The Application of Automated Chain Line Pattern (CLiP) Matching to Identify Paper Mouldmate Candidates in Rembrandt's Prints

C. Richard Johnson, Jr., Cornell University/ Rijksmuseum
William A. Sethares, University of Wisconsin
Margaret Holben Ellis, New York University/ Morgan Library & Museum
Saira Haqqi, New York University
Reba Snyder, Morgan Library & Museum
Erik Hinterding, Rijksmuseum
Idelette van Leeuwen, Rijksmuseum
Arie Wallert, Rijksmuseum
Dionysia Christoforou, Rijksmuseum
Jan van der Lubbe, Technische Universiteit Delft
Nadine Orenstein, Metropolitan Museum of Art
Angela Campbell, Metropolitan Museum of Art
Georg Dietz, Independent Paper Expert

October 2014

Summary

This document describes an on-going project initiated in late 2012 to investigate the use of chain line pattern (CLiP) matching, rather than watermarks, for the detection of mouldmates, i.e., sheets made from the same papermaking mould, in Rembrandt's prints. We investigate the application of computer-based, image processing tools to mark, measure, and compare the idiosyncratic intervals of chain lines as recorded in beta-radiographs of these prints in the hunt for mouldmates.

Background

The study of Rembrandt's prints has occupied scholars for over two centuries. With several thousand in existence today, the study of his printing papers occupies a prominent place within this scholarship. Rembrandt's prints were predominantly executed on antique *laid* papers.[1] Until the widespread adoption of the papermaking machine in the early nineteenth-century, paper was made by scooping up macerated and suspended paper pulp from a vat using a rectangular mould comprised of a porous screen surrounded by a removable wooden frame. Prior to 1750, the screen was fabricated from fine, densely spaced horizontal rows of *laid* wires secured by thicker, more widely spaced vertical *chain* wires.[2] When the mould was plunged into the vat and lifted out, the wires acted as a sieve, filtering out the pulp in thinner and thicker accumulations depending upon how much interference they produced as the water drained through. The crisscrossed pattern of chain and laid lines is thus replicated in the final sheet of paper. Because it is thinner in areas corresponding to the wire grid, the laid and chain line pattern can be easily seen when the paper is held up to the light. Two papers will have identical laid and chain line patterns if they have been formed at the same time in the same manner on the same mould – hence they are called *mouldmates*. Because each papermaking mould was made by hand, there are small variations between the precise intervals of chain lines from one screen to the next. Thus, the sequence of intervals between chain lines is considered a viable feature for identifying mouldmate candidates. When there exists an extensive corpus of an artist's work on paper, as is the case with Rembrandt, the identification of mouldmates can help in establishing production chronology, indicate periods of intense activity, and suggest paper preferences or workshop practices.

To date, paper mouldmates have been identified primarily by the ability to exactly superimpose images of each paper's *watermark*. The watermark was made from a wire bent into a simple shape, for example, a star, shield, or monogram that was stitched onto the heavier supporting chain lines. The watermark could designate the paper's manufacturer, country of origin, date, size, or function. Just like the laid and chain wires, the watermark wire impeded the rate and quantity of pulp as it drained through the screen; hence, an image of the watermark was also left in the paper.

As stated above, watermarks and laid and chain line patterns are easily viewed via transmitted light. In fact, transmitted light photography has frequently been used to record paper morphology. However, the heavily inked designs of the prints themselves frequently prohibit an unobstructed reading of the paper structure. Furthermore, the resulting photograph is not to scale, making the superimposition of watermarks and measuring of chain line intervals unreliable. For the same reasons, hand-drawn tracings of the watermark, chain line intervals, and laid line frequency are invalid for the purpose of comparison.

A more exact and convenient technology for accurately recording paper structure is the use of beta-radiography or low-energy x-radiography. The resulting radiograph is to scale and the weak radiation easily penetrates most printing inks, even those to which radio-opaque lead driers were added to hasten drying times. **Figure 1** illustrates a beta-radiograph image of the watermark found in Rembrandt's *Medea, or the Marriage of Jason and Creusa* (B112iv). It is known as a *foolscap* and depicts the head of a jester wearing a five-pointed ruff surmounting a cross emerging from three spheres. Twenty-one variants of foolscap watermarks found in Rembrandt prints have been catalogued; one has nine subvariants, of which one appears here.[3] This particular subvariant is found in sixteen different prints by Rembrandt.[4] A hand-drawn rendering of this watermark also appears in **Figure 1**.

