of RPFT: We recall the following well-known theorem.

Theorem 1: The function

$$F^{\mathbf{Q}}(\mathbf{v}) = \sum_{i=1}^{n+k-1} \sum_{j=i+1}^{n+k} q_{ij} g\left[d(x_i, y_i, x_j, y_j)\right] \tag{13}$$

is convex if $g(z) : \mathbb{R}^+ \to \mathbb{R}$ is convex and non-decreasing.

Proof: Please refer to [14], p. 434. ■

Consider the function $F^{\mathbf{Q}}(\mathbf{v})$ in (13). Let

$$g\left[d(x_i, y_i, x_j, y_j)\right] = \left[1 - Q\left(\sqrt{2\gamma\left(d\left(x_i, y_i, x_j, y_j\right)\right)} \right) \right]^{-b} \tag{14}$$

To determine whether $F^{\mathbf{Q}}(\mathbf{v})$ in (12) and (13) is convex on some domain, by **Theorem** 1, we only need to determine whether $g(z)$ is convex and non-decreasing on that domain.

Observation 1. The function $g(z)$ is convex and non-decreasing on the interval $(0, rd_0)$, $\forall r$ such that $rd_0 > R$.

Observation 1 is analyzed with more detail in Appendix A. Figure 2 shows a plot of $g(z)$ when $P_t = 10[W]$, $R \approx 110[m]$, $b = 256[bits]$, $d_0 = 1[m]$.

Given function $F^{\mathbf{Q}}(\mathbf{v})$ in (12) and $R \le rd_0$, the RPFT problem becomes

$$\mathbf{v}' = \arg\min_{\mathbf{v}} \left\{ F^{\mathbf{Q}}(\mathbf{v}) \right\} \text{ s.t. } d(x_i, y_i, x_j, y_j) \le R, \forall q_{ij} > 0 \tag{15}$$

The constraints inequalities in (15) are convex too ([14]).

Hence, for any fixed matrix $\mathbf{Q}$ and a network graph where links are constrained within transmission range R, we can solve (15) and find the positions of the relay nodes that minimize the network communication cost. Standard convex optimization algorithms such as the steepest gradient descent with constraints [14] can be used to solve the RPFT problem.

C. Optimal Brute Force Solution to the GRPP

Going back to the GRPP, we can now present a brute force combinatorial solution. Given a graph G of fixed nodes, its

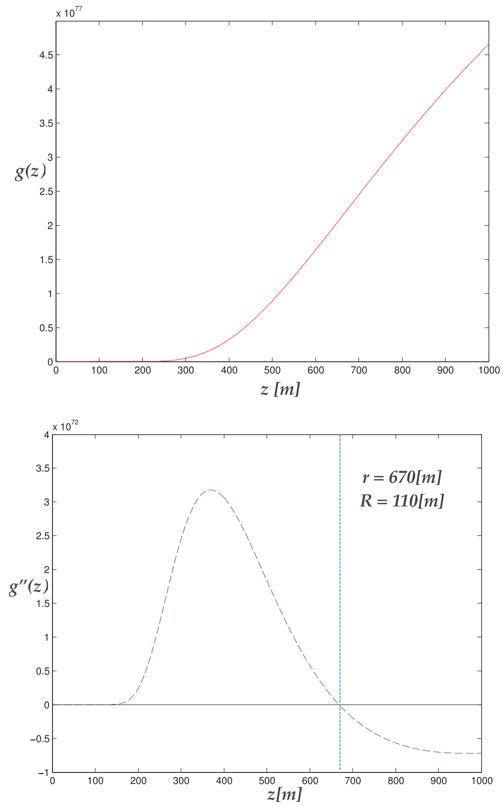

Fig. 2. $g(z)$ behavior for $b = 256$[bits], $d_0 = 1$, $R = 110$[m].

$$\begin{array}{c} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

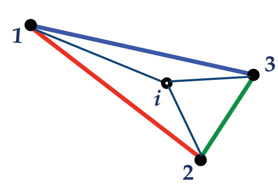

Fig. 3. The given network on three source nodes and one relay node has 4x4x4 = 64 distinct possible single-path routing traffic patterns, given demand matrix W. Equivalently, collection $\mathbf{Y}$ contains 64 elements. Although large, the number of elements in $\mathbf{Y}$ is finite.

weighted adjacency matric C, demand matrix W, and a number k of relay nodes, the algorithm outputs as a solution the optimal coordinates $\mathbf{v}^*$ of the k relay nodes and the optimal routing Y^* in the resulting network G' on vertex set $N \cup K$. The pair $(\mathbf{v}^*, Y^*)$ minimizes the network communication cost.

Let Y be a set of routing paths on the vertices $N \cup K$ connecting all $\langle sd \rangle$ pairs. Y can be found by running a variant of Floyd-Warshall's algorithm. Let $\mathbf{Y}$ be the collection of all possible sets Y. $\mathbf{Y}$ has finite number of elements depending on n, k, and the maximum transmission radius R (assuming that the flow for each pair $\langle sd \rangle$ is routed on a single path). For example, fig. 3 illustrates a network on three source nodes and one relay node. Assuming each source-destination commodity is routed on a single path, there are 64 possible traffic patterns in the resulting network.

The brute force algorithm (**BruteForceMin**) then follows:

1) for each set of routing paths $Y \in \mathbf{Y}$, generate the matrix $\mathbf{Q_Y} = [q_{ij}]_{(n+k)(n+k)}$ using (7) and solve the constrained convex optimization problem (15) for $\mathbf{Q_Y}$, finding $\mathbf{v}'$ minimizing $F^{\mathbf{Q_Y}}(\mathbf{v}')$;
2) pick the solution $\mathbf{v}^*$ and the corresponding Y^* that yields the minimum $F^{\mathbf{Q_Y}}(\mathbf{v}')$ over all different routing paths $Y \in \mathbf{Y}$:

$$(\mathbf{v}^*, \mathbf{Y}^*) = \arg\min_{\mathbf{v}', Y} F^{\mathbf{Q_Y}}(\mathbf{v}') \tag{16}$$

The cardinality of $\mathbf{Y}$ is exponential of the number of vertices in $N \cup K$ and thus solving the GRPP using **BruteForceMin** is not practical for larger networks. In the next sections, we characterize the inherent computational hardness of the GRPP. We pose the GRPP as a maximization problem and provide an efficient, practical heuristic algorithm for it.

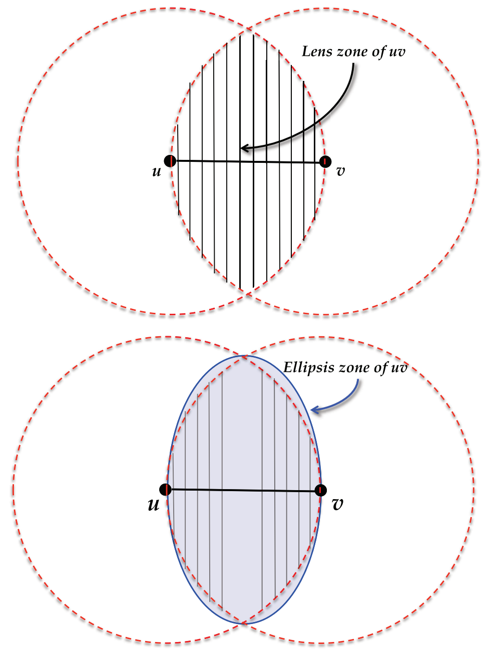

Fig. 4. **Top**: Lens zone of link uv. **Bottom**: Ellipse zone of uv w/ major axis a and minor axis b.

IV. Zones, Overlaps and Feasible Regions

A. Defining Zones and Overlaps

In this section, we define a set of geometrical constraints on the possible optimal positions of the relay nodes. This allows us to define a feasible set of regions in the network area, where relay nodes can be initially placed. We show that listing these feasible regions is at least APX-hard.

Observation 2. Given a link uv in G and a relay node i, the communication cost of uv can be reduced if and only if i is positioned within the lens formed by the overlap of the two circles with centers respectively u and v, each with radius d_{uv}.

The lens of link uv is shown in fig. 4 (**left**). We approximate the lens with a corresponding ellipse zone of major axis a and minor axis b, as shown in fig. 4 (**right**). The ellipse approximation is chosen for ease of presentation[2] and allows us to state

[2]It is not hard to show that the ellipse to lens approximation is tight in the sense that the ratio of lens zone to ellipse zone's areas is a constant c, where $0.9 < c < 0.903$ for all major and minor radii $\frac{a}{2}$, $\frac{b}{2}$.

