Analysis of Multipath Routing, Part 2: Mitigation of the Effects of Frequently Changing Network Topologies

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***Abstract*—In this paper, we extend the analysis of multipath routing presented in our previous work, so that the basic restrictions on the evaluation and optimization of that scheme can be dropped (e.g., disjoint paths and identical paths in terms of failure probability). In that work, we employed *Diversity Coding* in order to provide increased protection against frequent route failures by splitting data packets and distributing them over multiple disjoint paths. Motivated by the high increase in the packet delivery ratio, we study the increase we can achieve through the usage of multiple paths in the general case, where the paths are not necessarily independent and their failure probabilities vary. For this reason, a function that measures the probability of successful transmission is derived as a tight approximation of the evaluation function P_{succ}. Given the failure probabilities of the available paths and their correlation, we are able to find in polynomial time the set of paths that maximizes the probability of reconstructing the original information at the destination.**

***Index Terms*—Ad hoc networks, ad hoc routing, alternate-path routing, diversity coding, multipath routing, network-fault tolerance, quality of service.**

I. Introduction

IN THIS PAPER, we extend our work in [1], where we proposed a multipath scheme for mobile *ad hoc* networks based on diversity coding [2]. Data load is distributed over multiple paths in order to minimize the packet drop rate, achieve load balancing, and improve end-to-end-delay. We evaluate our scheme by calculating the probability that a transmission from the source results in successful packet reception at the destination. The probability function of successful reception is analytically derived and data is split over multiple paths in such a way that the function is maximized.

The majority of the protocols proposed for routing in *ad hoc* networks utilizes one route at a time in order to send data from a source to a destination node, although multiple routes may be kept in a cache. Examples of such protocols can be found in [3] for dynamic destination-sequenced distance-vector routing, [4] for dynamic source routing, [5] for *ad hoc* on-demand distance vector routing, [6] for temporally ordered routing algorithm, and [7] for zone routing protocol. Multipath routing has been proposed in [8]–[11], and [12]. However, these schemes either use one primary route, and route traffic to alternative routes when it fails, or distribute traffic among the available routes, hoping that the packet delivery ratio will increase through load balancing. No analytical model was offered in order to justify the selection of a specific set of routes, since there is no consideration of the route failure characteristics. For a more detailed reference see [1].

In [1], multiple paths are utilized in order to increase the probability of successful transmission of data packets, denoted as P_{succ}. The basic assumption for the network model is that the mean time of packet transmission is much smaller than the mean time between variations in network topology. If this assumption holds, then one can assume that the probability that one or more path links fail is constant during the transmission of a packet. In other words, one can assume that the topology of the network will not change significantly while a packet is being transmitted. In this paper, we develop an approximation for P_{succ}, so that the complex cases we did not deal with in [1] can be tackled. In particular, we offer a simple approximation method in order to maximize P_{succ} by effectively choosing the optimal path set to be utilized, in the case where the paths are not necessarily independent and their failure probabilities vary. In [13], Chan and Chan also try to evaluate and optimize the effect of diversity coding on the success probability, but they only focus on its asymptotic behavior. Thus, they only provide a bound for the evaluation function (accurate only when hundreds of paths are used) and optimized accordingly. In contrast, we provide a tight approximation and use that approximation for optimization.

Our paper is organized as follows. Section II provides a short description of the proposed scheme and the definition of the successful transmission probability function P_{succ}, the function used for the evaluation of the scheme. In Section III, we derive an approximation of P_{succ}, so that we can extend the results obtained in [1] to more general cases. More specifically, we find which of the available paths must be used so that P_{succ} is maximized. In Section V, we present various examples of evaluation scenarios of our scheme. Finally, in Section VI, we present conclusions and we set the goals for future work.

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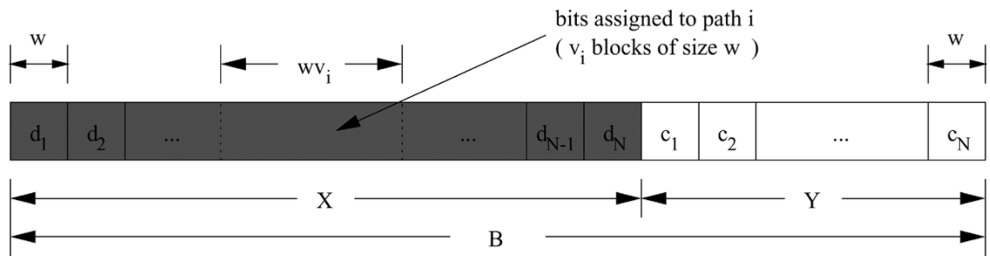


Fig. 1. Information and overhead packet fragmentation.

II. Brief Description of the Scheme

In this section, we briefly describe the scheme that we proposed in [1]. The general case of multipath transmission is considered, in which $n_{\max}$ disjoint paths are available for a packet transmission. Each path is treated as a pure erasure channel and it is associated with some failure probability p_i, which is defined as the probability that, at the transmission attempt time, the path is down. The failure probabilities of the available paths are organized in the probability vector $\underline{p} = [p_i]$, $i = 1, \ldots, n_{\max}$, so that $p_i \leq p_{i+1}$. Given $\underline{p}$, we also define $\underline{q} = [q_i]$, $q_i = 1 - p_i$, $i = 1, \ldots, n_{\max}$, which is the vector of the success probabilities. Throughout this paper, we use $\underline{p}$ and $\underline{q}$ interchangeably.

In [14], a path availability model for wireless *ad hoc* networks is proposed, which provides us with some indication on how the quality of the network can be expressed in terms of the probability vector $\underline{p}$. In addition, there are other protocols, such as associativity-based routing (ABR) [15] and signal-stability based routing (SSA) [16], that quantify the stability of the routes in a network using various criteria, based on network measurements.

Our scheme splits the original X-bit packet into N blocks and adds Y overhead bits, which are partitioned into M blocks of overhead (calculated using linear transformations from the original N blocks with M-for-N diversity coding, as explained in [2]). This process is depicted in Fig. 1. All blocks are equal in size (w bits). If the total number of bits (information plus overhead) is B, then the overhead factor r is defined as

$$r = \frac{B}{X} = \frac{b}{x} \tag{1}$$

where b and x take integer values and the fraction b/x cannot be further simplified, i.e., the greatest common divisor of b and x is 1.

In our scheme, we assume an allocation vector $\underline{v} = [v_i]$, where v_i is the number of blocks allocated to path i, $i = 1, \ldots, n \leq n_{\max}$, and where n is the number of paths that are used in practice (out of the $n_{\max}$ available ones). If z_i is the number of blocks that actually reaches the destination through path i, then

$$\Pr\{z_i = v_i\} = q_i$$
$$\Pr\{z_i = 0\} = p_i.$$

This is so, because we assume that if a path fails, then all the blocks sent over the path are lost (recall the pure erasure channel assumption). M-for-N diversity coding can reconstruct the original X-bit information packet, provided that at least N blocks reach the destination. Therefore, we can define P_{succ} in terms of the number of paths that are actually used and the corresponding allocation vector

$$P_{\text{succ}}(n, \underline{v}) = \Pr\left\{ \sum_{i=1}^{n} z_i \geq \frac{\sum_{i=1}^{n} v_i}{r} \right\}. \tag{2}$$

In Section III, we derive a tight approximation for P_{succ}, and we use it in order to determine the optimal path set, i.e., the one that maximizes P_{succ}.

III. Approximation of P_{succ}

As we explained in [1], it is evident that neither the optimal number of paths, nor the optimal allocation vector, can be calculated in the general case where the probability vector is nonuniform. The main problem is the complexity of P_{succ} in terms of continuity and the required computation time. Since P_{succ} is not continuous because of the presence of unit-step functions in its formula, its derivative is not defined everywhere. Moreover, the time required to calculate P_{succ} is exponential, which simply means that real-time computation is impossible to perform.

To address the above problems of P_{succ} evaluation, we will present an approximation of P_{succ} based on the following observations: 1) the binomial distribution can be approximated by the normal distribution and 2) the sum of n independent, normally distributed random variables follows the normal distribution.

In Section III-A, we will find an approximation for P_{succ} in the simplest case in which all paths are characterized by the same probability of failure p and the allocation vector is uniform, i.e., all paths carry one block. In Section III-B, we derive an approximation for an arbitrary path probability vector. Section IV presents the approximation of P_{succ} in the general case where the allocation vector is arbitrary. A generalized algorithm for the calculation of the optimal block allocation is also given. Finally, in Section IV-A, we calculate P_{succ} when the paths we are using are correlated, i.e., they have nodes or links in common.