In Rembrandt scholarship, emphasis has been placed on watermark recording and identification as a research tool.[5] A significant problem, however, is the prevalence of non-watermarked papers. Indeed, among Rembrandt prints, approximately two-thirds lack a watermark or even a fragment of one.[6]

Chain lines, not just the two or three spanning the watermark, have long been promoted as key identifying features of paper and, more specifically, the unique chain spacing sequence across a full sheet of paper has been proposed as an identifier in the absence of a watermark.[7] The drawback of using chain line spacing sequences for identifying mouldmates, however, is the difficulty of accurately measuring and systematically recording them.[8] Furthermore, accumulating a statistically meaningful library of chain line spacings for comparison represents enormous time and effort with a high probability of human error. Utilization of computer-based image processing raises the possibility of an automated approach.[9]

Four PhD dissertations at TU Delft have addressed this route, with the most recent examining the possibility of characterizing and exploiting average laid line density – the number of laid lines per centimetre – and chain line spacings.[10] Presumably, unique chain line spacing sequences in handmade moulds can be used to 'fingerprint' a paper made from one specific mould. To test this hypothesis the problem was viewed as the application of two basic image/signal processing tasks. The first task was to locate the chain lines; the second was to use the unique chain line patterns to identify potential mouldmates.

Three streams of concurrent research commenced, involving signal processing engineers, conservators, curators, art historians, and graduate students.

- Firstly, the image processing algorithms needed to be developed and tuned in order to test the thesis that chain line pattern (CLiP) matching alone could be used to reduce a large library of images of chain line patterns of different papers to a manageable set of identified paper mouldmate candidates.
- Secondly, a potential complication needed to be addressed. It was pointed out that either the intaglio process used to create Rembrandt's prints or subsequent conservation treatments, such as washing and flattening, could potentially alter the chain line configuration.[11] Could these creative and conservation processes have an impact on chain line pattern spacings?
- Finally, CLiP matching needed to be applied to hundreds of existing beta-radiographs of Rembrandt prints. This necessitated collecting images, initially from three participating institutions, the Morgan Library & Museum, the Rijksmuseum, and the Metropolitan Museum of Art.

A Visual Description of Chain Line Pattern (CLiP) Matching

To provide a visualization of the computational procedure pursued in assessing the degree of similarity of a pair of chain line pattern spacings, we begin with the chain line pattern of (a portion of) a particular mould, as illustrated in **Figure 2**. Observe the similar, but unequal, spacings between nearly, but not exactly, parallel lines. The fourth line from the left (or right) in **Figure 2** is the only line that is noticeably tilted.

Consider two pieces of paper made separately on this screen, the chain line patterns for which appear in **Figure 3**. The thicker spot near the top of the leftmost line in the top of **Figure 3** can be seen as the same artefact near the top of the leftmost line in **Figure 2**. A similar thick spot/artefact near the centre of the second line from the left in **Figure 2** can be related to this feature near the bottom of the second line from the left in the top pattern in **Figure 3** and to the same feature in the upper half of the first line in the bottom pattern in **Figure 3.** This indicates that paper A is from the upper left portion of the mould and paper B is shifted to the right one line and down relative to paper A. This observation is unknown to the procedure.

Our approach to checking mould-mate status for these two papers (while unaware that they are mould-mates) lines up the leftmost line in each image and then considers the similarity of the chain space sequence of the two images. Full assessment of a pattern match will require match examination for vertical and horizontal shifts. To avoid having to compare and horizontally shift, we instead extract all possible patterns of four adjacent lines form the original images, which yields four chain line patterns in **Figure 4**. Overlaying the chain line patterns of the four possible configurations between the two sets of spacing triples is illustrated in **Figure 5**.

If A1 matches B1, then we know they share the leftmost line in the chain line pattern of **Figure 2** and the subsequent four chain lines to the right. If A1 matches B2, then the leftmost line in paper A in **Figure 3** matches the line to the right of the leftmost in paper B in **Figure 3**. Similarly, A2 matching B1 indicates that the leftmost line in paper B is the same chain line as the line to the right of the leftmost in paper A. Finally, A2 matching B2 indicates the same relative configuration as a match between A1 and B1.

Of the four images in **Figure 5** only the match between the second spacing triple in paper A, i.e. A2, and the first spacing triple in paper B, i.e., B1, in the bottom left of **Figure 5** appears close. The three essentially vertical lines are matched precisely. The single pair of noticeably tilted lines is separate, but parallel and close. Such a pair of parallel tilted lines suggests that a vertical relative shift will achieve a complete match, as shown in **Figure 6**, which realizes their original configuration in **Figure 2**.

This sequence of comparisons omits the various rotations and flips that would be needed to examine all possibilities of paper orientation during printing and during the beta-radiography. These flips are included in the full procedure developed to test the potential utility of relying on matching chain spacing sequences alone to produce mouldmate candidates.[12] For our library of images with (partial) watermarks including at least four chain lines, we developed Mathematica code that (i) allows us to mark points on the chain lines including near the edges of the image of the paper, (ii) extracts the sub-images of adjacent chainspace triples, and (iii) computes a similarity measure mimicking the visual fit procedure outlined above that is used to rank the closeness of the patterns of pairs of chainspace triples. The same approach could be applied to chainspace quadruples or quintuples.