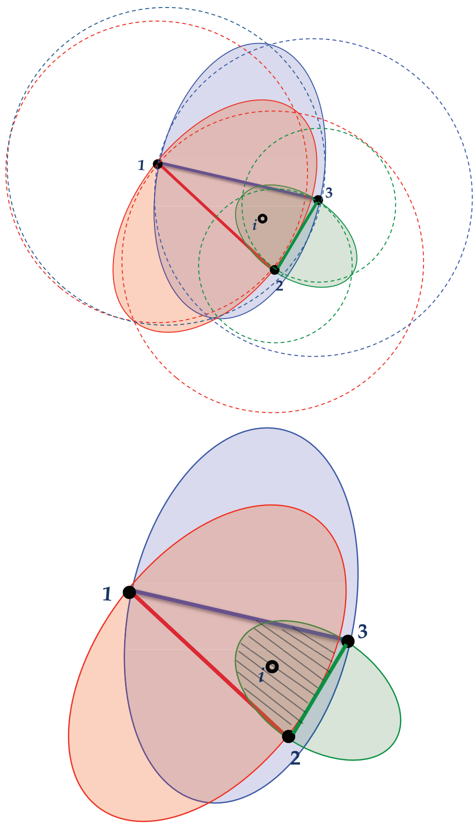

Fig. 5. Lens zones construction of links 12, 13, and 23 (**top**); resulting elliptical overlapping zones of the links in the same network (**bottom**). Relay node i is positioned in the overlap region $\theta_{\{12,13,23\}}$ of zones $\theta_{12}, \theta_{13}, \theta_{23}$ (shaded region) and reduces the communication cost of all three links in the network, iff their traffic is routed through i.

properties of the feasible relay placement regions section IV-B with fewer geometrical technicalities. We denote this approximate ellipse zone of link uv by θ_{uv}, and refer to it as the zone of link uv. Let $\theta_{u'v'} \diamond \theta_{uv}$ denote the statement "the zone of link $u'v'$ overlaps with the zone of link uv". Let A be a set of links, such that $\theta_{u'v'} \diamond \theta_{uv}$, $\forall uv, u'v' \in A$. The region formed by the overlap of all zones of links in A is denoted by θ_A. $\theta_A > 0$ *iff* the overlap region has area greater than 0.

Observation 3. Given a set of links $A \subseteq E$, their zones, and a relay node i, i can reduce the communication cost of every link in A if and only if i is placed in θ_A, assuming $\theta_A > 0$.

Observation 3 follows from *Observation* 2, since i is in the zone of every link in A, when placed within θ_A. Figure 5 demonstrates such an arrangement, where i is placed in θ_A, $|A| = 3$.

Consider the set $X(A) = \{\theta_S : S \subseteq A, \theta_S > 0\}$. In the example of fig. 5, $A = \{12, 13, 23\}$ and $X(A) = \left\{\theta_{\{12\}}, \theta_{\{13\}}, \theta_{\{23\}}, \theta_{\{12,23\}}, \theta_{\{12,13\}}, \theta_{\{13,23\}}, \theta_{\{12,13,23\}}\right\}$. Notice that a zone overlaps with itself. According to *Observation* 3, the relay node i can only reduce the communication cost of all the links 12, 13, and 23 if and only if i is placed in the overlap of the set of zones $\theta_{\{12,13,23\}} \in X(A)$.

Observation 4. Consider the set $Y^* \in \mathbf{Y}$ of routing paths connecting all $\langle sd \rangle$ pairs in the relay network G'. Let A be a subset of the utilized links in Y^* from the initial network G. For a given relay node i, from *Observations* 2 and 3, $\forall u, v$ such that

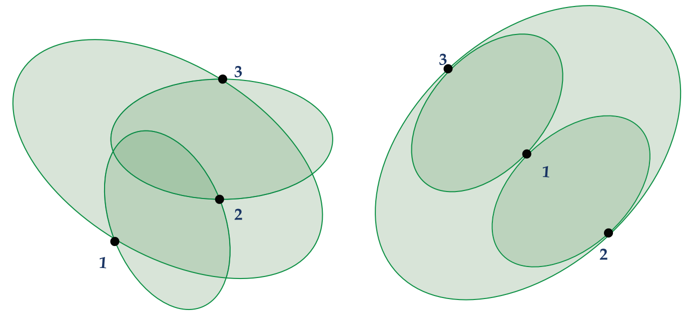

Fig. 6. $S_1 = S_2 = \{\theta_{12}, \theta_{13}, \theta_{23}\}$. $\theta_{S_1} \in X(E)$ for the graph G (**left**); $\theta_{S_2} \notin X(E)$ for the graph G (**right**). Zones θ_{12}, θ_{13} have positive overlap for the graph on the **left**, and zones θ_{12}, θ_{13} have zero overlap for the graph on the **right**.

$uv \in A$, if $ui \in Y^*$ and $iv \in Y^*$ then $d_{ui} < d_{uv}$ and $d_{iv} < d_{uv}$. The latter is equivalent to placing i in θ_A. By contraposition, if i is not placed in θ_A, $\exists\, u$ and v such that $uv \in A$, $ui \notin Y^*$ and $iv \notin Y^*$. Furthermore, $\exists$ a relay node i that if placed in θ_A, then the set of routing paths Y^* minimizing network communication cost and utilizing i should include links ui and iv, $\forall u, v$ such that $uv \in A$.

From *Observation* 4, for instance, if the zones of links u_1v_1 and u_2v_2 do not overlap, then the links u_1i, iv_1, u_2i, *and* iv_2 cannot all be in Y^*. Each set of routing paths containing all four of these links is suboptimal.

B. Feasible Regions and Their Independence

1) Feasible Regions: Based on *Observations* 1 - 4, in this section we describe a set of feasible regions that necessarily contains the optimal positions for the placement of relay nodes in the network. Let E be the set of links in the initial graph G. Each $\theta_S \in X(E)$, $S \subseteq E$, defines an overlap region. From *Observation* 4, if relay node i is placed outside θ_S it would be suboptimal for i to offload all links in S. θ_S provides an initial feasible site at which relay node i can be placed to offload all links in S. The relay nodes are optimally placed in a subset of the regions contained in $X(E)$. E.g., in fig. 5 (**bottom**) the overlap region $\theta_S \in X(E)$ associated with the set of links $S = \{12, 13, 23\}$ is shaded and contains relay node i. In this example, the routing Y^* in the relay network is not hard to compute. Given i's placement in θ_S and the traffic routed through i, i's optimal position is found by solving the RPFT with input $\mathbf{Q}_{\mathbf{Y}^*}$.

Definition 2: Let $A \subseteq E$ be a set of links connecting some subset of fixed nodes in G. The *set of feasible regions* is the set $X(A) = \{\theta_S : S \subseteq A, \theta_S > 0\}$.

2) Finding the Set of Feasible Regions is at least APX-Hard: Notice that some zones of links in E may overlap while others may not have common overlap region. For instance, fig. 6 shows an example of a network with two sets of links S_1 and S_2, each containing three links. There we have $\theta_{S_1} \in X(E)$ and $\theta_{S_2} \notin X(E)$, since $\theta_{S_1} > 0$ while $\theta_{S_2} = 0$. For the network shown in fig. 5, $X(E) = \left\{\theta_{\{12\}}, \theta_{\{13\}}, \theta_{\{23\}}, \theta_{\{12,23\}}, \theta_{\{12,13\}}, \theta_{\{13,23\}}, \theta_{\{12,13,23\}}\right\}$.

Obtaining $X(E)$ is a prerequisite for finding optimal placement for the relay nodes. In general, given a graph G the construction of the set $X(E)$ is a problem, which has a non-obvious solution. Can we design an efficient algorithm generating the set of overlap regions that belong to $X(E)$? This is the FEASIBLEREGIONSSET problem.

We claim that the FEASIBLEREGIONSSET **problem is at least** APX**-hard.**

Our proof strategy is straightforward: we show that solving a particular APX-hard problem cannot be harder than solving the FEASIBLEREGIONSSET problem. In other words, we *reduce* a particular APX-hard problem to the FEASIBLEREGIONSSET problem. The particular APX-hard problem we consider is the clique problem in a class of ellipse-ellipse intersection graphs.

For completion, we recall the definition of an intersection graph directly from [30]. "Let $\mathcal{M}$ be a collection of sets. The intersection graph of $\mathcal{M}$ is the abstract graph $G_{\mathcal{M}}$ whose vertices are the sets in $\mathcal{M}$, and two vertices are connected by an edge if the corresponding sets intersect." In the case of an ellipse-ellipse intersection graph $G_{\mathcal{E}}$, $\mathcal{M} = \mathcal{E}$ is a collection of ellipses in the plane. The set of vertices $V(G_{\mathcal{E}}) = \mathcal{M} = \mathcal{E}$ and for all $i, j \in V(G_{\mathcal{E}})$, $ij \in E(G_{\mathcal{E}})$ iff ellipses i and j intersect. $E(G_{\mathcal{E}})$ is the set of edges in $G_{\mathcal{E}}$.

More specifically, let the ratio of the major to the minor axis of an ellipse be ρ and consider the following three problems: ELLIPSEρCLIQUE, ELLIPSE $\leq \rho$CLIQUE, and FILLEDELLIPSEρCLIQUE, as in [30].

