ECE4960 **Spring 2013**

Calculation of the Reuse Partitioning Gain

Zygmunt J. Haas
Wireless Networks Lab
School of Electrical and Computer Engineering
Cornell University, Ithaca, NY 14853
e-mail: *haas@ece.cornell.edu*
wnl.ece.cornell.edu

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We refer to the inner cells as tier-0 network and to the cell areas outside the inner cells as tier-1 network.

The reuse factor of tier-1 is 7. Thus, since $\frac{D}{R} = \sqrt{3 \cdot N} = \sqrt{21} = 4.6$:

$$\frac{D_1}{R_1} = 4.6 \tag{1}$$

Similarily, since $\sqrt{3 \cdot 3} = 3$:

$$\frac{D_0}{R_1} = 3. \tag{2}$$

Moreover, since the tier-0 network has to have the same SIR as tier-1 and since the SIR is determined by the ratio of $\frac{D}{R}$ only, we also have:

$$\frac{D_0}{R_0} = 4.6 \tag{3}$$

Consequently, using (2) and (3), we obtain that

$$\frac{\frac{D_0}{R_1}}{\frac{D_0}{R_0}} = \frac{R_0}{R_1} = \frac{3}{4.6} \overset{\text{def}}{=} \alpha \tag{4}$$

Now, assume that there are altogether C_{total} number of channels in the whole system. These channels are divided as follows: C_0 to tier-0 and C_1 to tier-1; i.e.,

$C_{total} = C_0 + C_1$. (Note, C_0 and C_1 are channels assigned to **all** the cells in the tier-0 and the tier-1 networks, respectively.) Because tier-0 has reuse of 3 and tier-1 has reuse of 7, the actual numbers of channels in a **single** cell are: $\frac{C_0}{3}$ and $\frac{C_1}{7}$ for tier-0 and for tier-1, respectively. We label $c_0 = \frac{C_0}{3}$ and $c_1 = \frac{C_1}{7}$.

We assume that the users are uniformly distributed throughout the whole area. Thus, the fraction of the number of channels assigned to a tier-0 cell relative to the total number of channels assign to a cell is equal to the ratio of the tier-0 cell area to the total cell area. I.e.,

$$\frac{c_0}{c_0 + c_1} = \frac{\frac{C_0}{3}}{\frac{C_0}{3} + \frac{C_1}{7}} = \frac{\frac{3\sqrt{3}}{2} \cdot R_0^2}{\frac{3\sqrt{3}}{2} \cdot R_1^2} = (\frac{R_0}{R_1})^2 = \alpha^2 \tag{5}$$

Since, $C_{total} = C_0 + C_1$, we get:

$$\frac{\frac{C_0}{3}}{\frac{C_0}{3} + \frac{C_{total} - C_0}{7}} = \alpha^2 \tag{6}$$

Solving the last equation yields:

$$C_0 = \frac{\alpha^2}{7(\frac{1}{3} - \frac{\alpha^2}{3} + \frac{\alpha^2}{7})} C_{total} \tag{7}$$

Now, since α=0.65, then $C_0 = 0.24 C_{total}$ and $C_1 = 0.76 C_{total}$.

To calculate the improvement in capacity, we calculate the number of users that could be supported in a regular $N = 7$ network and in the two-tiered network. In a regular $N = 7$ network, the number of channels **per cell** is: $\frac{C_{total}}{7}$. In the two-tiered network, the number of channels per cell is: $\frac{0.24 C_{total}}{3} + \frac{0.76 C_{total}}{7}$. Thus the improvement is:

$$\frac{\frac{0.24 C_{total}}{3} + \frac{0.76 C_{total}}{7}}{\frac{C_{total}}{7}} = 1.32 \; . \tag{8}$$

In other words, the improvement is about 32% of the network capacity.