Differentiating Paper Mouldmates from Paper Twins

The first challenge was to establish that CLiP matching could be used to identify mouldmates in lieu of manually superimposing watermarks. For this a collection of antique laid papers known to have come from one mould is required. It would be virtually impossible to guarantee such a homogeneous accumulation of like papers in a collection of individual prints and drawings; however, our bibliophile colleagues have long known that bound books routinely contain mouldmates. Book and manuscript scholars have recognized for some time that bound volumes of paper naturally reflect the industrial process of papermaking. Using an alternating pair of moulds most efficiently produces handmade paper.[13] As seen in **Figure 7**, the vatman plunges one mould into the tub of paper pulp, brings it up, and passes the mould to his helper. Having done so, he picks up a second *twin* mould, which has now been emptied by his helper, and dips it into the tub, and passes it back to his helper, who has already emptied and returned the first one, so continuing the rhythmic cycle of making individual sheets of paper. The second twin mould was watermarked in the same manner as the first. However, as has been pointed out, small variations existed between the wires of the watermarks, as well as the chain line sequences. Since a day's production was processed sequentially and *en masse* - pressed, dried, sized, finished, graded, and packaged - it is likely that the final ream of paper will contain papers from at least two moulds. That is, the ream will contain both mouldmates (papers formed from the same mould) and twins (papers formed in the alternate mould). By extension, a book made from that ream will presumably contain both mouldmates and twins from a limited number of moulds belonging to one mill.

Such a volume was found in the collection of the Conservation Center, Institute of Fine Arts, New York University (**Figure 8**). It is a partial sixteenth-century blank ledger made from a northern Austrian paper made at the Kremsmunster paper mill on the River Krems. The Wurm family worked the mill from its foundation by the Benedictine monastery of Kremsmunster in 1542 until the paper mill finally closed in 1860. Two versions of its watermark, a crowned double-eagle holding a shield with the letter K, can be seen in **Figure 8.** The paper appears to date from the 1570s or 1580s.[14] Throughout the entire textblock only two versions of the watermark appear indicating that it is comprised of papers made from only two moulds – A and B.

Table I is a collation chart of the ledger, which illustrates the sequence of leaves made from the Kremsmunster paper from Mould A and Mould B. Beta-radiographs were taken of 28 different pages of the ledger. Chain line pattern matching successfully matched the mouldmates and differentiated them from their twins.

TABLE I
Collation Chart of Ledger

Key:	Mould A	V
	Mould B	U
	V or U	Leaf of a gathering[15]

V	Innermost leaf of gathering
V	Leaf of gathering outside innermost leaf
V	Leaf of gathering inside outermost leaf
V	Outermost leaf of gathering
1	Gathering number counted from front of book

V	U	V	U	V	V	U
V	U	V	V	U	U	V
U	V	V	U	V	V	U
U	U	V	U	V	U	U
1	**2**	**3**	**4**	**5**	**6**	**7**
						V
V	U	V	U	U	U	U
U	U	U	U	U	U	V
U	U	U	U	V	U	U
V	U	U	V	U	U	V
8	**9**	**10**	**11**	**12**	**13**	**14**
U	U	V	V	V	U	V
U	U	U	V	U	V	U
V	V	V	V	U	U	U
U	U	U	U	V	U	V
15	**16**	**17**	**18**	**19**	**20**	**21**
U	U	V	U	V	U	V
V	V	U	U	U	U	U
U	U	V	V	V	U	V
V	V	V	V	V	U	U
22	**23**	**24**	**25**	**26**	**27**	**28**

This first phase established the validity of automated CLiP matching for differentiating papers made from two moulds without watermark comparison.

The Impact of Creative and Conservation Processes on Chain Line Patterns

A concern raised by paper conservators and printmaking experts about this approach to mouldmate identification is the potential for the chain line pattern geometry to be altered by the wetting and pressure undergone by the paper during the intaglio printing process and certain subsequent conservation treatments, such as washing and flattening. Paper that undergoes intaglio printing is first fully wet and then subjected to tremendous pressure when run through the press. Papers that are washed and flattened during conservation treatments likewise could exhibit modified chain line intervals. The implications are obvious. If printing processes and typical conservation treatments affect the morphology of paper, then it is impossible to match papers that have undergone such procedures. This also means that some conservation treatments, such as washing and flattening, can lead to a very specific loss of information that must be taken into account before a decision is made to implement certain treatments.

To examine this concern, a series of experiments were designed to evaluate the impact of intaglio printing and typical paper conservation treatments. A separate group of papers from the same book were then subjected to printing and conservation treatments that included wetting and drying.

This group included four leaves in the first gathering, with every other page being numbered at the top right corner. Beta-radiographs of all pages were taken before their removal from the book, with the position of the beta plate marked in pencil on each page. These radiographs were marked with the page number, the felt/wire side of the paper, the mould (A or B), and labelled BT (Before Treatment).

Two leaves (one with pages numbered 257 and 258, the other with pages numbered 256 and 259) were dry-cleaned with fine white vinyl eraser crumbs and immersed in deionized water for one hour. On removal from the bath, they were placed on a blotter to air dry for fifteen minutes, after which they were placed between blotters under approximately 70 lbs. of weight. They were left there for one week, after which they were removed and beta-radiographs again taken of each page, with care taken to place the beta plate in the same area which it had occupied for the first round of radiography. These radiographs were labelled with the same information as above, but marked AT (AW) for After Treatment (After Washing).