ELLIPSEρCLIQUE is the problem of finding a maximal complete subgraph of $G_{\mathcal{E}}$, where all the ellipses in $V(G_{\mathcal{E}})$ have minor to major axis ratio of exactly ρ. The range of ρ is $1 < \rho < \infty$. The problem considers ellipses circumferences, without their interiors (i.e., if an ellipse is enclosed by another ellipse, the two do not intersect).

Similarly, ELLIPSE $\leq \rho$ CLIQUE is the problem of finding a maximal complete subgraph of $G_{\mathcal{E}}$, where all the ellipses in $V(G_{\mathcal{E}})$ have minor to major axis ratio of at most ρ. The range of ρ is $1 < \rho < \infty$. The problem again considers ellipses without their interiors.

The FILLEDELLIPSEρCLIQUE is the problem of finding a maximal complete subgraph of $G_{\mathcal{E}}$ where all the ellipses in $V(G_{\mathcal{E}})$ have minor to major axis ratio of exactly ρ. The range of ρ is $1 < \rho < \infty$. This problem considers ellipses with their interiors (i.e., if an ellipse is enclosed by another ellipse, the two intersect).

Theorem 2: For every $\rho > 1$ the problem ELLIPSEρCLIQUE is APX-hard. Furthermore FILLEDELLIPSEρCLIQUE is at least APX-hard.

Proof: See p.280 and Theorem 1 in [30]. ■

Consider the FILLEDELLIPSE $\sqrt{3}$ CLIQUE problem, (i.e. $\rho = \sqrt{3} > 1$) .

Theorem 3: There exist polynomial time reduction from FILLEDELLIPSE$\sqrt{3}$CLIQUE to FEASIBLEREGIONSSET.

Proof: We start by defining the zone graph $G_Z(V_Z, E_Z)$ of the wireless network graph $G(V, E)$. Associate a vertex $v_{ij} \in V_Z$ to each zone θ_{ij}, $ij \in E$. Any two vertices $v_{ij}, v_{i'j'} \in V_Z$ are connected by an edge $e \in E_Z$ if and only if $\theta_{ij} \diamond \theta_{i'j'}$. The zone graphs of three sample wireless network graphs on fixed nodes are given in fig. 7.

Suppose we are given any input graph, $G_{\mathcal{E}}$, to the FILLEDELLIPSE $\sqrt{3}$ CLIQUE problem. We can construct graph

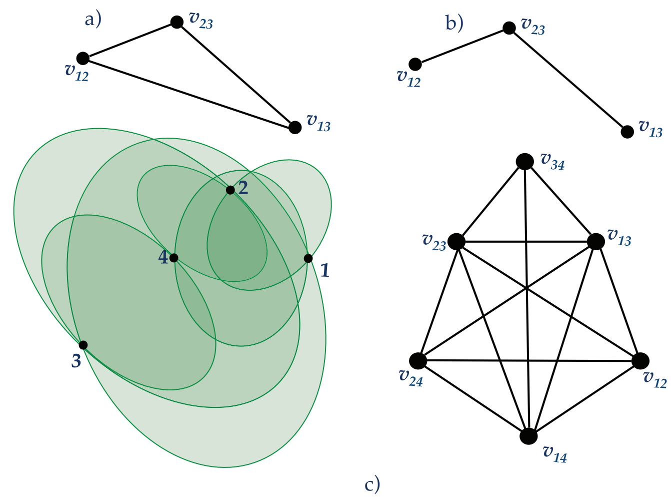

Fig. 7. Zone graphs of three network instances: a) zone graph of the network in fig. 6 (**left**); b) zone graph of the network in fig. 6 (**right**); c) network on 4 nodes (**left**) and its zone graph (**right**). The zone graphs quickly become more complex as the number of nodes in the corresponding networks increases.

$G(V, E)$ with corresponding zone graph $G_Z = G_{\mathcal{E}}$ in polynomial time of $|V(G_{\mathcal{E}})|$.

We start with an empty graph $G(V, E)$ where $V = \{\}$ and $E = \{\}$; as before, V is the set of fixed source/destination nodes in the graph, and E is the set of edges in the graph. We iterate through all ellipses in $V(G_{\mathcal{E}})$. For each ellipse e in $V(G_{\mathcal{E}})$, we add two fixed nodes i and j to graph G, so that now $V = V \cup i, j$ and $E = E \cup (i, j)$. We place nodes i and j in the plane at the minor vertices of ellipse e (the minor vertices are easily found from a description of the ellipse e, e.g., its equation). Note that each existing link (i, j) in $G(V, E)$ has a corresponding elliptical zone and each elliptical zone has a corresponding link (i, j) in E. The set of elliptical zones of $G(V, E)$ corresponds exactly to the vertices of zone graph G_Z, by our definition of zone graph above; and by construction, the set of elliptical zones of $G(V, E)$ corresponds exactly to the set of ellipses in $V(G_{\mathcal{E}})$. (Notice that in fig. 4 nodes u and v are positioned at the minor vertices of the zone θ_{uv}). That is $V(G_Z) = V(G_{\mathcal{E}})$. Furthermore, by our definition of a zone graph, any two vertices in G_Z are connected iff their respective elliptical zones intersect. The latter is exactly equivalent to the definition of edge sets in intersection graphs, and hence $E(G_Z) = E(G_{\mathcal{E}})$.

Since $V(G_Z) = V(G_{\mathcal{E}})$ and $E(G_Z) = E(G_{\mathcal{E}})$ the resulting zone graph G_Z of G is identical to $G_{\mathcal{E}}$. It was constructed in polynomial time of $V(G_{\mathcal{E}})$.

All that remains to be proven is the connection between the clique complex of the zone graph G_Z and the set of feasible regions $X(E)$. Namely, finding all sets in $X(E)$ is equivalent to finding all maximal cliques of G_Z.

Consider the clique complex $\chi(G_Z)$ of the graph G_Z and let $V_S \subseteq V_Z$ be the set of nodes associated with the links in $S \subseteq E$.

We next show that $V_S \in \chi(G_Z)$ if and only if $\theta_S \in X(E), \forall V_S \subseteq V_Z$.

Let $\theta_S \in X(E)$. We have $\theta_{ij} \diamond \theta_{i'j'}, \forall ij, i'j' \in S$. Then, v_{ij} and $v_{i'j'}$ are connected by an edge $e \in E_Z$, $\forall v_{ij}, v_{i'j'} \in$

V_S. Therefore, V_S is a clique in G_Z, and $V_S \in \chi(G_Z)$. Conversely, let $V_S \in \chi(G_Z)$, then the vertices in V_S form a clique in G_Z. We have that v_{ij}, and $v_{i'j'}$ are connected by an edge $e \in E_Z, \forall v_{ij}, v_{i'j'} \in V_S$. By the definition of $G_Z, \theta_{ij} \diamond \theta_{i'j'}, \forall ij, i'j' \in S$. This implies $\theta_S \in X(E)$.

There is equivalence between the elements in $X(E)$ and the elements in $\chi(G_Z)$. Listing all sets in $X(E)$ is equivalent to listing all cliques of G_Z. Listing all cliques of G_Z is at least as hard as listing all maximal cliques in G_Z. ■

Solving the FEASIBLEREGIONSSET (i.e. finding all sets in $X(E)$) is equivalent to finding all maximal cliques of G_Z. In turn, finding all maximal cliques of G_Z yields a maximal complete subgraph, and respectively a solution to the FILLEDELLIPSE $\sqrt{3}$ CLIQUE with input $G_{\mathcal{E}}$. We conclude that FEASIBLEREGIONSSET is at least APX-hard via polynomial reduction from FILLEDELLIPSE $\sqrt{3}$ CLIQUE.

Hence, even listing a discrete finite set of potential regions where relay nodes could feasibly be placed is at least APX-hard. Interestingly, in [31], the authors show that intersection graph of n convex compact sets whose sides are parallel to k different directions contains a polynomial number of cliques (at most n^k). The latter fact could be exploited in works on special cases of relay placement, where the collection of zones can be partitioned in k sets each containing zones with parallel axes. The resulting clique problem can be solved more efficiently. Recently, in [32], the authors proof that the clique problem in ray intersection graphs is NP-hard. Techniques similar to theirs may have various applications for more constrained cases of relay placement problems. We discuss this further in section VIII.

Corollary 1: $X(E)$ is an independence system, therefore $X(E)$ is the intersection of m matroids, where m is finite.

Proof: From Theorem 3, $X(E)$ is equivalent to the clique complex $\chi(G_Z)$ of the zone graph G_Z. The clique complex of a graph is an independence system [15]. Therefore $X(E)$ is an independence system. It is well known that any independence system is an intersection of a finite number of matroids [16]. ■

Next, we pose the GRPP as a set function maximization problem, subject to independence constraints. This perspective provides intuition for heuristics solving approximately the GRPP.