A. Uniform Probability Vector, Uniform Allocation Vector Case

As a starting point, we will find an approximation for the simplest case; we assume a uniform probability vector (i.e., all paths have the same probability of failure p) and a uniform allocation vector (i.e., all paths are assigned equal number of blocks). In [1], we showed that in the case where a uniform allocation vector is used, and the probability vector is uniform as well, the probability function is simplified to the following expression:

$$P_{\rm succ}^{(u)}(n)=\sum_{k=m}^{n}\binom{n}{k}q^{k}p^{n-k} \tag{3}$$

where $m=\lceil n/r\rceil$.

In order to derive the approximation for (3), we assume the random variable Z, which represents the number of successful paths out of n available ones. Z follows the binomial distribution

$$\Pr\{Z=k\}=\binom{n}{k}q^{k}p^{n-k}. \tag{4}$$

Z can be approximated by a Gaussian distribution of mean value $\mu=nq$ and standard deviation $\sigma=\sqrt{npq}$ (see [17]). The probability density function is

$$\begin{aligned}f_Z(z)&=\frac{1}{\sigma\sqrt{2\pi}}\cdot\exp\left[-\frac{(z-\mu)^2}{2\sigma^2}\right]\\&=\frac{1}{\sqrt{2\pi npq}}\cdot\exp\left[-\frac{(z-nq)^2}{2npq}\right].\end{aligned} \tag{5}$$

By integrating (5), we get an approximation for the $P_{\rm succ}$ as it is defined in (3). The lower bound in the sum is m, and, as explained in [18], if we shift it by -1/2, we derive a better approximation of $P_{\rm succ}$, denoted as $P_a^{(u)}$

$$P_a^{(u)}(n)=\int_{m-1/2}^{+\infty}f_Z(z)dz=\frac{1}{2}+\frac{1}{2}\cdot erf\left(\frac{nq-\lceil\frac{n}{r}\rceil+\frac{1}{2}}{\sqrt{2npq}}\right). \tag{6}$$

The validity of the approximation and an estimation of the error it introduces are discussed in the Appendix. We also offer an alternative, higher-order approximation, which is more accurate, but requires a tedious computation effort. The error function $\operatorname{erf}(\cdot)$ that appears in (6) is defined as

$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x\exp(-u^2)du. \tag{7}$$

In Fig. 2, both $P_{\rm succ}$ and its approximation, calculated using (6), are presented for $r=3/2$ and $q=0.8$. Extended simulations showed that the approximation is very tight when the number of paths is sufficiently large. This is so, when

$$n>b \tag{8}$$

where b is defined in (1).

There are two important observations we can make about (6). First, the approximation function described by that equation is defined not only for integer values of n, but it can be also calculated for real values. Second, we can define a new function where n can take any real value greater than or equal to 1. This function is "made" continuous simply by replacing $\lceil n/r\rceil$ with n/r and, thus, it is called the continuous version of the approximation P_a, denoted as P_c

$$P_c^{(u)}(n)=\frac{1}{2}+\frac{1}{2}\cdot\operatorname{erf}\left(\frac{nq-\frac{n}{r}+\frac{1}{2}}{\sqrt{2npq}}\right). \tag{9}$$

This function passes through the local maxima of P_a, which approximately coincide with the local maxima of $P_{\rm succ}$. This is so, because $\operatorname{erf}(\cdot)$ is an ascending function and $\lceil n/r\rceil\geq n/r$. Therefore, $P_a(n)=P_c(n)$, only when $\lceil n/r\rceil=n/r$, or, equivalently, when $n=kb$, where k is an integer and b is defined in (1). If $\lceil n/r\rceil>n/r$, then $P_a(n)<P_c(n)$. Consequently, the global maximum candidates for P_a are the following values:

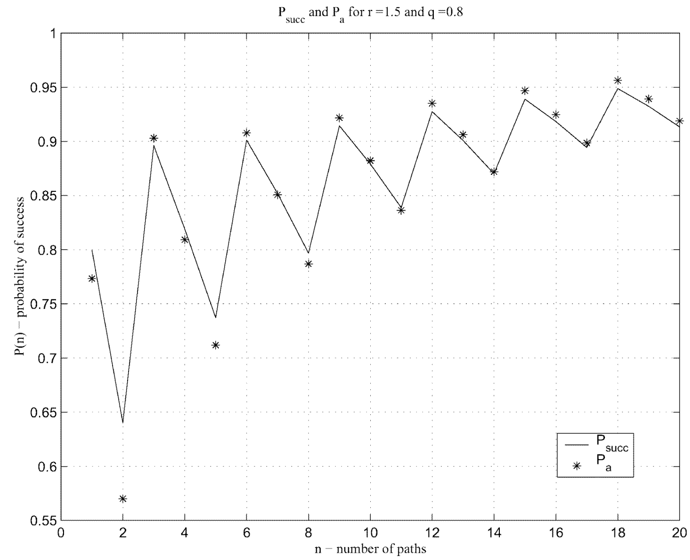


Fig. 2. $P_{\rm succ}$ and P_a for $r=3/2$ and $q=0.8$.

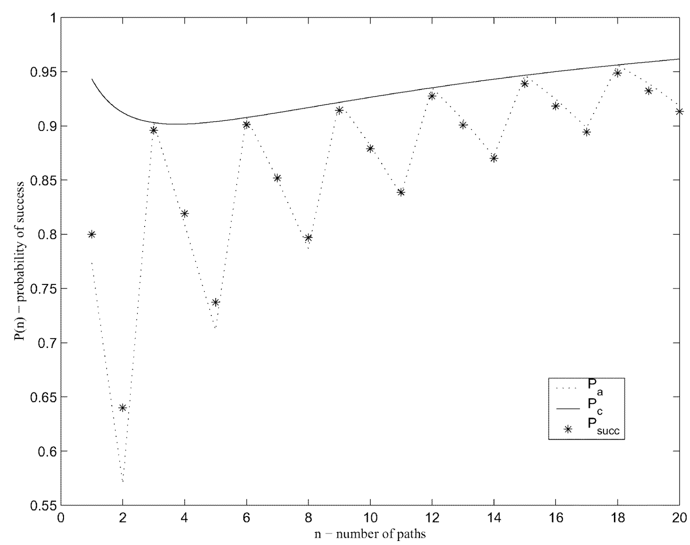


Fig. 3. $P_{\rm succ}$, P_a, and P_c for $r=3/2$ and $q=0.8$.

$$n=k\cdot b, k\geq 1. \tag{10}$$

Since the derivative of P_c is defined for all $n\geq 1$, we can use it to calculate the global minimum and maximum of the function. Calculating the global maximum of P_c will give us a good approximation of the optimal number of paths. We only have to take as the optimal number of paths the value $n=kb$, k integer, that is closest to the value calculated when P_c is maximized. This will be the global maximum of P_a. In Fig. 3, we study the behavior of $P_{\rm succ}$, P_a, and P_c for $r=3/2$ and $q=0.8$.

The derivative of P_c with respect to n is

$$\frac{dP_c^{(u)}(n)}{dn}=\frac{nq-\frac{n}{r}-\frac{1}{2}}{2n\sqrt{2\pi npq}}\cdot\exp\left[-\frac{(nq-\frac{n}{r}+\frac{1}{2})^2}{2npq}\right]. \tag{11}$$

In Fig. 4, we have drawn P_c and its derivative. The left vertical axis holds the values for P_c and the right axis holds the values

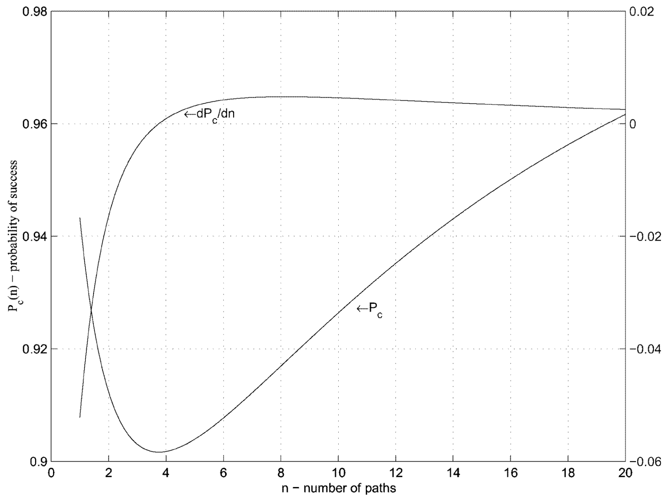


Fig. 4. P_c and derivative for $r = 3/2$ and $q = 0.8$.