The other two leaves (one with pages numbered 254 and 261, the other with pages numbered 255 and 260) were folded into folios and immersed in deionized water for twenty minutes. They were then blotted. Each folio was placed over an un-inked intaglio plate and run through a small intaglio press under newsprint and felts, as in any print shop. For one folio, page 254 was placed facedown against the plate; for the other, page 255 was placed facedown against the plate. The edge towards the head of the book was first through into the press.

After going through the intaglio press, the folios were unfolded and pinned to a wooden board to dry, as would have been common in traditional print shops. Once they were dry, each of the pages was radiographed again with the beta plate in the same position on the page as it had been in during the first round of radiography. The radiographs were again labelled with page number, mould, and felt/wire side, as well as the caption AP (After Printing).

The printed pages were then immersed for an hour in deionized water, air dried for around ten minutes and blotted. They were then placed under weight in the same manner as the unprinted pages were earlier, and allowed to dry for a week. Upon removal from the drying stack, each of the pages was beta-radiographed and the radiographs labelled as before, but with the caption AP+AW or After Printing and After Washing.

All of the above can be seen in the following table:

TABLE II

Page	Mould	Label
254	B	BT
		AP
		AP+AW
255	B	BT
		AP
		AP+AW
256	A	BT
		AT (AW)
257	A	BT
		AT (AW)
258	A	BT
		AT (AW)
259	A	BT
		AT (AW)
260	B	BT
		AP
		AP+AW
261	B	BT
		AP
		AP+AW

The Before and After images were scanned and the endpoints of the chain lines were located semi-automatically. From the locations of the endpoints the relative angles and the shortest distances from a measurement point on the leftmost verticalized chain line were computed. The pairs of relative angle and separation distance from the leftmost chain line to the three or four to the right of it in the original sheet were then compared to the same descriptors of the After Treatment/Printing beta-radiographs. To account for possible flips of the paper during the collection of beta-radiographs all four possible flips were evaluated: (i) no flip, (ii) left-right flip followed by up-down flip, i.e. 180^{o} rotation, (iii) left-right flip, and (iv) up-down flip. The basic result is that the angles were modified by less than one degree and the spacings by less than two per cent. These alterations are within the thresholds we found corresponded to the similarity measured between two separately marked, chain-line-sharing images from the same sheet of paper. Once again chain line pattern recognition matched the mouldmates belonging to Mould A and Mould B. More details on this match evaluation procedure can be found in a companion paper, where the potential that some chain lines may be crooked or bent is discussed.[16] Such lack of straightness may interfere with the proper operation of a mouldmate identification scheme that relies on the chain lines being straight. In our current dataset, non-straight chain lines occur in less than ten per cent of the papers.

Identification of Rembrandt Mouldmates

Automated CLiP matching was carried out on a collection of over ninety beta-radiographs of Rembrandt prints belonging to the Morgan Library & Museum. A chain line pattern match was found between Rembrandt's *Medea, or the Marriage of Jason and Creusa*, (B112iv, **Figure 1**) and a left-right flipped version of *The Artist's Mother in Widow's Dress and Black Gloves* (B344) (**Figure 9**). The close match between the (shifted) chain lines of these two images (with the proper flips) is shown in **Figure 10**. Although this match was identified solely from the close similarity of their chain line patterns, the match of the watermarks can verify it, though B344 has only a fragment of the watermark that is fully contained in B112iv.

The authorship of B344 has been questioned for many years. Most scholars after Adam Bartsch, the eighteenth-century cataloguer of Rembrandt prints, agree that B344 is by a pupil of Rembrandt, who was most likely copying an earlier print, dated circa 1631, *The Artist's Mother Seated at a Table, Looking Right* (B343).[17] The impressions, the beta-radiographs of which appear in **Figures 1** and **9**, are known by the watermark to be restrikes from ca. 1650 (the *Medea* was originally etched in 1648, and *The Artist's Mother in Widow's Dress* probably around 1635). The significance of the chain line pattern match of the beta-radiographs in **Figures 1** and **9** is that it places the pupil's print firmly among Rembrandt's original prints, which makes it very likely that the pupil's print originated in Rembrandt's studio. If it was indeed made around 1635, one can look for pupils who were with Rembrandt at that time.

With the addition of images from the Metropolitan Museum of Art we have formed a collection of 129 scanned beta-radiographs that is now available through the Netherlands Institute for Art History (RKD).[18] In this enlarged set we found another interesting pair that exhibited a chain line pattern match: an impression of the second state of *Christ Crucified Between the Two Thieves: The Three Crosses* (B78) in the collection of the Morgan Museum & Library and an impression of the third state at the Metropolitan Museum of Art. The fact that impressions of these two states can be on paper mouldmates is affirmed with the identification of the shared watermark: Strasbourg Bend variant D'.a.a.[19]

The Way Forward

Our efforts in the immediate future will be centered upon applying our CLiP matching software to a treasure trove of over 1000 images of watermarks and countermarks in the laid paper supports of Rembrandt's etchings, to which the Dutch University Institute for Art History in Florence recently provided us access.[20] As this set of images is drawn from over a dozen print collections there are a significant number of mouldmates. This work will provide a realistic test of the strategy of identifying mouldmate candidates using CLiP matching.