C. Optimal Assignment: A Maximization Perspective

Let the communication cost reduction ϕ_K be the difference between the communication cost in the initial network $G(N, E)$ and in the relay network $G'(N \cup K, E')$:

$$\phi_K = \sum_{ij \in E} f_{ij} - \sum_{ij \in E'} f_{ij} \tag{17}$$

Given $X(E)$, the GRPP problem becomes an optimal assignment problem of elements/zones in $X(E)$ to relay nodes. Each element in $X(E)$ may be assigned to 0, 1 or more relay nodes. Rename the set of overlaps $X(E)$ to J. Let the pair (i, j), $i \in K$ and $j \in J$, denote the assignment of relay node i to the overlap polygon j. Let $\mathbf{\Phi} = \{(i, j) : i \in K, j \in J\}$ be the set of all possible assignment pairs, and $\mathbf{\Phi}_j = \{(i, j) : i \in K\}$, $j \in J$. The maximization problem below is equivalent to the GRPP:

$$O^* = \arg\max_{O \subseteq \mathbf{\Phi}} \left\{ \phi(O) : |O \cap \mathbf{\Phi}_j| \leq k, \, j \in J \right\} \tag{18}$$

Since we only relabeled $X(E)$ to J in (18), $\mathbf{\Phi}$ in (18) is an independence system from *Corollary* 4. The function $\phi(O)$ is defined here as the maximum communication cost reduction in the network G achieved by placing a subset of the K relay nodes within a combination of zones in the set $X(E)$, according to the assignment pairs $(i, j) \in O \subseteq \mathbf{\Phi}$. The function $\phi : O \to \mathbb{R}^+$ is not given explicitly, but it can be computed.

We start from (17) and note that maximizing the communication cost reduction is equivalent to minimizing the communication cost in G'. Given each $(i, j) \in O$ for a particular set O, we position a relay node i within the overlap $j \in J \Leftrightarrow \theta_S \in X(E)$ at a random point in θ_S. Suppose the traffic matrix $\mathbf{Q}$ of the initial network G is given. This can be obtained by running Floyd-Warshall's algorithm on the weighted adjacency matrix C of G. The communication cost in G, given the resulting routing, is computed using (9). We apply *Observation* 4 and offload the traffic from all links in S only through the relay nodes placed in θ_S. Given this constraint, let Y_O be a set of routing paths on the vertices $N \cup K$, while the relay nodes are placed according to the assignment pairs in O. Let $\mathbf{Y_O}$ be the collection of all possible sets Y_O for a fixed O. Run the **BruteForceMin** algorithm with input $\mathbf{Y} = \mathbf{Y_O}$.

This yields the minimum communication cost $F^{\mathbf{Q_Y}}(\mathbf{v})$ in the network G' for the fixed O. Equivalently, we have $\phi(O)$.

The **BruteForceMax** algorithm follows:

1) For each $O \subseteq \mathbf{\Phi}$, compute $\phi(O)$ as described above.
2) Pick O^* that maximizes $\phi(O)$ over all $O \subseteq \mathbf{\Phi}$.

Similarly to **BruteForceMin**, the **BruteForceMax** algorithm is exponential on the number of vertices in $N \cup K$. The formulation of the GRPP as maximization problem does not in itself reduce the complexity of the solution. However, we know that (18) is a maximization problem over an independence system $\mathbf{\Phi}$. Typically, greedy algorithms perform well in that context. The **RePlace** algorithm follows such Greedy strategy.

V. THE *RePlace* HEURISTIC

Given the exponential running time of the discussed brute force algorithms and the hardness of the exact computation of the feasible regions in $X(E)$, in this section, we resort to designing an efficient heuristic for the GRPP.

Based on the initial network $G(N, E)$ and demand matrix $\mathbf{W}$, we start by computing the adjacency cost matrix $\mathbf{C}$ of G using (6). We find routing Y and traffic matrix $\mathbf{Q}$ by running, for instance, all pairs shortest paths routing algorithm over the weighted adjacency matrix $\mathbf{C}$ of G. Suppose we can enumerate all sets $\theta_S \in X(E)$. (We describe a procedure to approximately compute the set $X(E)$ of overlap regions in V-A.)

Given input G, $\mathbf{C}$, Y, $\mathbf{W}$, $\mathbf{Q}$, $X(E)$ and K, the goal is to find k positions at which to place the relay nodes and the routing through the resulting relay network, so that a best-effort solution to (18) is computed. The **RePlace** algorithm given by **Algorithm 1** relies on a Greedy heuristic to achieve that.

Suppose P is the set of placed relay nodes after iteration t. v_t contains the coordinates of the nodes in P. Correspondingly,

$G'(N \cup P, E_t)$ is the resulting network with weighted adjacency matrix $C_t = [c_{ij}]_{(n+t)|(n+t)}$, where $E_0 = E$. Let F_t be the communication cost in the network after iteration t. Then, ϕ_t is the communication cost reduction:

$$\phi_t = F_t - F_0 = \sum_{ij \in E_t} f_{ij} - \sum_{ij \in E_0} f_{ij} \tag{19}$$

At each iteration t of the **RePlace** algorithm, we place a new relay node p at a location within the overlap region $\theta_S \in X(E)$ that leads to the maximum ϕ_t (lines 10 - 38, **Algorithm 1**). To determine θ_S maximizing ϕ_t at each iteration t, **RePlace** probes all overlap regions prior to placing each relay node p.

Namely, at iteration t, for each $\theta_S \in X(E)$, relay node p is temporarily placed at a random location in θ_S (line 11, **Algorithm 1**). The network routing is updated according to *Observation* 4, to account for the placement of p (lines 13 - 21, **Algorithm 1**). Next, node p's position within θ_S is optimized by solving (RPFT), with respect to the updated network routing (lines 22 - 28, **Algorithm 1**). Notice that the positions of all relay nodes placed until iteration t are also optimized, within the constraints of their respective overlap regions. Then, the estimated communication cost reduction ϕ_S from placing p in overlap θ_S is obtained (line 28, **Algorithm 1**). Eventually, p is placed in overlap θ_S leading to maximum ϕ_t at iteration t. **RePlace** terminates when all k relay nodes are placed and returns the Greedy solution $\mathbf{v_g}$ for the positions of the relay nodes, along with the corresponding routing Y_g in the network.

A. *RePlace Implementation*

To simplify geometric computations, we approximate the ellipse zones with rhombus zones as shown in fig. 8. The rhombus approximation is chosen for representation purposes only; any convex polygon approximation to the link zones would suffice. Given graph G, the collection $X(E)$ is approximately found via Monte Carlo based approach. Suppose M points are sampled on the plane uniformly at random. Checking whether each of the M points is positioned within any given link's rhombus zone is efficient. Let m be a sample point and S_m be a set of zones that contain m. Combining the sets S_m for each m yields approximate set $X'(E)$. The larger M the more complete the approximate solution $X'(E)$ of $X(E)$. We can efficiently find the overlap polygons of the zones forming $\theta_S \in X'(E)$. To do that, we adapt the clipping algorithm of [17]. Given a set of convex zones as input, we use clipping to compute the intersection polygons of the zones in $X'(E)$.

VI. Numerical and Simulation Results

A. *Numerical Evaluation*

The performance of **RePlace** is shown in fig. 9. In small networks, we directly compare **RePlace** to the optimal solution obtained by the **BruteForceMax** algorithm. Notice that the two algorithms perform remarkably close. Each data point represents the communication cost reduction achieved by placing k relay nodes in a network of n nodes and is averaged over 50 random network topologies. $X(E)$ is computed exactly by

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Algorithm 1. RePlace

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input: $G, C, Y, \mathbf{W}, \mathbf{Q}, X(E)$, and K

output: $\mathbf{v_g}, Y_g$: vector of relay node locations and a set of routing paths

1: $t \leftarrow 0$; $P \leftarrow \emptyset$; $Y_t \leftarrow Y$; $\mathbf{v}_t \leftarrow \emptyset$; $C_t \leftarrow C$; $E_t \leftarrow E$

2: **while** $|P| \leq |K|$ **do**

3: $p \leftarrow rand(K \setminus P)$// pick a relay node from the set of relay nodes that are not placed yet

4: $P \leftarrow P \cup p$

5: $E^S \leftarrow E_t$

6: $t \leftarrow t + 1$

7: $\phi_{max} \leftarrow 0$

8: $\mathbf{v_t} \leftarrow v_{t-1}$

9: $Y_t \leftarrow Y_{t-1}$

10: **for all** $\theta_S \in X(E)$ do

11: $(x_p, y_p) \leftarrow (x_{rand}, y_{rand})$// tentatively place$p$ at a random location in θ_S

12: // estimatecommunication cost reduction of placing p in θ_S

13: **for all** $v \in N \cup P$ **do**

14: **if** $d_{vp} \leq R$ **then**

15: $c_{vp} = \frac{1}{1-r_{vp}}$// updatecommunication cost on all links that now use p

16: $E^S \leftarrow E^S \cup vp$// update tentative edge set

17: **end if**

18: **end for**

19: $C^S \leftarrow C_t$

20: $Y^S \leftarrow$ FloydWarshall$(G(N \cup P, E^S), C^S)$// get all-pairs shortest paths Y^S using Floyd-Warshall on $G(N \cup P, E^S)$ with matrix C^S