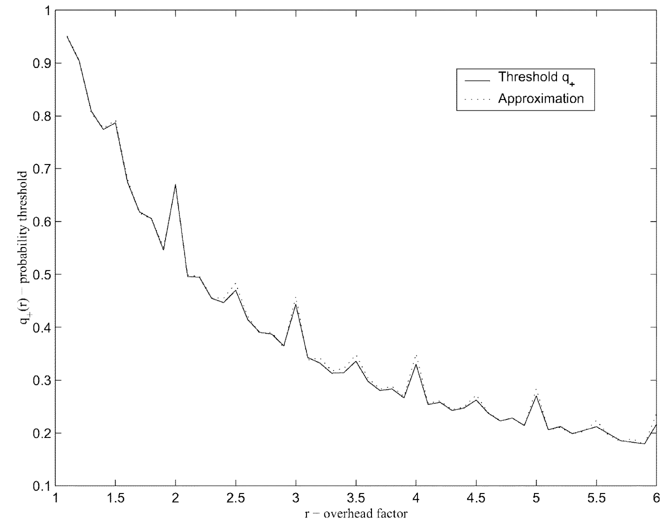


Fig. 5. Threshold q_+ and its approximation.

for the derivative. As expected, the derivative is zero at the value of n that minimizes P_c.

Our first goal is to find the threshold value of the probability q of each path beyond which P_c is ascending for $n \geq 1$. However, what we actually need is the value of q beyond which P_{succ} is ascending within the subspace of its local maxima. Therefore, we will extract an approximation of that value given the fact that P_c is a tight approximation of P_{succ} for $n > b$, in the subspace of the local maxima defined by (10). We will compare the results of this method with the accurate results produced in [1]. First, we give the definition of the set of potential global maxima

$$K = \{n = k \cdot b | n \leq n_{\max} \wedge k = 1, 2, , \ldots, \}. \tag{12}$$

We define the threshold value q_+ as the following:

1) for all $q \geq q_+$ and for all $n_1, n_2 \in K$, with $n_2 \geq n_1$: $P_{\text{succ}}(n_2) \geq P_{\text{succ}}(n_1)$;
2) for all $q < q_+$ there are $n_1, n_2 \in K$, with $n_2 \geq n_1$, for which $P_{\text{succ}}(n_2) < P_{\text{succ}}(n_1)$.

Using P_c, we can find an approximation for q_+, because for all $n \in K$, $P_c(n) = P_a(n)$ and P_a is a tight approximation of P_{succ}. The approximation of the threshold is calculated as the number q_+ for which the following two properties hold:

1) the root n_0 of dP_c/dn for all $q \geq q_+$ is less than or equal to $b + 1$;
2) the root n_0 of dP_c/dn for all $q < q_+$ is greater than b.

The value of P_c at $n = n_0$ is its global minimum, if $q > 1/r$, as it will be shown promptly. The value b is chosen so as to ensure the validity of the approximation [see (8)].

We will now work on finding an analytical approximation for q_+ using the derived continuous approximation of P_{succ}. P_c has a global minimum at $n = n_0$, where its derivative is zero, if $q > 1/r$

$$n_0(r, q) = \frac{1}{2\left(q - \frac{1}{r}\right)}. \tag{13}$$

If the minimum point n_0 is less than or equal to $b + 1$, then P_c is ascending for all $n > b$, that is for all n that yield a good approximation [see (8)]. Therefore, an approximation for q_+ is obtained when the rightmost part of (13) is set to $b + 1$:

$$q_+(r) = \frac{1}{2(b+1)} + \frac{1}{r}. \tag{14}$$

In Fig. 5, we can compare q_+ and its approximation described by (14).

The following are the main conclusions of this section.

1) If $q \geq q_+(r)$, then P_c is ascending for $n > b$ and, therefore, the optimal number of paths is the maximum number in the set of potential global maxima K, which is
$$n_{\text{opt}}(r) = \max_{n \in K}\{n\} = b \cdot \left\lfloor \frac{n_{\max}}{b} \right\rfloor.$$
2) If $q < 1/r$, then P_c is descending with respect to n and so is the set of local maxima of P_{succ}. The optimal number of paths for this case is
$$n_{\text{opt}}(r) = b.$$
3) If $1/r \leq q < q_+(r)$, then the optimal number is found at
$$n_{\text{opt}}(r) = \max{}^{-1}\left\{P_c(b), P_c\left(b \cdot \left\lfloor \frac{n_{\max}}{b} \right\rfloor\right)\right\}.$$

In Fig. 6, we show a comparison of $n_0(q)$ and the point where P_{succ} exhibits its maximum, for $r = 3/2$.

Using the approximation analysis presented in this section, we will proceed with the derivation of an approximation formula for P_{succ} when the assumption of a uniform probability vector is dropped.

B. Uniform Allocation Vector Case

In this section, we derive an approximation for P_{succ} in the case of an arbitrary path probability vector $\underline{p}$. The allocation vector remains uniform. First, we define a random variable Z_i, which represents the average number of successful transmissions out of k attempts to transmit over path i. Z_i follows the binomial distribution, and, as it was explained in Section III-A, we can approximate it using the normal distribution

$$f_{Z_i}(z_i) = \frac{1}{\sigma_i\sqrt{2\pi}} \cdot \exp\left[-\frac{(z_i - \mu_i)^2}{2\sigma_i^2}\right] \tag{15}$$

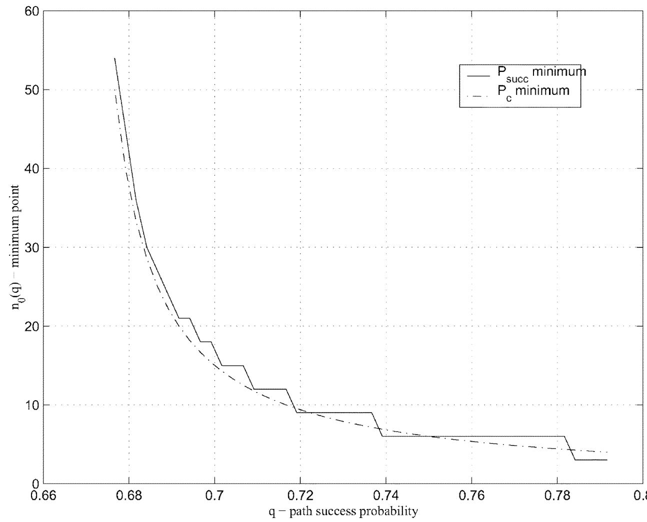


Fig. 6. Minimum points for $r = 3/2$.

where the mean value is $\mu_i = q_i$ and the standard deviation is $\sigma_i = \sqrt{p_i q_i}$. We define the random variable Z that represents the number of paths that succeed, averaged over k independent trials

$$Z = \sum_{i=1}^{n} Z_i. \tag{16}$$

Z is the sum of independent variables that follow the normal distribution; therefore, Z follows the normal distribution as well. The probability density function of Z is given by the following:

$$f_Z(z) = \frac{1}{\sigma(n)\sqrt{2\pi}} \cdot \exp\left[-\frac{(z-\mu(n))^2}{2\sigma^2(n)}\right] \tag{17}$$

where the mean value is $\mu(n) = \sum_{i=1}^{n} q_i$ and the standard deviation is $\sigma(n) = \sqrt{\sum_{i=1}^{n} p_i q_i}$. The approximation of P_{succ} is derived in the same way as in Section III-A by integrating f_Z

$$P_a^{(u)}(n) = \frac{1}{2} + \frac{1}{2} \cdot \mathrm{erf}\left(\frac{\mu(n) - \lceil \frac{n}{r} \rceil + \frac{1}{2}}{\sigma(n)\sqrt{2}}\right). \tag{18}$$

Extended simulations showed that the approximation is tight, when the number of paths is sufficiently large. This is so, when $n > b$, as it was pointed out in (8). As an example, in Fig. 7, we have drawn both P_{succ} and its approximation, calculated using (18), for $r = 3/2$. The probability vector is $q_i = 0.8$, for $1 \le i \le 10$, and $q_{i+10} = 0.8 - 0.02 \cdot i$, $1 \le i \le 10$.

As in Section III-A, we define the "continuous" version of P_a by replacing $\lceil n/r \rceil$ with n/r. Of course, n can take only integer values because functions μ and σ involve summations indexed from 1 to n

$$P_c^{(u)}(n) = \frac{1}{2} + \frac{1}{2} \cdot erf\left(\frac{\mu(n) - \frac{n}{r} + \frac{1}{2}}{\sigma(n)\sqrt{2}}\right). \tag{19}$$

This function passes the local maxima of P_a at positions $n_0 = kb$, k integer [see (10)], which approximately coincide with the local maxima of P_{succ} at the same positions. In Fig. 8, P_a and P_c are drawn on the same graph for the same values of r and $\underline{q}$ that was used to produce Fig. 7.