Illustration and Table Captions

Figure 1
Medea, or the Marriage of Jason and Creusa (B112iv) with a beta-radiograph of its foolscap watermark. A hand-drawn version of the watermark is on the right. The chain lines are the vertical features that are spaced approximately one inch apart (see top ruler) in the beta-radiograph. Thanks are extended to Reba Snyder for providing the beta-radiograph of B112. (Etching Photo Credit: The Morgan Library \& Museum, New York. RvR 178. Photography by Graham S. Haber, 2014.)

Figure 2
Illustrative Full-Mould Chain Line Pattern

Figure 3
Hypothetical Paper Samples: top: A (upper left in mould); bottom: B (lower right in mould)

Figure 4
Extracted Chainspace Triplets from 2 Samples:
upper left: A1 - first chain line quadruple in A;
upper right: A2 - second chain line quadruple in A;
bottom left: B1 -first chain line quadruple in B;
bottom right: B2 - second chain line quadruple in B

Figure 5
Chainspace Triplet Comparisons:
upper left: A1 and B1; upper right: A1 and B2;
lower left: A2 and B1; lower right: A2 and B2

Figure 6
Shifted Best Triplet Match (actually lines 2, 3, 4, 5 numbered from left of chain line pattern in Figure 2).

Figure 7
The vatman on the left lifts a mould onto which a freshly made sheet of paper has formed. The workmen to the right transfer the sodden sheet from the old onto a growing pile of papers. Heavy presses used to remove the remaining water from the paper appear in the background. (Diderot, *Encyclopédie*, Plate X).

Figure 8
A sixteenth-century blank ledger book contains papers (top) from only two moulds, A (bottom left) and B (bottom right), which can be differentiated by their watermarks and chain line patterns. (Beta-radiographs courtesy S. Haqqi).

Figure 9
Pupil of Rembrandt, *The Artist's Mother in Widow's Dress and Black Gloves* (B344) is shown on the left. A beta-radiograph of the area near the watermark is shown on the right. Thanks are

extended to Reba Snyder for providing the beta-radiograph of B344. (Etching Photo Credit: The Morgan Library & Museum, New York. RvR 459. Photography by Graham S. Haber, 2014.)

Figure 10
Extracted chain line ‘optimum’ overlay for B112iv (quadruplet #59) and B344 (quadruplet #304).

Table I
Collation Chart of Ledger

Table II
Beta-radiographs of untreated and treated sample papers.

References

Ash, Nancy, and Shelley Fletcher. *Watermarks in Rembrandt's Prints*. Washington: National Gallery of Art, 1998.

Bartsch, Adam. *Catalogue Raisonné de toute les estampes qui forment l’oeuvre de Rembrandt et ceux de ses principeaux imitateurs*. Vienna: A. Blumauer, 1797.

Björklund, George. *Rembrandt’s Etchings: True and False*. Stockholm: Esselte Aktiebolag, 1955.

Carter Hailey, R. "The Dating Game: New Evidence for the Dates of Q4 Romeo and Juliet and Q4 Hamlet." *Shakespeare Quarterly* 58, no. 3 (2007): 367-87.

Diderot, Denis. *Encyclopédie, ou, dictionnaire raisonné des sciences, des arts et des métiers*. Elmsford, N.Y.: Pergamon Press, 1969.

Hind, Arthur M. *A Catalogue of Rembrandt’s Etchings – Chronologically Arranged and Completely Illustrated*. London: Methuen and Co., 1923.

Hinterding, Erik. *Rembrandt as an Etcher: The Practice of Production and Distribution*. 3 vols. Ouderkerk aan den IJssel: Sound & Vision Publ., 2006.

Hinterding, Erik and Jaco Rutgers, *The New Hollstein Dutch and Flemish Engravings, Etchings and Woodcuts, ca. 1450 – 1700. Rembrandt*. 7 volumes. Ouderkerk aan den IJssel: Sound & Vision Publ. 2013.

Hollstein, F.W.H. *Hollstein’s: Dutch and Flemish Engravings, Etchings and Woodcuts, ca. 1450 – 1700*. 72 volumes. Amsterdam: M. Hertzberger, 1949 -– 1987, Roosendaal: Koninklijke Van Poll, 1988 – 1994. Rotterdam: Sound & Vision Interactive, 1995 – 2004, Ouderkerk aan den IJssel: Sound & Vision Publ., 2005 – 2010.

Hollstein, F.W.H. *The New Hollstein: Dutch and Flemish Etchings, Engravings, and Woodcuts, ca. 1450 – 1700*. Amsterdam: M. Hertzberger, 1949 - 2014.