21: $Y \leftarrow Y^S$// update tentative network routing after placing p in θ_S

22: **for all** $ij \in E^S$ **do**

23: // compute tentative traffic q_{ij} on link ij per eq. (7); set entry $q_{ij} \in \mathbf{Q}_Y$

24: $q_{ij} \leftarrow \sum_{\langle sd \rangle \in Y^{ij}} w_{sd}$

25: **end for**

26: $v_S \leftarrow$ ConstrainedDescent$(F^{\mathbf{Q}_Y}(v_t \cup (x_p, y_p)))$// Solve the RPFT problem eq. (15) w/ input $\mathbf{Q}_Y$ via constrained gradient descent; the positions of all relay nodes already placed may tentatively shift within the constraints of their regions

27: $F^S \leftarrow F^{\mathbf{Q}_Y}(\mathbf{v}^S)$

28: $\phi^S \leftarrow F_0 - F^S$// communication cost reduction

29: **if** $\phi_{max} \leq \phi^S$ **then**

30: $\phi_{max} \leftarrow \phi^S$// communication cost decreases after placing p in θ_S, update solutions

31: $\mathbf{v}_t \leftarrow \mathbf{v}^S$

32: $Y_t \leftarrow Y^S$

33: $E_t \leftarrow E^S$

34: **end if**

35: $E^S \leftarrow E_{t-1}$

36: $C_t \leftarrow C_{t-1}$

37: $\phi_t \leftarrow \phi_{max}$

38: **end for**

39: **end while**

40: $\mathbf{v_g} \leftarrow \mathbf{v}_t$

41: $Y_g \leftarrow Y_t$

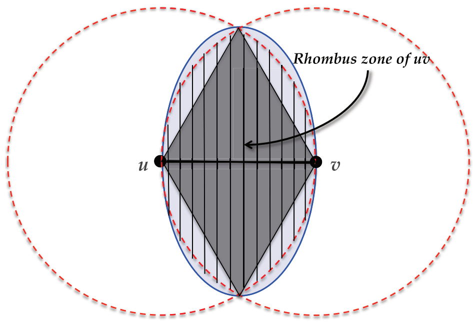

Fig. 8. Rhombus zone of uv.

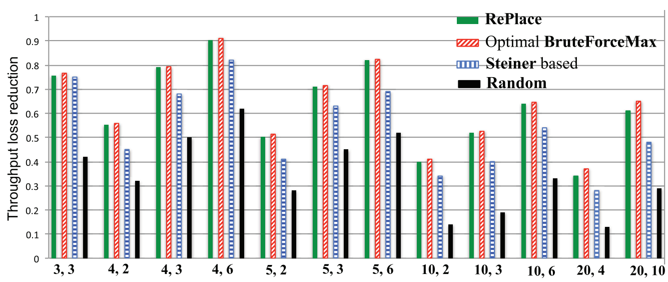

Fig. 9. Communication cost reduction performance in small networks for varying numbers n and k of fixed and relay nodes ($x - axis$: n, k): **RePlace**, optimal **BruteForceMax**, **random**, and **Steiner**-tree based placement. The **RePlace** heuristic achieves solutions remarkably close to the optimal ones and markedly better than alternative solutions.

brute force for the input of the **BruteForceMax** algorithm. We compare the results for the optimal and **RePlace** solutions for networks of size up to $n = 20$ and $k = 10$ nodes. Figure 10 lists a few typical relay network topologies found by the two algorithms for different values of n and k. In these examples, **RePlace** matches exactly the optimal solution in terms of both routing and relay nodes' positions.

B. JiST/SWANS Simulations

To investigate the effect of bandwidth limitation, interference, collisions, link asymmetries, etc., we simulate **RePlace** in the full stack network simulator JiST/SWANS. Table I summarizes the simulation setup parameters. We consider the metrics communication cost reduction and average packet delay (i.e. the delay of a packet successfully delivered at a destination, averaged across all such packets on all network paths). These metrics are investigated in networks of different sizes and varying number of relay nodes. Each data point in the simulation figures represents the mean network performance over a 100 different random network instances.

1) Communication Cost Reduction: The communication cost reduction obtained by **RePlace** (vis-a-vis the base case where no relay nodes are deployed) is shown in fig. 11. The communication cost metric accounts for the number of dropped packets due to low SINR at each receiver in the network. We observe, similarly to the work in [4], that in smaller networks operating in mid-SNR regime, interference does not affect critically the packet loss in the network. As long as the relay nodes are placed optimally, so that signal degradation due to

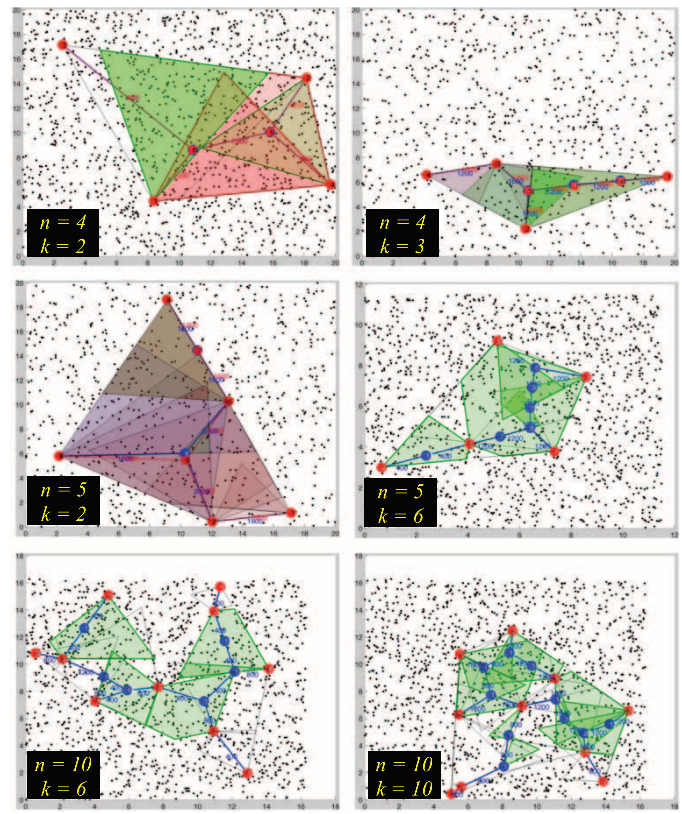

Fig. 10. The output of the **RePlace** and the optimal **BruteForceMax** algorithms on typical random network topologies. The relay nodes placed by the **RePlace** algorithm are marked with blue circles; the relay nodes placed by the **BruteForceMax** algorithm are marked with red squares. In these cases the solutions, both routing and positions of nodes, output by the two algorithms are the same and the respective relay nodes placements overlap. Hence, only one solution is visible at a time. For $k = 2$ and 3, all zone overlaps bounded by the convex hull of the network are shown. For $k = 6$ and 10, only the zone overlaps selected for the optimal placement of the relay nodes are visible in green. The black dots are the sample points used for computing the approximate set $X(E)$.

TABLE I
PARAMETERS OF THE JiST/SWANS SIMULATION

Simulator	JiST/SWANS v1.0.6
Radio frequency	2.4GHz
Channel bandwidth	1Mb/s
Path loss exponent α	3 (e.g. [12], [11], [4])
R	$\approx$ 10[m]
Area	Varying
Propagation model	Free Space
MAC Layer	IEEE 802.11b
b	256[bits]
n	[5,50]
k	[2,30]

separation distance is minimized, packet loss can be reduced significantly. For instance, in a network of 20 nodes we can position 6 relay nodes and achieve almost 70% reduction of dropped packets as shown in fig. 11.

2) Average Packet Delay: The communication cost reduction leads to substantial decrease of average packet delay in the network when relay nodes are optimally deployed. This is due to the retransmission backoff mechanism in the IEEE 802.11 MAC protocol. Reducing the number of retransmissions effectively reduces the packet delay. The delay ratio $DR_{k,n}$ is defined

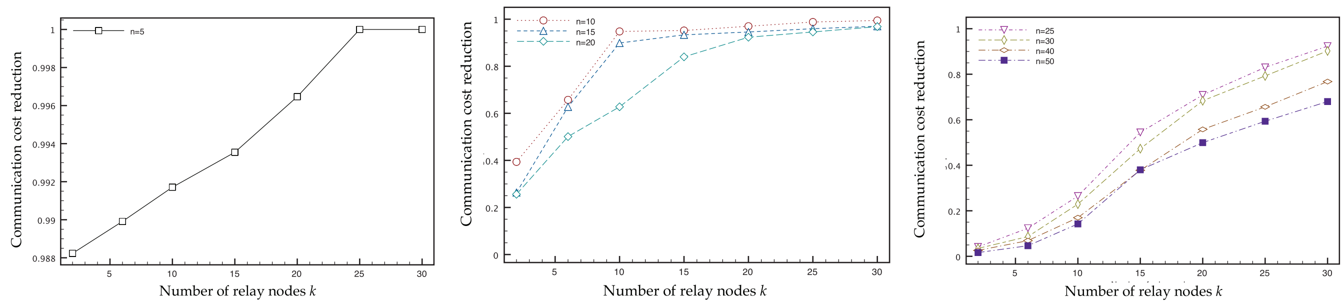

Fig. 11. The fractional difference between the number of retransmissions due to dropped packets when relay nodes' positions are optimized in comparison to the base case with no relay nodes placed. As the ratio k/n varies there are three different trends: **left**, **middle** and **right**. As expected for large k/n, the reduction is almost 100%. The communication cost reduction rate is highest for smaller values of k/n. The communication cost reduction *rate* becomes lower for larger k/n.