The function $\mathrm{erf}(\cdot)$ (i.e., the error function) is a monotonically ascending function, so, in order to maximize P_a, it is sufficient

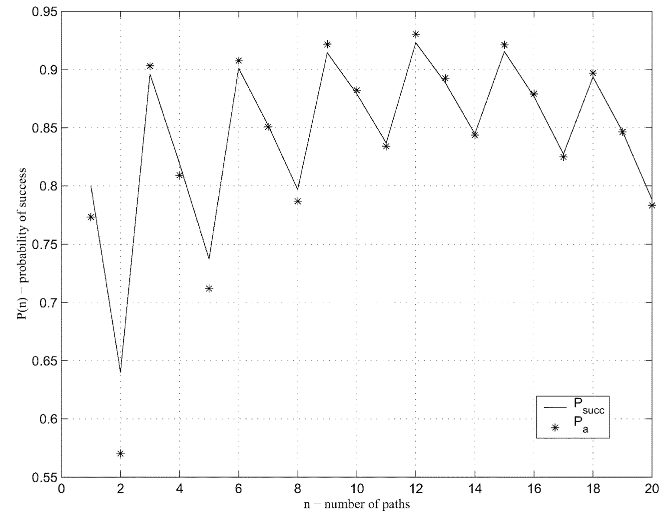


Fig. 7. P_{succ} and P_a for $r = 3/2$.

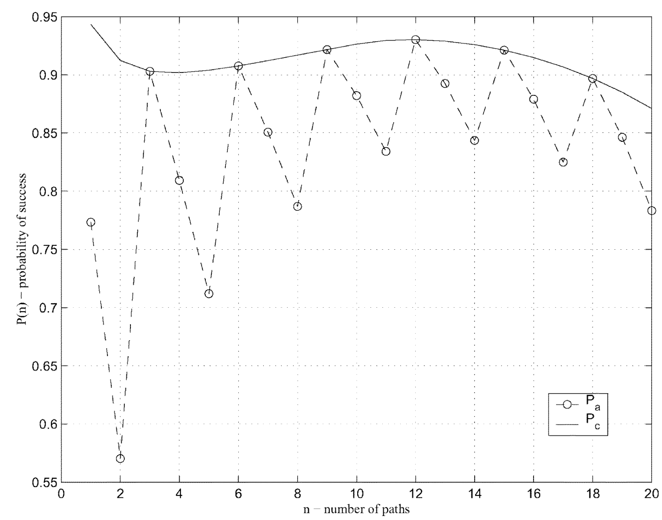


Fig. 8. P_a and P_c for $r = 3/2$.

to maximize the expression that this function takes as its argument. Therefore, the optimal number of paths is given by the following:

$$n_{\mathrm{opt}} = \max_{n=kb}^{-1}\left\{\frac{\mu(n) - \frac{n}{r} + \frac{1}{2}}{\sigma\sqrt{2}}\right\}. \tag{20}$$

The maximum of P_c is found at the point where its derivative, which is given by the following equation, is zero. Taking into account that the derivative of μ is $d\mu/dn = q_n$, and that the derivative of σ is $d\sigma/dn = p_n q_n / 2\sigma$, we can calculate the derivative of P_c

$$\frac{dP_c^{(u)}(n)}{dn} = \frac{2\left(q_n - \frac{1}{r}\right)\sigma^2 - \left(\mu - \frac{n}{r} + \frac{1}{2}\right)p_n q_n}{2\sigma^3\sqrt{2\pi}} \cdot \exp\left[-\frac{\left(\mu - \frac{n}{r} + \frac{1}{2}\right)^2}{2\sigma^2}\right]. \tag{21}$$

In Fig. 9, P_c and its derivative are presented for the same probability vector we used in Fig. 8. The derivative is zero at $n = 12$, and, therefore, the global maximum is found at $n_{\mathrm{opt}} = 12$. In Fig. 10, we present a more complex case where P_c has more than one points that are candidates for a global maximum.

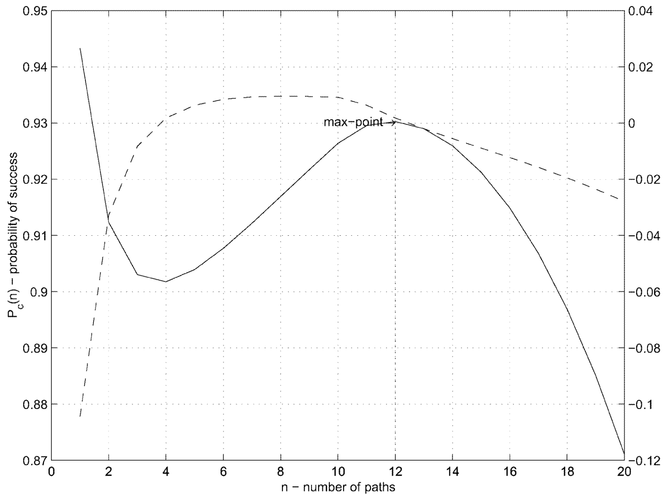


Fig. 9. P_c and its derivative for $r = 3/2$.

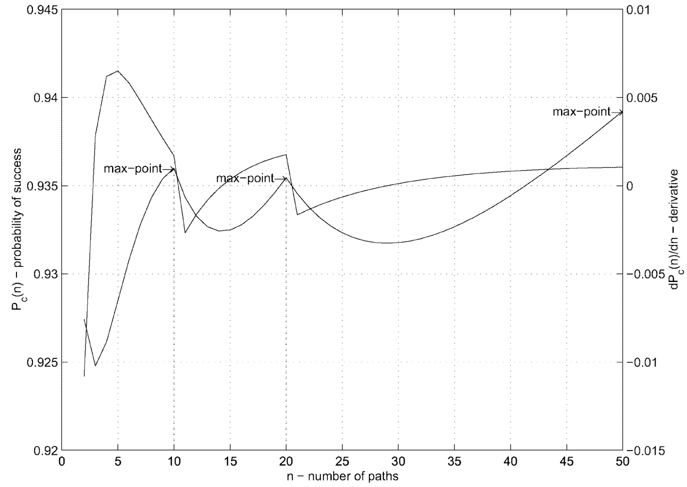


Fig. 10. P_c and its derivative for $r = 2$.

The conclusion of this section is that the problem of finding the maximum and minimum of P_c (and, therefore, a good approximation of the maximum and minimum of P_{succ}) is reduced to the problem of finding the roots of its derivative. The roots n_0 can be calculated from the following:

$$2\left(q_{n_0} - \frac{1}{r}\right)\sigma^2 = \left(\mu - \frac{n_0}{r} + \frac{1}{2}\right)p_{n_0}q_{n_0}. \tag{22}$$

We only have to check those values of n_0 that can give local maxima of P_{succ} as explained in Section III-A by (10). In the next section, we present a method for the computation of the optimal block allocation, i.e., the uniformity of the block allocation is dropped.

IV. Optimal Block Allocation

In this section, we study the optimal allocation of blocks to the multitude of paths, based on the approximation formula (presented later in this section) for the general case in which the allocation vector is not necessarily uniform. We assume n paths, the path probability vector $\underline{p}$ and the block allocation vector $\underline{v} = [v_i]$, $1 \le i \le n$. The blocks v_i allocated to path i can take only integer values, greater than or equal to one.[1] However, as we will see promptly, in order to solve the maximization problem, we will assume that the v_i can take real values (greater than or equal to one), so that we can easily solve the corresponding continuous maximization problem, instead of the integer one. In the end, the "continuous" solution will be adjusted, in order to adhere to the integer constraints of the problem.

If we go through the same approximation analysis that we followed in the previous sections, we can calculate the approximation P_a

$$P_a(\underline{v}, n) = \frac{1}{2} + \frac{1}{2}\cdot erf\left(\frac{\mu(\underline{v}) - \left\lceil \frac{1}{r}\sum_{i=1}^{n} v_i \right\rceil + \frac{1}{2}}{\sigma(\underline{v})\sqrt{2}}\right) \tag{23}$$

[1]The probability of success P_{succ} and its approximation P_a are presented as functions of the vector $\underline{v}$ and n. This means than for fixed n, $v_i \ge 1$ for all $i \le n$.

where

$$\mu(\underline{v}) = \sum_{i=1}^{n} v_i q_i, \tag{24}$$

$$\sigma(\underline{v}) = \sqrt{\sum_{i=1}^{n} v_i^2 p_i q_i}. \tag{25}$$

As mentioned before, in the integer version of the approximation, i.e., P_a, the blocks v_i are integers with $v_i \ge 1$. We now define the continuous version of the approximation P_c by dropping the integer constraint imposed on the v_is. Also, without loss of generality, we assume that the last path receives one block, i.e., $v_n = 1$. In other words, we normalize all v_i's over v_n, which is less than or equal to all other v_is. This is so, because it is the path with the highest failure probability, and as such, it cannot receive more blocks than any other path with lower failure probability. After the block values are normalized, we get $v_i \ge 1$, for $1 \le i \le n-1$, and $v_n = 1$. Having all these remarks in mind, we define the continuous approximation as

$$P_c(\underline{v}, n) = \frac{1}{2} + \frac{1}{2}\cdot erf\left(\frac{\mu'(\underline{v}) + \frac{1}{2}}{\sigma(\underline{v})\sqrt{2}}\right) \tag{26}$$

where we defined μ' as

$$\mu'(\underline{v}) = \mu(\underline{v}) - \frac{1}{r}\sum_{i=1}^{n} v_i = \sum_{i=1}^{n} v_i\left(q_i - \frac{1}{r}\right). \tag{27}$$