Hunter, D. *Papermaking: The History and Technique of an Ancient Craft*. New York: Dover Publications, 1978. (Unabridged republication of the second edition published by Alfred A. Knopf in 1947)

Johnson, Jr., C.R., W.A. Sethares, M.H. Ellis, and S. Haqqi. “Hunting for Paper Moldmates among Rembrandt’s Prints.” Submitted for publication. *IEEE Signal Processing Magazine*. 2014.

Johnson, D.H, E. Hendriks, and C.R. Johnson, Jr. “Interpreting Canvas Weave Matches.” *Art Matters* 5 (2013): 48-52.

Liedtke, W., C.R. Johnson, Jr., and D.H. Johnson. “Canvas Matches in Vermeer: A Case Study in the Computer Analysis of Fabric Supports.” *Metropolitan Museum Journal* 47 (2012): 99-106.

Loeber, E.G. *Paper Mould and Mouldmaker*. Amsterdam: Paper Pub. Society (Labarre Foundation), 1982.

Münz, Ludwig. *Rembrandt’s Etchings*. London: Phaidon Press, 1952.

Rijksbureau voor Kunsthistorische Documentatie. *Image Sharing*. http://english.rkd.nl/Services/image-sharing

Schwartz, Gary. *Rembrandt: All the Etchings Reproduced in True Size*. London: Oresko Books, 1977.

Stevenson, Allan H. “Chain-Indentations in Paper as Evidence.” *Studies in Bibliography* 6 (1954): 181-195.

---. "Watermarks are Twins." *Studies in Bibliography* 4 (1951-2): 57-91.

Van Staalduinen, M., J.C.A. Van Der Lubbe, E. Backer, and P. Paclik. "Paper Retrieval Based on Specific Paper Features: Chain and Laid Lines." In *Multimedia Content Representation, Classification and Security: International Workshop Proceedings*, edited by B. Gunsel, A.K. Jain, A.M. Tekalp, and B. Sankur, 346-353. Vol. 4105. Berlin: Springer, 2006.

Van Staalduinen, M. “Content-based Paper Retrieval Towards Reconstruction of Art History.” PhD thesis, Technische Universiteit Delft, 2010.

Van der Lubbe, J.C.A., E.P. van Someren, M.J.T. Reinders. “Dating and Authentication of Rembrandt's Etchings with the Help of Computational Intelligence.” In *Proceedings of International Cultural Heritage Informatics Meeting: Cultural Heritage and Technologies in the Third Millennium (ichim01)* 2 (September 2001): 485-492.

Vander Meulen, D.L. “The Identification of Paper without Watermarks: The Example of Pope’s ’Dunciad’.” *Studies in Bibliography* 37 (1984): 58-81.

Van Tilborgh, L., T. Meedendorp, E. Hendriks, D.H. Johnson, C.R. Johnson, Jr., and R.G. Erdmann. “Weave Matching and Dating of Van Gogh’s Paintings: An Interdisciplinary Approach.” *The Burlington Magazine* CLIV (February 2012): 112-122.

White, Christopher and Karel G. Boon. *Rembrandt’s Etchings: An Illustrated Critical Catalogue*. New York: Abner Schram, 1969.

Endnotes

[1] Many of Rembrandt's later etchings were executed on vellum and non-western papers. A few have been printed posthumously on fine silk.
[2] After 1750, a smoother surfaced paper was developed by using a fabric-covered screen also in a mould surrounded by a removable frame. This paper, called *wove*, eventually superseded *laid* paper. The first European book to be printed on wove paper was an edition of *Publii Virgilii Maronis Bucolica, Georgica, et AEneis* published by John Baskerville in 1757, as noted in D.Hunter, *Papermaking*, p.127.
[3] Hinterding, *Rembrandt as an Etcher*, 2:116-150.
[4] Ibid.
[5] Ash and Fletcher, *Watermarks in Rembrandt’s Prints*, 11.
[6] Hinterding, *Rembrandt as an Etcher*, 3:27.
[7] Vander Meulen, “Identification of Paper,” 58-59.
[8] Carter Hailey, “The Dating Game,” 377.
[9] Van der Lubbe et al., “Authentication of Rembrandt’s Etchings,” 481.
[10] Van Staalduinen et al., “Paper Retrieval”; and Van Staalduinen, “Content-based Paper Retrieval.” The use of average laid line density is similar to the problem of average thread density mapping to identify canvas rollmates among paintings from their x-radiographs, as described in van Tilbough et al., “Weave Matching”; Liedtke et al., “Canvas Matches in Vermeer”; and Johnson et al., “Interpreting Canvas Weave Matches.”
[11] Thanks are extended to Ad Stijnman and Peter Bower for their comments to the authors on this issue.
[12] Johnson, et al., “Hunting for Paper Moldmates.”
[13] Stevenson, “Watermarks are Twins,” 60-61.
[14] Thanks are extended to forensic paper historian Peter Bower for his assistance in identifying this paper.
[15] A gathering is formed by stacking several sheets of paper and folding them in half. In the case of the NYU ledger, each gathering is made from four sheets of paper. The gatherings were then stacked and sewn together to form the text block.
[16] Johnson et al., “Hunting for Paper Moldmates.”
[17] Hind 1923 (H91) first claimed that it is probably not by Rembrandt; Münz 1952 (MZ327) attributes it to Ferdinand Bol; Björklund 1955 (BB Rej. 71) rejects it; White and Boon 1969

(W&B 344) attribute it to a pupil, possible Karl van der Pluym, and finally the New Hollstein (NHD 91 copy a) to an artist from Rembrandt's studio. Thanks are extended to Per Rumberg of the Morgan Library & Museum, for this attribution chronology.