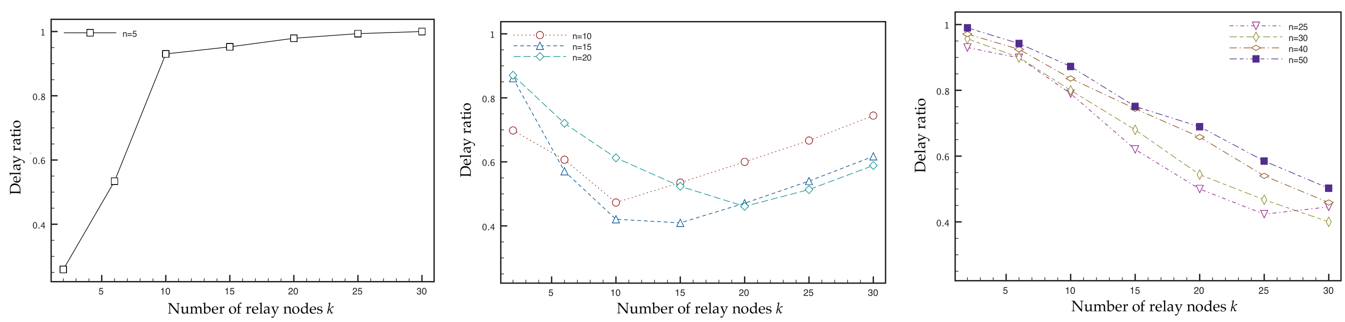

Fig. 12. Delay ratio $DR_{k,n}$. The delay, after optimized placement of the relay nodes, is less compared to the base case with no relay nodes, for the different values of n and k shown.

as **average end-to-end packet delay in a network of** n **fixed nodes with** k **relay nodes** *divided by* **the average packet delay in the network of** n **fixed nodes without relay nodes**. If the ratio is less than 1, the delay is lower after the addition of the relay nodes. The plots of $DR_{k,n}$ for the different values of k and n are shown in fig. 12. Notice that for all values of n and k, the achieved delay when nodes are deployed utilizing **RePlace** is less than the delay when no relay nodes are deployed.

Figure 12 (**left**) depicts the delay ratio for large ratio k/n. In these cases the communication cost reduction saturates, the average path length grows longer and the delay increases as more relay nodes are added to the network. Furthermore, the delay increases due to the RTS/CTS mechanism on the MAC layer as the number of relay nodes in the network grows. A similar effect is observed in fig. 12 (**middle**) for $k/n > 1$: the delay increases as more relay nodes are added in these cases. In contrast, fig. 12 (**right**) shows the *decrease* of delay, as more relay nodes are added while $k/n < 1$. For the set of simulated network instances (100 random topologies per data points), in terms of delay, the optimal ratio k/n is approximately 1. Although not necessarily prescriptive for all network deployment cases, this ratio of relays to fixed source nodes may be a good "rule of thumb" for sparser networks over the IEEE 802.11 MAC layer.

3) Performance Comparison: We investigate the performance of two strategies alternative to **RePlace**, for placing relay nodes in wireless networks. The **Steiner tree**-based strategy follows the General Steiner Tree (GST) model discussed in section I above. GST is utilized in [3] and [4], among other works. The **DoubleStage** strategy has been recently suggested in [18]. **DoubleStage** constructs a routing tree in the original graph (sans relay nodes) by finding a shortest path connecting a source-destination pair. The length of each edge in the path models the energy required for successful transmission of a packet on the corresponding wireless link. Hence, the resulting shortest path minimizes the communication cost. This is the first stage of the algorithm. In the second stage, the authors position k relay nodes iteratively. Each relay node's position is determined, so that the communication cost in the network is minimized given the routing found in the first stage of the algorithm. The routing is not updated during the placement of the relay nodes. This inherently leads to potential inefficiencies of the **DoubleStage** algorithm's output.

The performance of the **Steiner tree** and **DoubleStage** algorithms relative to the **RePlace** algorithm is shown in fig. 13 (**left**: communication cost; and **right**: delay ratio). The number k of placed relay nodes is the same for each of the three schemes. **RePlace** outperforms the other two schemes for varying numbers of fixed nodes in the network. Noticeably, unlike traffic oblivious algorithms as the network size grows **RePlace** maintains its performance gains while the routing paths become longer introducing more complex traffic patterns. The **DoubleStage** scheme performance degrades less as network size increases since routing is optimized in the first stage of the algorithm; however, routing paths are not updated as more and more relay nodes are placed leading to loss of efficiency as the network grows.

Given a topology of fixed source nodes, the relay locations can be computed offline. However, for completion, we discuss the run-times of the three algorithms discussed in this

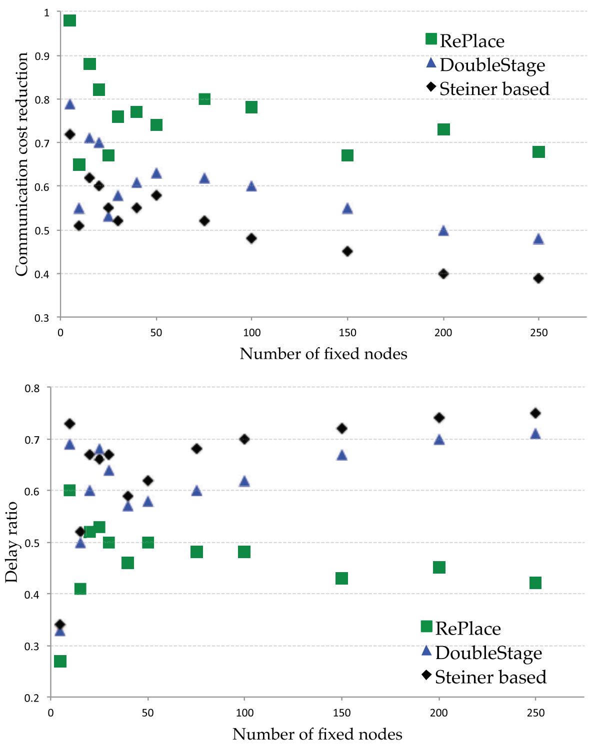

Fig. 13. **RePlace** vs. **DoubleStage** vs. **Steiner-tree** based approaches. **Left**: *Communication cost reduction*. Higher values of communication cost reduction are better. **Right**: *Delay ratio*. Lower values of the delay ratio are better. Average node degree is kept constant as the network grows. The three schemes are given equal budgets k of relay nodes.

section. The centralized **DoubleStage** algorithm in [18] runs in $\lambda * O(n^2)$ time steps, where λ is the number of iterations required for convergence in the second stage of the algorithm. The authors do not provide analysis of the convergence rate λ towards the optimal positions of the relay nodes in the network plane. In the case of the general Steiner tree problem, one can utilize different constant approximation algorithms, with varying run-times. Here, our **Steiner**-based algorithm employs the well-known Graph Iterative 1-Steiner Point (GI1S) algorithm ([20]) with approximation ratio 1.5[3]. Our GI1S implementation is the classical one found in [20] and has a run-time of $O(n|G| + n^4 log(n))$, where $|G|$ is the total number of edges and vertices in the input graph G: the union of the set N of fixed source nodes ($n = |N|$); the set V of vertices corresponding to the candidate relay node locations (e.g. a Manhattan grid spanning the network area); and the set of edges interconnecting N and V. In practice we observed that the **Steiner**-based and **RePlace** algorithms had similar and somewhat longer run-times than **DoubleStage** algorithm.

VII. Related Work

A number of works study the placement of relay nodes to provide connectivity in the network (e.g. [21]–[24] and references therein). The problem addressed by the authors in this setting is different from the problem considered in our work. Given a network of fixed nodes, the goal is to place a minimum number of relay nodes so that the induced overall network topology is connected. In some cases (e.g. [22]) to accomplish that, the authors assume that the relay nodes have larger transmission radius than the fixed nodes; in other cases (e.g. [21]) a constant transmission radius is given as an input to the problem, and the number of required rely nodes is found so that the network is connected. The authors of [23] consider a heterogeneous network where a minimum number of relay nodes are utilized to connect sensor nodes to more powerful base stations via directed paths. In [24], the authors extend this result allowing constraints to be placed on the discrete sets of locations available for relay node placement (e.g. due to natural obstacles). Although the network lifetime may be increased as a result of the improved network connectivity, the above works approach the use of relay nodes in a different setting than the one presented in this paper. Here the relay network topology is explicitly optimized with the goal of reducing communication costs. Relays' communication radius is not altered and the physical communication model from [6] is assumed rather than the protocol model assumed by the above studies.