In order to find the extrema of P_c, we calculate the points where the partial derivatives with respect to the v_is are all zero, and where $1 \le i \le n-1$ and $v_i \ge 1$. We employ the following simple substitution, so that we can get rid of the inequality constraints:

$$v_i = 1 + \epsilon_i^2 \ge 1 \tag{28}$$

where ϵ_is are all real numbers. A simple observation can save us a lot of tedious calculations; in order to maximize P_c, it is sufficient to maximize the expression inside the error function

$\text{erf}(\cdot)$, because this function is monotonically increasing. We denote the expression to be maximized as f:

$$f(\underline{v}, n) = \frac{\mu'(\underline{v}) + \frac{1}{2}}{\sigma(\underline{v})}. \tag{29}$$

The optimal solution can now be calculated at the point where the gradient of f with respect to the ϵ_is is zero

$$\frac{\partial f}{\partial \epsilon_i} = 0, 1 \leq i \leq n - 1. \tag{30}$$

Let us first calculate the partial derivatives of the mean value and the standard deviation, as defined in (27) and (25)

$$\frac{\partial \mu'}{\partial \epsilon_i} = 2\epsilon_i \left(q_i - \frac{1}{r}\right) \tag{31}$$

$$\frac{\partial \sigma}{\partial \epsilon_i} = \frac{2\epsilon_i (1 + \epsilon_i^2) p_i q_i}{\sigma}. \tag{32}$$

The partial derivatives of f with respect to the ϵ_is are

$$\frac{\partial f}{\partial \epsilon_i} = 2\epsilon_i \cdot \frac{\left(q_i - \frac{1}{r}\right)\sigma^2 - \left(1 + \epsilon_i^2\right) p_i q_i \left(\mu' + \frac{1}{2}\right)}{\sigma^3}. \tag{33}$$

When the gradient is zero, and in the case that $\epsilon_i = 0$

$$v_i = 1, \tag{34}$$

or, in the case that $\epsilon_i \neq 0$

$$v_i = 1 + \epsilon_i^2 = c_i \cdot \frac{\sigma^2}{\mu' + \frac{1}{2}} > 1 \tag{35}$$

where

$$c_i = \frac{q_i - \frac{1}{r}}{p_i q_i}. \tag{36}$$

The optimal solution $\underline{v}$ given the number of paths n is in the following format:

$$\underline{v}(k, n) = [v_1, v_2, \ldots, v_k, 1, 1, \ldots, 1],\ 0 \leq k < n \tag{37}$$

where the k first elements of $\underline{v}$ are obtained from (35). The feasible solutions of (35) are those for which $v_k \geq 1$, and they represent the local extrema of f (equivalently, the local extrema of P_c). The optimal solution $\underline{v}_{\text{opt}}$ is the maximum of all the local extrema, that is for all k and n

$$\underline{v}_{\text{opt}} = \max_{0 \leq k < n \leq n_{\max}}{}^{-1} \{f(\underline{v}(k, n))\}. \tag{38}$$

We could determine which of the local extrema correspond to local maxima versus local minima, by calculating the Hessian matrix of f, defined as a $n \times n$ matrix $H(\underline{\epsilon})$, $\underline{\epsilon} = [\epsilon_i]$, whose ijth entry is $\partial^2 f / \partial \epsilon_i \partial \epsilon_j$. If $H_k(\underline{\epsilon})$ is the kth leading principal minor of $H(\underline{\epsilon})$, then a local extremum corresponds to a local maximum if $H_k(\underline{\epsilon})$ is nonzero and has the same sign as $(-1)^k$. However, computing the Hessian matrix is time consuming and, therefore, we choose to calculate all local extrema and then pick the maximum.

The only task left now is to calculate $\underline{v}(k, n)$, which can be accomplished by making use of (35), for $1 \leq i \leq k$, if $k > 0$. In the case of $k = 0$, we simply get the uniform allocation of one block per path. For $i = k$, (35) yields

$$v_k = c_k \cdot \frac{\sigma^2}{\mu' + \frac{1}{2}}. \tag{39}$$

If we divide (35) by (39), we can express all v_is in terms of v_k

$$v_i = \lambda_i v_k, \lambda_i = \frac{c_i}{c_k}. \tag{40}$$

Now, we can return to (39) and substitute the v_is using (40), in order to calculate v_k. In addition, we should not forget that for $k + 1 \leq i \leq n$, $v_i = 1$

$$v_k = c_k \cdot \frac{v_k^2 \sum_{i=1}^{k} \lambda_i^2 p_i q_i + \sum_{i=k+1}^{n} p_i q_i}{v_k \sum_{i=1}^{k} \lambda_i \left(q_i - \frac{1}{r}\right) + \sum_{i=k+1}^{n} \left(q_i - \frac{1}{r}\right) + \frac{1}{2}}. \tag{41}$$

From (40), we can observe that $c_k \lambda_i^2 p_i q_i = \lambda_i (q_i - 1/r)$, so

$$v_k^2 \sum_{i=1}^{k} \lambda_i \left(q_i - \frac{1}{r}\right) + v_k \left[\sum_{i=k+1}^{n} \left(q_i - \frac{1}{r}\right) + \frac{1}{2}\right]$$
$$= v_k^2 \sum_{i=1}^{k} \lambda_i \left(q_i - \frac{1}{r}\right) + c_k \sum_{i=k+1}^{n} p_i q_i. \tag{42}$$

The coefficients of v_k^2 cancel each other out and, therefore, v_k is simply given by

$$v_k = \frac{c_k \sum_{i=k+1}^{n} p_i q_i}{\sum_{i=k+1}^{n} \left(q_i - \frac{1}{r}\right) + \frac{1}{2}}. \tag{43}$$

This solution is valid only when $v_k \geq 1$, because if $v_k < 1$, then $\epsilon_i^2 < 0$. This cannot hold, because ϵ_i is a real number. The rest of the v_is, for $i < k$, are computed using (40). Finally, after the vectors $\underline{v}(k, n)$ are found for all k and n, and the global maximum $\underline{v}_{\text{opt}}$ is determined using (38), we only have to convert the continuous solution to an integer one, in which all v_is are integers obtained from the corresponding real value using rounding and, in addition, the expression inside the ceiling function in (23) takes on an integer value. If the latter is not true, then the effective overhead ratio r' (i.e., the number of total blocks sent, divided by the minimum required number of blocks that must be received, so that the original signal can be reconstructed) would be less than the overhead ratio r employed by

$$r' = \frac{\sum_{i=1}^{n} v_i}{\left\lceil \frac{1}{r} \sum_{i=1}^{n} v_i \right\rceil} < \frac{\sum_{i=1}^{n} v_i}{\frac{1}{r} \sum_{i=1}^{n} v_i} = r. \tag{44}$$

In the rest of this section, we will give an example of how the calculation of the optimal block allocation is carried out in practice. We assume $r = 2$, $n_{\max} = 9$, and $\underline{q} = [0.85, 0.7, 0.7, 0.7, 0.7, 0.7, 0.7, 0.7, 0.7]$. We will calculate the optimal block allocation for $n = 9$. In Table I, we list the values of v_k, for $1 \leq k < 9$, as they were calculated using (43). Only for $k = 1$, the value is greater than 1. Thus, the following is the only feasible solution, together with the uniform solution (obtained, if we choose $\epsilon_i = 0$ for all i):

- $\underline{v}(8, 9) = [2.1961, 1, 1, 1, 1, 1, 1, 1, 1]$, with $P_c = 0.9709$;
- $\underline{v}(0, 9) = [1, 1, 1, 1, 1, 1, 1, 1, 1]$, with $P_c = 0.9658$.

If we run the same algorithm for all possible n, we still find that the optimal solution is $\underline{v}_{\text{opt}} = [2.1961, 1, 1, 1, 1, 1, 1, 1, 1]$,

TABLE I
VALUES v_k OBTAINED FROM (43)

k	v_k
1	2.1961
2	0.7368
3	0.7059
4	0.6667
5	0.6154
6	0.5455
7	0.4444
8	0.2857

with $P_c = 0.9709$. This solution must be converted to an integer solution, which is obviously $\underline{v}_{\rm opt} = [2, 1, 1, 1, 1, 1, 1, 1, 1]$, with $P_a = 0.9708$. The exact value of the probability of success for $\underline{v} = \underline{v}_{\rm opt}$ is $P_{\rm succ} = 0.9613$.