[18] Rijksbureau voor Kunsthistorische Documentatie, *Image Sharing*.

[19] Hinterding, *Rembrandt as an Etcher*, 2:185.

[20] Thanks to Michael Kwakkelstein and Gert Jan van der Sman.

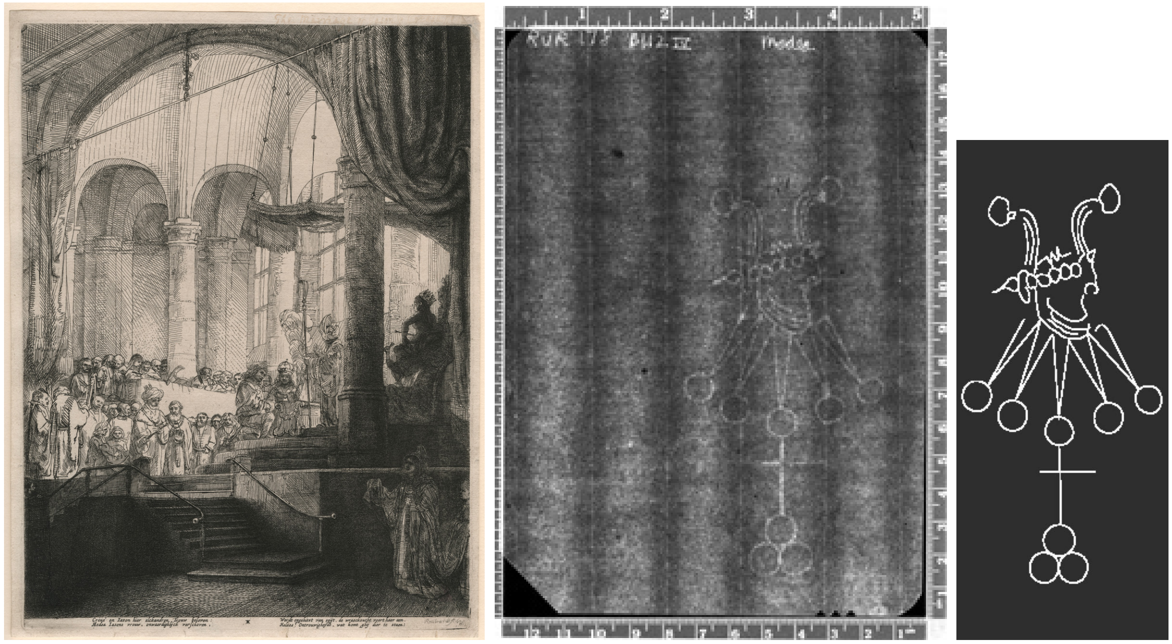

Figure 1: *Medea, or the Marriage of Jason and Creusa* (B112iv) with a beta-radiograph of its foolscap watermark. A hand-drawn version of the watermark is on the right. The chain lines are the vertical features that are spaced approximately one inch apart (see top ruler) in the beta-radiograph. Thanks are extended to Reba Snyder for providing the beta-radiograph of B112. (Etching Photo Credit: The Morgan Library & Museum, New York. RvR 178. Photography by Graham S. Haber, 2014.)