A different stream of technical literature (e.g. [5], [25], and [26]) considers the deployment of relay nodes to provide fault tolerance in networks guaranteeing m-connectivity between sensor nodes, or sensor nodes and base stations. In a comprehensive work, Patel et al. [27] study versions of these problems restricted to a setting where sensors, relay nodes and base stations may occupy only a discrete set of locations on a 2D grid of points. Minimizing congestion [28] and load balancing are other application areas where topology control in the form of relay node placement is utilized (e.g. [25]). However, network performance in terms of communication cost is not an objective there.

Targeting problems more similar to the one considered in this paper, there are studies in the area of controlled mobility, where nodes in the network may adjust their locations to optimize certain network performance metrics (e.g. [29], [8], [18]). In these settings relay nodes may move on a continuum of points for optimal performance. For instance, the authors of [29] allow a set of k mobile nodes to adjust their location and provide a wireless communication backbone (i.e. comprising relay nodes) that minimizes energy expenditure by increasing the reliability of links. The authors propose a solution finding the lowest energy level of a dynamical physical system modeling the problem. However, in their setting routing is not considered. Similarly, in [8] the authors construct a wireless backbone communication network of relay nodes maximizing links' quality using Steiner tree approximations. They study the performance of their schemes with different routing patterns, but the location of the relay nodes is independent of the routing and traffic loaded on the links. More recently, the authors of [18] employ mobile relays to minimize both energy due to transmissions and the movement of the relay nodes. Their algorithm assumes that the routing tree does not change during nodes' mobility. This is analogous to the RPFT problem defined in section III-B above, where the assumption is that the routing pattern is fixed. In

[3]The best known approximation ratio of 1.39 at present is due to Byrka et al. [19]; although their algorithm is polynomial, its run-time depends on the performance of the ellipsoid method and the performance of the corresponding separation oracle.

our case, the optimal positions of the relay nodes is obtained by convex optimization. In contrast, the algorithm presented in [18] does not guarantee an optimal solution.

Finally, a number of works on sensor networks have considered a form of sensor nodes placement in order to optimize network communication cost. For instance, in [4], the authors utilize Steiner tree model to place relay nodes in a sparse network and increase links' reliability. They observe that in the latter setting the log-normal path-loss model is rather accurate and interference does not contribute significantly to reduce the PRR. This conclusion is also corroborated by the simulations presented here in section VI-B. The authors of [3] consider the placement of relay nodes, so that overall link cost is minimized while the gathered information by sensor nodes is maximized. Their algorithm approximates a General Steiner Tree to suggest locations for the communication relay nodes. In both works, routing in the network and its influence on communication costs and relay nodes' positions is not accounted for.

VIII. Discussion

We study the general problem of placing optimally a set of k relay nodes offloading traffic from n source nodes in the network. Different versions of the problem have been tackled by a number of works in the technical literature and in various settings. The solutions suggested by state-of-the-art schemes, in the context of minimizing network communication cost, are based on constructing Steiner tree and placing relay nodes at the location of the Steiner tree vertices. As we demonstrate, this strategy does not account for the fundamental feedback between routing, traffic load on links, and locations of relay nodes in the network. Correspondingly, the resulting relay node placement model is not necessarily optimal, and the resulting solutions possess intrinsic inefficiencies. We formulate a novel topology control problem for communication cost minimization in relay networks, considering the interplay between routing and relay nodes' locations. We give an optimal algorithm, which however is not practical for larger networks. We show that even listing a set of feasible optimal sites for the relay nodes is at least APX-hard. Hence, we suggest a heuristic algorithm: **RePlace**. In the case of small network instances **RePlace** matches the optimal solution or attains a solution very close to the optimal (within 2-3%). We simulate **RePlace** and demonstrate that it reduces almost completely the communication cost due to retransmissions of packets on low quality links. The delay in the resulting relay network topology is reduced by (35%) as well.

In the case of relay placement where the network routing is specified and does not change as relay nodes are added, we show the problem can be optimally solved using standard convex optimization algorithms. This may be of independent interest in certain practical applications as discussed in [18], for instance.

The maximization problem formulation of the general relay placement problem given in (18) generalizes a number of cases, in which a constant approximation algorithms to the optimal solutions are known. Some of them may have practical significance. Below, we provide a brief sample of such results. The different cases result from placing different constraints on the communication cost reduction function $\phi(O)$ defined in (18). We assume here that $X(E)$ is given as part of the input.

- *Case 1*: *$\phi(O)$ is nonmonotone and submodular function over the independence system* $\mathbf{\Phi}$. There is a Greedy algorithm that iteratively selects assignment pairs $(i, j) \in \mathbf{\Phi}$ and provably achieves a constant factor $((m-1)/m^2 + \epsilon)$-approximation, $\forall \epsilon > 0$ [33]. m is the number of matroids intersecting to form the independence system $\mathbf{\Phi}$.

Potential application problem is the location of network integration points in wireless networks. This may be viewed as a special case of the GRP where relay/integrator nodes do not communicate with each other but only communicate with the fixed nodes. Also, each fixed node communicates with exactly one integrator/relay node. The case of placing a single relay node in the network also falls in this category. The online problem of placing relay nodes given one at a time (where the positions of the previous relay nodes cannot be altered) falls in this category as well, given that the routing through the already placed relay nodes is not affected by the addition of new relay nodes.

- *Case 2*: *$\phi(O)$ is monotone and submodular function over the independence system* $\mathbf{\Phi}$. Similarly to the above case there is a well-known constant approximation Greedy algorithm. The constant factor is $1/(m+1)$ [16]. A potential application problem would be the integrators placement problem, where communication cost on the network links between integrators and fixed nodes is relatively uniform.

- *Case 3*: *$\phi(O)$ is nonmonotone and submodular function and the independence system $\mathbf{\Phi}$ is an intersection of a single matroid (i.e. $\mathbf{\Phi}$ is a matroid)*. In this case, there is a $1/3.23$ constant approximation algorithm ([34]). Of course, this case is rather restrictive and may model only very specific practical scenarios.

- *Case 4*: *$\phi(O)$ is monotone and submodular function and the independence system $\mathbf{\Phi}$ is a matroid*. There is a well known $1/2$-approximation algorithm in this case (e.g. [16]). This is the most restrictive of all cases in terms of modeling potential practical scenarios, and is included here only for completion.

We leave open the investigation of special cases of the relay placement problem that permit such constant approximation algorithms.

Limitations:

- *Deployment area*: notice that the problem formulation and algorithms presented in the paper may be applied in deployment areas containing geographical obstacles (e.g. lakes, rivers, etc.), infrastructural obstacles (e.g. buildings, fenced areas, etc.) or other areas constraining network deployment. In these cases, we only require that the shapes modeling these obstacles are convex, so that the constraints, resulting from applying the currently used clipping algorithm ([17]) for finding zone overlaps, are convex. A clipping algorithm feasible for non-convex zone overlaps would obviate this assumption.
- *Link model*: the log-distance path-loss link model presented in section II-A does not explicitly account for interference. Section VI-B evaluates the performance of **RePlace** in the context of interference and corroborates

prior results in the technical literature: the log-distance path-loss model accurately characterizes links in sparsely deployed outdoor sensor networks (e.g. [4]); scenarios characterized by large-scale fading (e.g. [12]); or networks relying on interference avoidance MAC layer protocols. These are the settings in which our results apply readily. Network deployments dominated by small scale fading and interference may require adjustment of the present link model.

- *Network sparsity*: related to the above item, as the network density grows, interference effects become more dominant, rendering the log-distance path-loss model less accurate. In this work, we consider sparse networks where average node degree is relatively low (e.g. approximately four neighbors per node). As observed in various works on relay placement (e.g. [26]) as the network density increases the utility of relay nodes for survivability guarantees, throughput or link performance improvements decreases.

Appendix
Convexity of the RPFT Problem

From Theorem 1, one only needs to show that the function $g(z)$ is convex and non-decreasing for $z \in (0, rd_0)$, where $r > R$. For clarity of presentation, assume $d_0 = 1$.