Finally, the algorithm we developed in this section in order to find an approximation of the maximum of $P_{\rm succ}$ can be summarized as follows. As a first step, we derive an extension of the approximation P_a (see (23)), so that the general case of nonuniform allocation is included. We find an approximation of the maximum of $P_{\rm succ}$ by calculating the maximum of P_a. The real extension of the problem of maximizing P_a is solved and all the local extrema $\underline{v}(k, n)$ are computed

$$\underline{v}(k, n) = [v_1, v_2, \ldots, v_k, 1, 1, \ldots, 1],\ 0 \le k < n \tag{45}$$

where

$$v_k = \frac{c_k \sum\limits_{i=k+1}^{n} p_i q_i}{\sum\limits_{i=k+1}^{n} \left(q_i - \frac{1}{r}\right) + \frac{1}{2}} \tag{46}$$

and for $i < k$, $v_i = c_i/c_k v_k$. The optimal solution $\underline{v}_{\rm opt}$ is the maximum of all local extrema, that is for all k and n

$$\underline{v}_{\rm opt} = \max_{0 \le k < n \le n_{\max}}{}^{-1} \{ f(\underline{v}(k, n)) \}. \tag{47}$$

Note that for $k = 0$ we get the uniform solution. This solution is rounded to the closest integer solution, so that the expression inside the ceiling function in (23) takes on an integer value.

A. Correlation Among Paths

In this section, we extend the formula of $P_{\rm succ}$ in order to cover the case where the multiple paths provided by a route discovery, update, repair, and maintenance mechanisms demonstrate some sort of correlation, as they might share nodes or links. As it is explained in [19], this is not an uncommon event due to short-term variations in the amount of local traffic and "topological bottlenecks." Although in [19] diversity of routes is increased through diversity injection, there might still exist correlated paths in the acquired multipath set, especially if we require so many routes that the "diversity capacity" of the network is exceeded.

As we argued in [1], the problem of finding the optimal distribution over the available paths is reduced to a path selection problem, because there is no point in reusing paths, i.e., sending more than one block on any path. Consequently, in our analysis we assume that n paths are used and each of them conveys one block.

The problem of developing an approximation for $P_{\rm succ}$ is a variant of the problem presented in Section III-B, where the random variable Z, defined in (16), is a sum of random variables Z_i [see (15)] that are not independent. Z is normally distributed with the same mean value

$$\mu(n) = \sum_{i=1}^{n} q_i \tag{48}$$

and a standard deviation given by the following:

$$\sigma^2(n) = \sum_{i=1}^{n} p_i q_i + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \operatorname{cov}(Z_i, Z_j) \tag{49}$$

where $\operatorname{cov}(Z_i, Z_j)$ is the covariance of random variables Z_i and Z_j

$$\operatorname{cov}(Z_i, Z_j) = E[Z_i Z_j] - E[Z_i]E[Z_j] = E[Z_i Z_j] - q_i q_j. \tag{50}$$

Let us assume two paths, denoted as i and j, which have some links in common. The behavior of the paths is modeled through the random variables Z_i and Z_j, as explained in Section III-B. Having that in mind, we can compute the mean value of $Z_i Z_j$ as

$$\begin{aligned} E[Z_i Z_j] &= \sum_{z_i=0}^{1} \sum_{z_j=0}^{1} z_i z_j \Pr\{Z_i = z_i, Z_j = z_j\} \\ &= \Pr\{Z_i = 1, Z_j = 1\}. \end{aligned} \tag{51}$$

The success probabilities of paths i and j are q_i and q_j, respectively. Also, the probability that all common links succeed is $q_{\rm ij}$ ($q_{\rm ij} \ge \max\{q_i, q_j\}$), hence

$$\Pr\{Z_i = 1, Z_j = 1\} = \frac{q_i q_j}{q_{\rm ij}}. \tag{52}$$

Using the equations derived earlier, we are able to calculate the covariance of random variables Z_i and Z_j

$$\operatorname{cov}(Z_i, Z_j) = \frac{p_{\rm ij}}{q_{\rm ij}} q_i q_j \tag{53}$$

where $p_{\rm ij} = 1 - q_{\rm ij}$. Thus, the approximation of $P_{\rm succ}$ is written again as

$$P_a^{(u)}(n) = \frac{1}{2} + \frac{1}{2} \cdot erf\left(\frac{\mu(n) - \left\lceil \frac{n}{r} \right\rceil + \frac{1}{2}}{\sigma(n)\sqrt{2}} \right) \tag{54}$$

and its continuous approximation as

$$P_c^{(u)}(n) = \frac{1}{2} + \frac{1}{2} \cdot erf\left(\frac{\mu(n) - \frac{n}{r} + \frac{1}{2}}{\sigma(n)\sqrt{2}} \right). \tag{55}$$

In the remainder of this section, we will demonstrate how the covariance between paths can be calculated, when they share

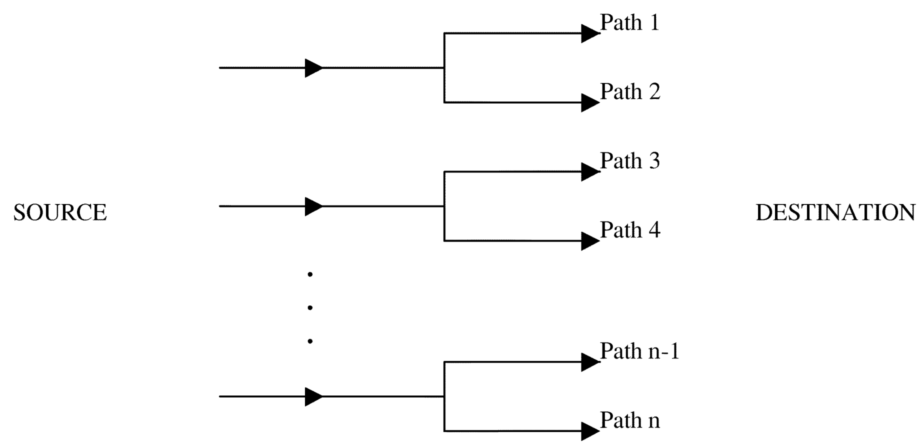


Fig. 11. Example of pairs of correlated paths.

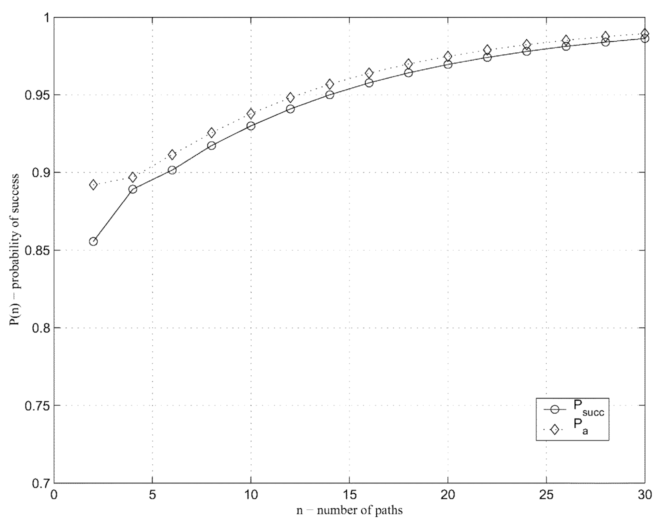


Fig. 12. $P_{\rm succ}$ and its approximation for $r = 2$, $q = 0.7$, and $q_0 = 0.9$.

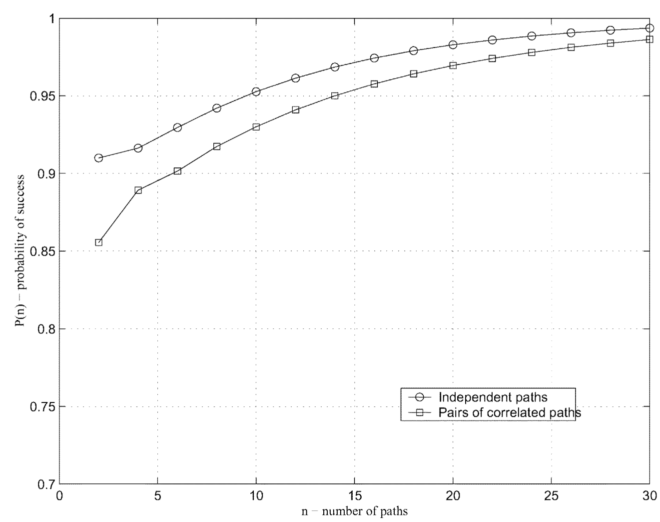


Fig. 13. $P_{\rm succ}$ for independent and correlated paths for $r = 2$, $q = 0.7$, and $q_0 = 0.9$.

links in common. We will also present a graph, so that the values of $P_{\rm succ}$ and its approximation can be compared.