To analyze the above claim, consider the function

$$g(z) = \left[1 - Q\left\{\sqrt{2\gamma(z)}\right\}\right]^{-b}$$

where $z \in \mathbb{R}^+$. Substituting γ from (2), we get $g(z)$ equals

$$\left[1 - Q\left\{\sqrt{2\left(P_t - a(1) - 10\alpha\log_{10}(z) - \eta(0,\sigma) - \chi(0,\sigma_1)\right)}\right\}\right]^{-b}$$

Let $A = P_t - a(1) - \eta(0, \sigma) - \chi(0, \sigma_1)$. A is a Gaussian r.v., however, at present A is treated a constant w.r.t. to z. Then

$$\begin{aligned} g(z) &= \left[1 - Q\left\{\sqrt{2\left[A - 10\alpha\log_{10}(z)\right]}\right\}\right]^{-b} \\ &= \left[\frac{1}{2} + \frac{1}{2}erf\left(\frac{\sqrt{2\cdot 10^{(A-10\alpha\log_{10}(z))/10}}}{\sqrt{2}}\right)\right]^{-b} \end{aligned}$$

Let $B = 10A/20$ and $a = \alpha/2$. We obtain $g(z) = \left(\frac{1}{2}\right)^{-b}\left[1 + erf\left(Bz^{-a}\right)\right]^{-b}$ and only need to consider the term $g_1(z) = \left[1 + erf\left(Bz^{-a}\right)\right]^{-b}$. If $g_1(z)$ is convex, so is $g(z)$. Observe that

$$\frac{dg_1(z)}{dz} = \frac{2abB}{\sqrt{\pi}} \cdot \frac{e^{-Bz^{-2a}}\left[1 + erf\left(Bz^{-a}\right)\right]^{-b-1}}{x^{a+1}} \tag{20}$$

and

$$\begin{aligned} \frac{d^2 g_2(z)}{dz^2} &= a\left[1 + erf\left(Bz^{-a}\right)\right]^{-b-1} e^{-B^2}z^{-2a} \\ &\times \left\{\frac{2(b+1)Be^{-B^2}z^{-2a}}{\left[1 + erf\left(Bz^{-a}\right)\right]\sqrt{\pi}} + \frac{2B^2}{z^a} - \frac{z^a(a+1)}{a}\right\} \end{aligned}$$

If $g_2''(z)$ is non-negative, $g(z)$ is convex. Notice that since $a > 0$, $b > 0$, and $B > 0$

$$a\left[1 + erf\left(Bz^{-a}\right)\right]^{-b-1} e^{-B^2 z^{-2a}} > 0$$

for all z. Hence, we are only interested in the inequality

$$\frac{2(b+1)Be^{-B^2 z^{-2a}}}{\left[1 + erf\left(Bz^{-a}\right)\right]\sqrt{\pi}} + \frac{2B^2}{z^a} - \frac{z^a(a+1)}{a} > 0 \tag{21}$$

In this form (21) does not have analytical solution expressed in simple functions. However, one can still reason about the convexity of the function $g(z)$. First, simplify (21) further by noting that $\left[1 + erf\left(Bz^{-a}\right)\right] < 2$ for any z, since Bz^{-a} is positive. We obtain

$$\frac{(b+1)Be^{-B^2 z^{-2a}}}{\sqrt{\pi}} + \frac{2B^2}{z^a} - \frac{z^a(a+1)}{a} > 0 \tag{22}$$

Observe that $B^2 z^{-2a} < 0.15$. Hence, via Taylor series expansion $e^{-B^2 z^{-2a}} \approx 1 - B^2 z^{-2a}$. This approximation is valid as z (or equivalently, the distance between two network nodes) increases, since $B = 10A/20$ and $a = \alpha/2$. Typically in wireless networks models $\alpha = 3$ and B is small in low SNR regimes. Rearrangement of terms in (22) yields

$$\frac{2\sqrt{\pi}B(b+1)}{z^a} - \frac{B^2(b+1)}{z^{3a}} - \frac{\sqrt{\pi}(a+1)}{aB} > 0 \tag{23}$$

Noting that $z^a > 0$, substituting $\alpha = 3, a = \alpha/2$ and letting $\zeta_1 = \frac{\sqrt{\pi}(a+1)}{aB}$, $\zeta_2 = 2\sqrt{\pi}B(b+1)$, $\zeta_3 = B^2(b+1)$ and $y = z^{3/2}$, we get $\zeta_1 y^3 - \zeta_2 y^2 + \zeta_3 < 0(\dagger)$ The solution intervals of ($\dagger$), after reverse substitution, are equivalent to the intervals where $g(z)$ is convex. The solutions of ($\dagger$) can be found explicitly, however the roots are complicated. Instead, one can gain intuition about the asymptotic behavior of the function $g(z)$ by observing its first derivative as z increases. Recall that

$$\frac{dg_1(z)}{dz} = \frac{2abB}{\sqrt{\pi}} \cdot \frac{e^{-Bz^{-2a}}\left[1 + erf\left(Bz^{-a}\right)\right]^{-b-1}}{x^{a+1}} \tag{24}$$

Notice that as z increases $\left[1 + erf\left(Bz^{-a}\right)\right]^{-b-1} \approx 1$. Then, asymptotically

$$\frac{dg_1(z)}{dz} = \frac{2abB}{\sqrt{\pi}} \cdot \frac{1 - B^2 z^{-2a}}{z^{a+1}} \to 0 \tag{25}$$

Hence, $g_2''(z) \to 0$. Thus, the function $g(z)$ is convex in this asymptotic regime. Intuitively this is correct since the $Q(\cdot)$, analogously $erf(\cdot)$, and the packet error rate functions converge to constants, as z increases. I.e., the received power is very low at a node that is large distance away from a transmitting node. Note that as z decreases the first two terms in (22) increase and the third term decreases.

Let $z \in (0, r)$ and respectively $z \in (0, rd_0)$ be intervals where the function is convex. Figure 14 demonstrates that (21) is indeed satisfied for large values of r depending on the transmit power and the packet size. The larger the transmit power

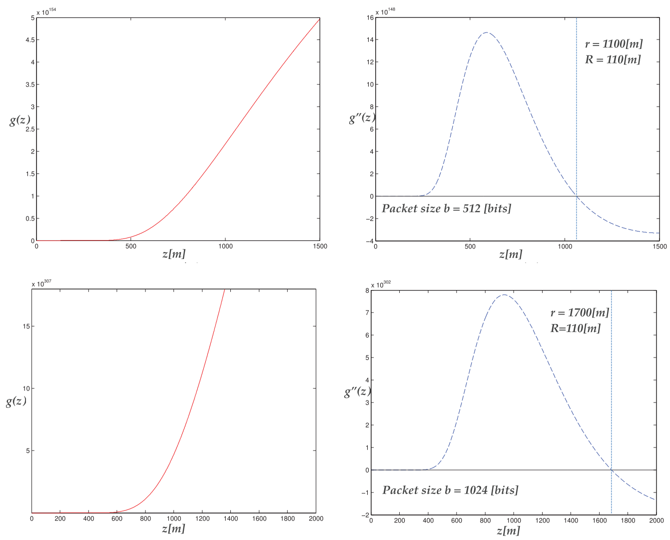

Fig. 14. $g(x)$ and $g''(x)$ for different packet sizes and fixed transmit power of 10[W]. Notice that the convexity interval $(0, r)$ increases as b grows. The transmission range, $R \approx 110$[m] is less than r for various values of BER and SNR.

and the packet size, the larger r. For instance, if the transmit power is 10[W] and the packet size is 512 bits the function $g(z)$ is convex within a range of $z = rd_0 = 1100[m]$, for packet size of 1024 bits; $g(z)$ is convex within a range of $z = rd_0 = 1700[m]$. $d_0 = 1[m]$ throughout. Also, to increase the transmission range, the transmit power is increased, which in turn increases the convexity interval $(0, r)$.

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Milen Nikolov received the undergraduate degrees in computer science and mathematics from the State University of New York at Brockport, Brockport, NY, USA, and the M.Sc. and Ph.D. degrees from the Department of electrical and Computer Engineering, Cornell University, Ithaca, NY. He is now a Postdoctoral Researcher with the Institute for Disease Modeling, Bellevue, WA, USA, where his focuses are malaria epidemiology and control models. He has been studying algorithms for networks with dynamic topologies, collaborative communication in information networks, and data-driven dynamic models of contagious and vector-borne diseases over spatially connected populations. His research interests include the wireless communications, information, and disease transmission networks.

Zygmunt J. Haas (S'84–M'88–SM'90–F'07) received the Ph.D. degree in electrical and computer engineering from Stanford University, Stanford, CA, USA, in 1988. In 1988, he joined AT&T Bell Laboratories in the Network Research Area. He then pursued research in wireless communications, mobility management, fast protocols, optical networks, and optical switching. In August 1995, he joined the Faculty of the School of Electrical and Computer Engineering, Cornell University, Ithaca, NY, USA, where he is now a Professor. He heads the Wireless Network Laboratory, a research group with extensive contributions and international recognition in the area of ad hoc networks and sensor networks. He has authored over 200 technical conference and journal papers and holds 18 patents in the areas of wireless networks and wireless communications, optical switching and optical networks, and high-speed networking protocols. His research interests include mobile and wireless communication and networks, modeling and performance evaluation of large and complex systems, and biologically-inspired networks. He has organized several workshops, delivered numerous tutorials at major IEEE and ACM conferences, and served as an Editor for several journals and magazines, including the IEEE/ACM TRANSACTIONS ON NETWORKING, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the *IEEE Communications Magazine*, and *Wireless Networks* (Springer). He has been a Guest Editor of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS issues and served as the Chair of the IEEE Technical Committee on Personal Communications. He was the recipient of a number of awards and distinctions, including best paper awards and the 2012 IEEE ComSoc WTC Recognition Award for "outstanding achievements and contribution in the area of wireless communications systems and networks."