In our example, we assume $n/2$ pairs of correlated paths. Each of the n total paths is composed of two links and has the same probability of success q. The n paths are organized in $n/2$ pairs, whose paths share one of their two links. The common links have a probability of success q_0. The topology is depicted in Fig. 11. From (53), it follows that

$$\mathrm{cov}(Z_i, Z_j) = \frac{p_0}{q_0} q^2 \tag{56}$$

where $k = 1, \ldots, n/2$ and $p_0 = 1 - q_0$ and, therefore, the standard deviation is

$$\sigma^2 = npq + n\frac{p_0}{q_0} q^2. \tag{57}$$

The mean value is $\mu = nq$, so the approximation of $P_{\rm succ}$ is

$$P_a^{(u)}(n) = \frac{1}{2} + \frac{1}{2} \cdot erf\left(\frac{nq - \left\lceil \frac{n}{r} \right\rceil + \frac{1}{2}}{\sqrt{2}\sqrt{npq + \frac{nq^2 p_0}{q_0}}}\right). \tag{58}$$

In Fig. 12, we show $P_{\rm succ}$ and P_a for $r = 2$, $q = 0.6$, and $q_0 = 0.9$. In Fig. 13, we compare the approximations of $P_{\rm succ}$ for the topology of Fig. 11 ($n/2$ pairs of correlated paths) and the one in which we have n independent paths at our disposal. As expected, the latter yields higher probability of success, but in both cases $P_{\rm succ}$ converges to 1, as n is large.

The algorithm we are using in order to select the set of paths, belongs to the family of greedy algorithms. The objective of the algorithm is to maximize the argument of the error function in (55)

$$g(L) = \frac{\mu(L) - \frac{|L|}{r} + \frac{1}{2}}{\sigma(L)} \tag{59}$$

where L is the selected set of paths and $|L|$ the size of this set, i.e., the number of used paths. It has to be pointed out that if there is correlation among the paths, there can be no ordering of the paths according to the failure probabilities. Therefore, starting by choosing the best path, we must always decide on the path we will use next. This is why a greedy algorithm is applicable. The formal definition of the process follows. Initially, the set of selected paths consists of the path with the maximum success probability

$$L^0 = \left\{ \max_{1 \le i \le n}{}^{-1}\{q_i\} \right\}, g^0 = g(L^0). \tag{60}$$

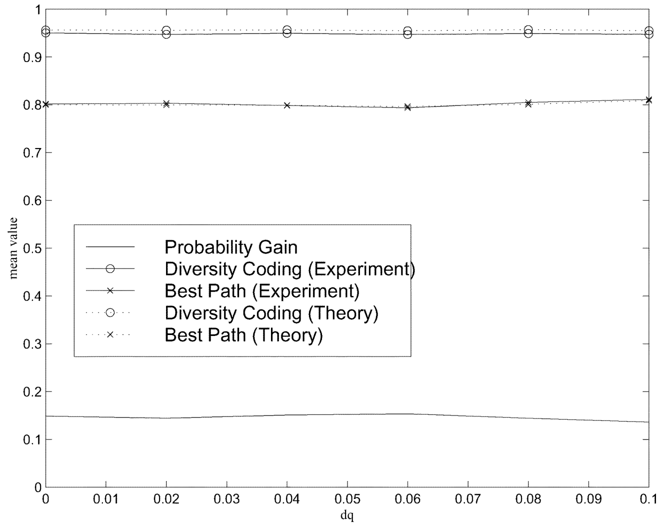


Fig. 14. Mean value of P_{succ} and probability gain.

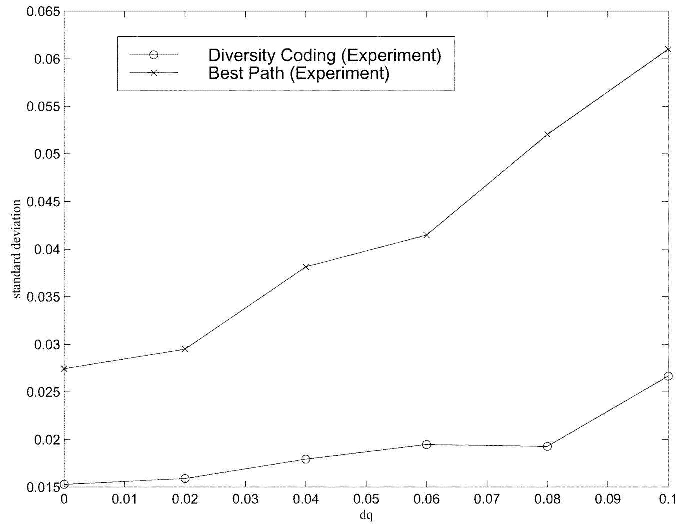


Fig. 15. Standard deviation of P_{succ}.

Given an already selected set L^k at iteration k, the next path l^{k+1} that is selected from among the remaining paths is the one that yields the highest value of function g

$$l^{k+1} = \max_{l \notin L^k}{}^{-1}\{g(L^k \cup \{l\}\} \tag{61}$$

and, based on that path, the next set and the corresponding value of function g are computed

$$L^{k+1} = L^k \cup \{l^{k+1}\}, g^{k+1} = g(L^{k+1}). \tag{62}$$

The process stops when k equals the number of paths. The number of paths is

$$n_{\text{opt}} = \max_{1 \le k \le n}{}^{-1}\{g^k\}. \tag{63}$$

Therefore, the optimal set is the one acquired at the n_{opt}th step of the algorithm

$$L_{\text{opt}} = L_{n_{\text{opt}}}. \tag{64}$$

V. Numerical Results

In this section, we present various numerical results of evaluation scenarios. In the first case, the multiple paths from source to destination are independent, whereas in the second one they share links in common. The overhead factor is $r = 3/2$ and the mean value of the path success probability is $q = 0.8$ in all the cases. The number of available paths is assumed to be $n_{\max} = 18$. Our scheme is compared to the simple scheme in which routing is performed over one single route, in terms of the successful transmission probability (P_{succ}).

A. Independent Paths

Our evaluation program constructs $n_{\max}$ independent paths. The success probability of each path is a random variable with mean value q. However, we assume an uncertainty in the probabilities q_i, uniformly spread over the range $[q - dq, q + dq]$. Thus, the sender is not aware of the exact probability values and, therefore, it cannot distinguish between the paths. The proposed scheme uses all the paths and the single path scheme picks up a route at random. In Fig. 14, we show the mean value of P_{succ} (both the value expected in theory and the one obtained from our program) for both schemes. It is clear that our scheme performs much better than the single path scheme. The gain in probability, defined as the increase in P_{succ}, is around 0.15. Moreover, as Fig. 15 proves, the standard deviation in the value of P_{succ} is significantly bigger in the case where one path is used. Thus, there is undesired fluctuation in the performance of the single path scheme, which, in turn, makes the task of guaranteeing quality of service (QoS) even harder. The performance of our scheme is only slightly affected in terms of stability.

TABLE II
EVALUATION PARAMETERS

Parameter	Value
h	8
dh	2
c	0..3
dc	0
q_l	0.972
dq_l	0.020

B. Correlated Paths

In the second part of the evaluation, we constructed n paths that have links in common. The following are the characteristics of the paths:

1) random path length of mean value h, uniformly distributed over the range $[h - dh, h + dh]$;
2) random number of common links of mean value c, uniformly distributed over the range $[c - dc, c + dc]$;
3) random link success probability of mean value q_l, uniformly distributed over the range $[q_l - dq_l, q_l + dq_l]$.

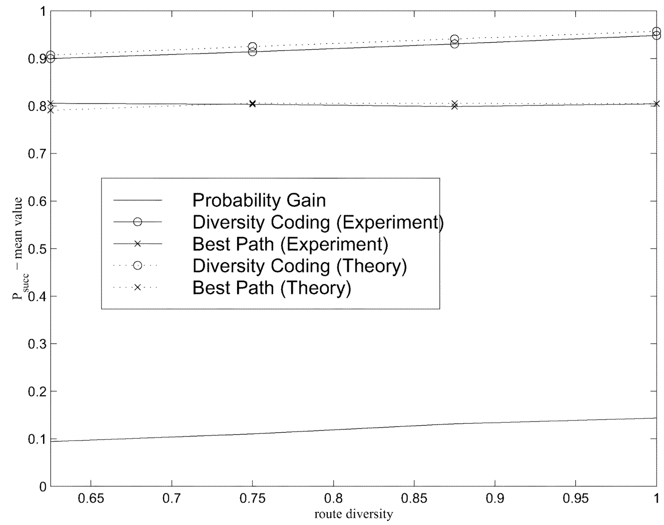


Fig. 16. Mean value of $P_{\rm succ}$ and probability gain.

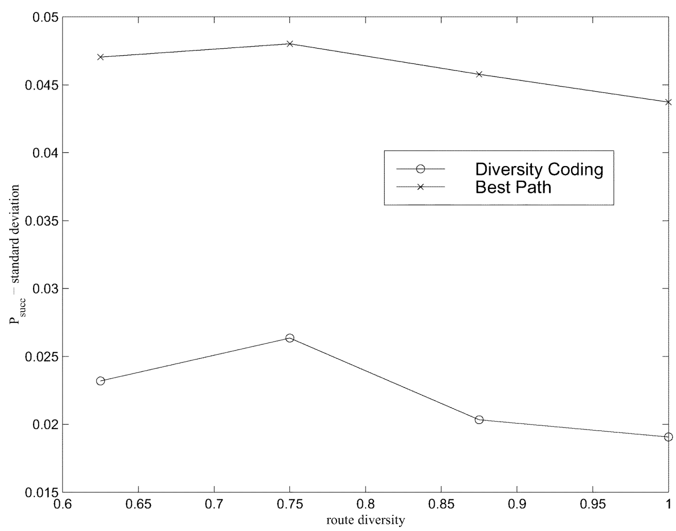


Fig. 17. Standard deviation of $P_{\rm succ}$.

In our experiment, we used $n_{\max} = 18$ paths with a mean success probability of $q = 0.8$. The rest of the evaluation parameters are shown in Table II.

The diversity was quantified as the ratio of the number of unique links among the multiple paths to the total number of links. In Figs. 16 and 17, we can observe the mean value and the standard deviation of $P_{\rm succ}$ for the two schemes. It is also implied in the graphs that the multitude of paths can compensate for the lack of diversity, even when it is as low as 65%. At the same time stability (i.e., lower standard deviation) is provided. As expected, $P_{\rm succ}$ in the multipath scheme increases when diversity increases, while in the single path scheme diversity plays no role at all.

VI. Conclusion and Future Work

In this paper, we extended the evaluation and optimization methods for the scheme proposed in [1]. Given the path failure probabilities, the overhead factor r, and the allocation of the original and overhead blocks to the $n_{\max}$ paths, we were able to approximate the probability function $P_{\rm succ}$, namely, the probability that the original packet can be reconstructed at the destination. We showed how to maximize $P_{\rm succ}$ (in terms of the block allocation) fast enough (Section III), so that the requirement for a real-time recalculation of the optimal solution, due to topology changes, could be met.

Our evaluation showed that our scheme can significantly increase the successful transmission probability, compared to a simple packet replication scheme. Even in the case where only limited knowledge about the probability vector is available (mean value and standard deviation), our scheme can still compute $P_{\rm succ}$ and, thus, evaluate its performance. Under some constraints on the path failure probabilities, it was found that the probability of a successful communication of packets between source and destination increases with the number of used paths. Moreover, this would effectively reduce transmission delay and traffic congestion through load balancing.

The proposed scheme can also be used to enforce error-rate QoS requirements, whenever the characteristics of the offered paths make it possible. In that case, we do not have to maximize $P_{\rm succ}$, but, instead, simply set it to the required probability (indicated by the QoS requirements) and then find the number of paths and the block allocation that satisfies it. This could make real-time data transmission feasible in an environment that is hostile to such type of communication. Moreover, by keeping track of the probability of success and by constantly comparing it with the QoS requirement, we obtain a metric that may be used in order to trigger new route discoveries, for example if $P_{\rm succ}$ tends to drop below the requirements.

By extending the definition of the path failure probabilities, we could enforce different classes of QoS requirements, such as maximum delay requirements. This can be done, by simply defining the path failure probability, as the probability that a packet will not arrive on time, i.e., within the maximum delay time, and, as a result, we assume it is lost.

We will take a moment here to comment on the impact the overhead associated with diversity coding has on the overall performance. The scheme proposed in this paper intends to mitigate the effects of frequent topological changes in *ad hoc* networks. In real-time traffic applications it is often unacceptable to lose connectivity because of changes in topology that render routes obsolete. Clearly, the kind of multipath routing proposed in this paper guarantees reliable end-to-end connectivity despite the inherent unreliability of the wireless *ad hoc* network. Of course, this is done at the price of increased traffic load, which means that fewer connections can be supported at a certain quality of service (as defined by $P_{\rm succ}$) and at a certain network capacity level. In other words, traffic load can always be controlled by accommodating fewer connections, without having to compromise on the quality of the connections.

Possible goals for future research include the following:

- evaluation of the proposed scheme when used for achieving load balancing and satisfying delay constraints;
- development of algorithms in order to estimate the probability vector $\underline{p}$ on a real-time basis;
- implementation of our scheme on top of existing routing protocols and comparative performance evaluation.

REFERENCES

[1] A. Tsirigos and Z. J. Haas, "Analysis of multipath routing, Part 1: The effect on the packet delivery ratio," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 138–146, Jan. 2004.

[2] E. Ayanoglu, C.-L. I., R. D. Gitlin, and J. E. Mazo, "Diversity coding for transparent self-healing and fault-tolerant communication networks," *IEEE Trans. Commun.*, vol. 41, pp. 1677–1686, Nov. 1993.

[3] C. E. Perkins and P. Bhagwat, "Highly dynamic destination-sequenced distance-vector routing (DSDV) for mobile computers," in *Proc. Conf. Communications Architecture, Protocols and Applications (ACM/SIGCOMM'94)*, Aug. 1994, pp. 234–244.

[4] D. Johnson and D. Maltz, "Dynamic source routing in *ad hoc* wireless networks," in *Mobile Computing*, T. Imielinski and H. Korth, Eds. Norwell, MA: Kluwer, 1996.

[5] C. Perkins. (1997) Ad hoc on demand distance vector (AODV) routing. Internet draft. [Online]draft-ietf-manet-aodv-00.txt

[6] V. D. Park and M. S. Corson, "A highly adaptive distributed routing algorithm for mobile wireless networks," in *Proc. 18th Annu. Joint Conf. IEEE Computer and Communications Societies (INFOCOM)*, 1999, pp. 1405–1413.

[7] Z. J. Haas and M. R. Pearlman. (1999) The zone routing protocol (ZRP) for ad hoc networks. Internet draft. [Online]draft-ietf-manet-zone-zrp-02.txt

[8] A. Nasipuri and S. R. Das, "On-demand multipath routing for mobile ad hoc networks," in *Proc. 8th Annu. IEEE Int. Conf. Computer Communications and Networks (ICCCN)*, 1999, pp. 64–70.

[9] J. Raju and J. J. Garcia-Luna-Aceves, "A new approach to on-demand loop-free multipath routing," in *Proc. 8th Annu. IEEE Int. Conf. Computer Communications and Networks (ICCCN)*, 1999, pp. 522–527.

[10] S. J. Lee and M. Gerla, "AODV-BR: Backup routing in ad hoc networks," in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, 2000, pp. 1311–1316.

[11] L. Wang, L. Zhang, Y. Shu, and M. Dong, "Multipath source routing in wireless ad hoc networks," in *Proc. Can. Conf. Electrical and Computer Engineering*, vol. 1, 2000, pp. 479–483.

[12] S. J. Lee and M. Gerla, "Split multipath routing with maximally disjoint paths in ad hoc networks," in *Proc. Int. Conf. Communications (ICC 2001)*, 2001, pp. 3201–3205.

[13] V. W. S. Chan and A. H. Chan, "Reliable message delivery via unreliable networks," in *Proc. 1997 IEEE Int. Symp. Information Theory*, Ulm, Germany, June 29– July 4, 1997.

[14] A. B. McDonald and T. Znati, "A path availability model for wireless ad-hoc networks," in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, vol. 1, 1999, pp. 35–40.

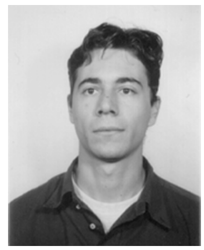
[15] C.-K. Toh, "Associativity-based routing for ad-hoc mobile networks," *Wireless Pers. Commun. J.*, vol. 4, pp. 103–139, Mar. 1997.

[16] R. Dube, C. D. Rais, W. Kuang-Yeh, and S. K. Tripathi, "Signal stability-based adaptive routing (SSA) for ad hoc mobile networks," *IEEE Personal Commun.*, pp. 36–45, Feb. 1997.

[17] W. B. Davenport Jr., *Probability and Random Processes: An Introduction for Applied Scientists and Engineers*. New York: McGraw-Hill, 1970.

[18] T. C. Fry, *Probability and its Engineering Uses*, 2nd ed. Princeton, NJ: Van Nostrand, 1965, pp. 229–234.

[19] M. R. Pearlman and Z. J. Haas, "Improving the performance of query-based routing protocols through 'Diversity injection'," in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, vol. 3, 1999, pp. 1548–1552.



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