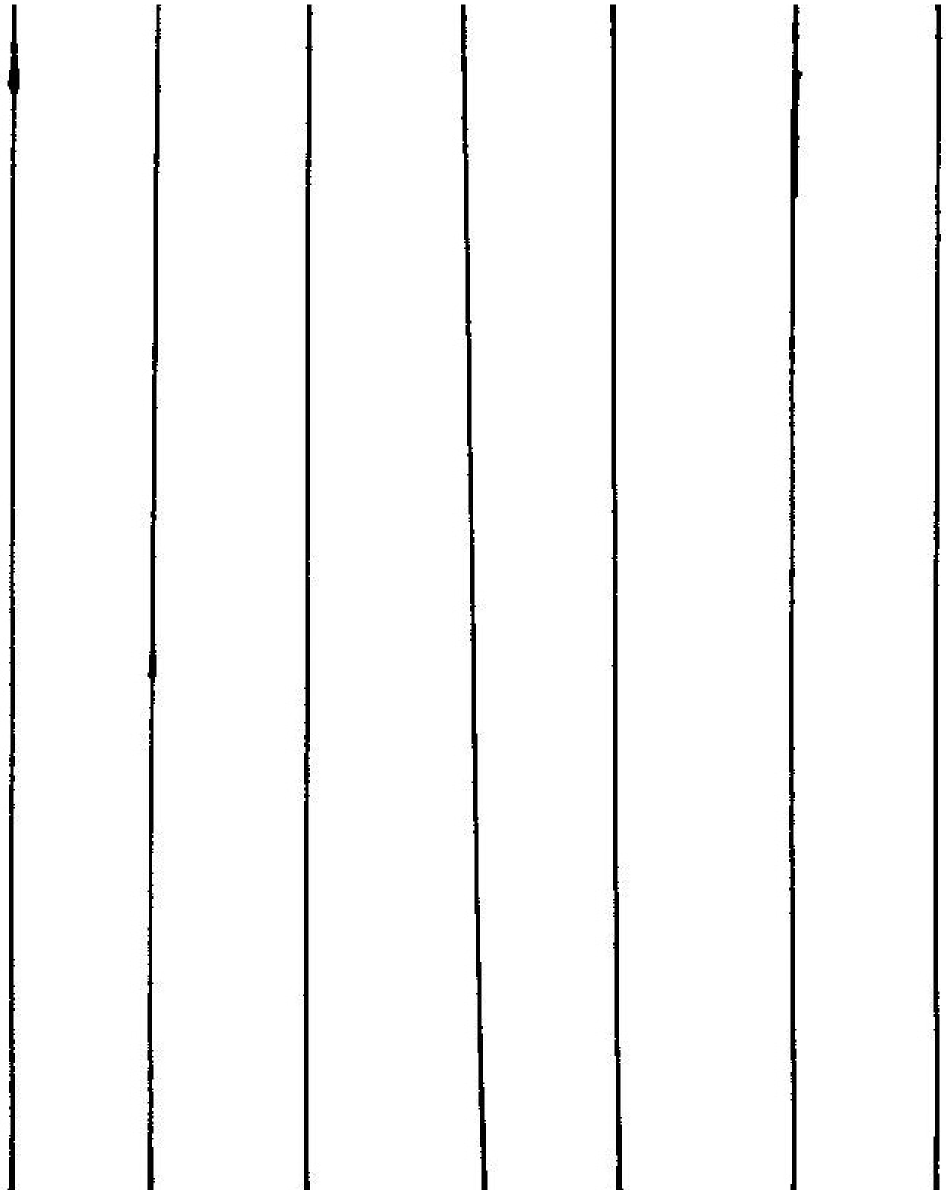

Figure 2: Illustrative Full-Mould Chain Line Pattern

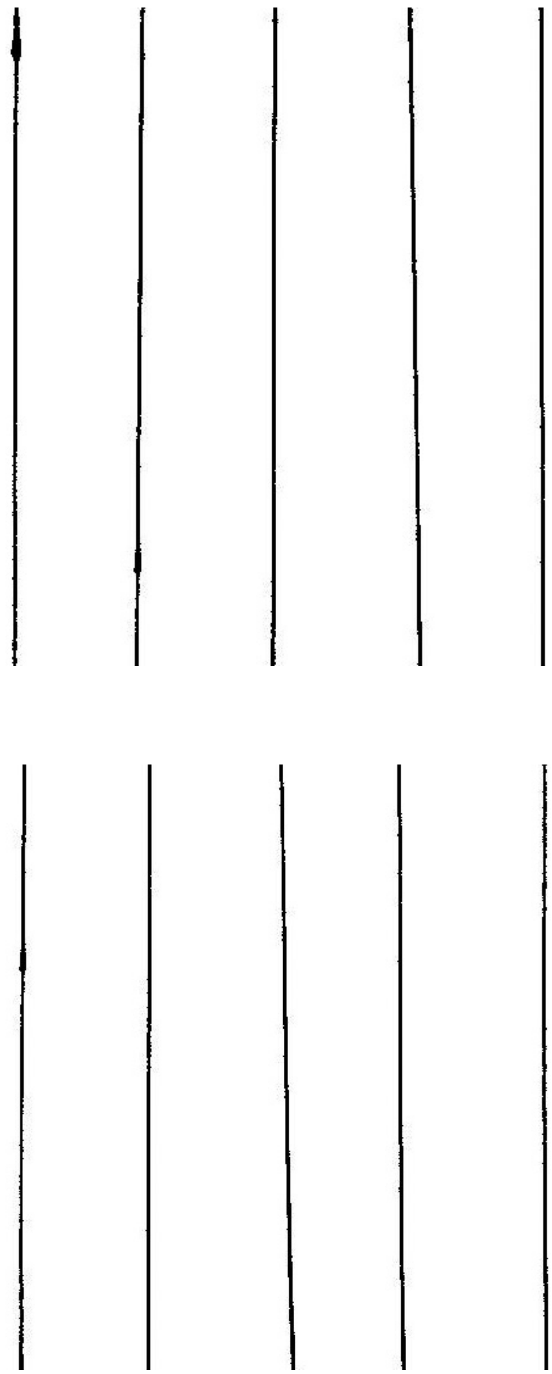

Figure 3: Hypothetical Paper Samples: top: A (upper left in mould); bottom: B (lower right in mould)

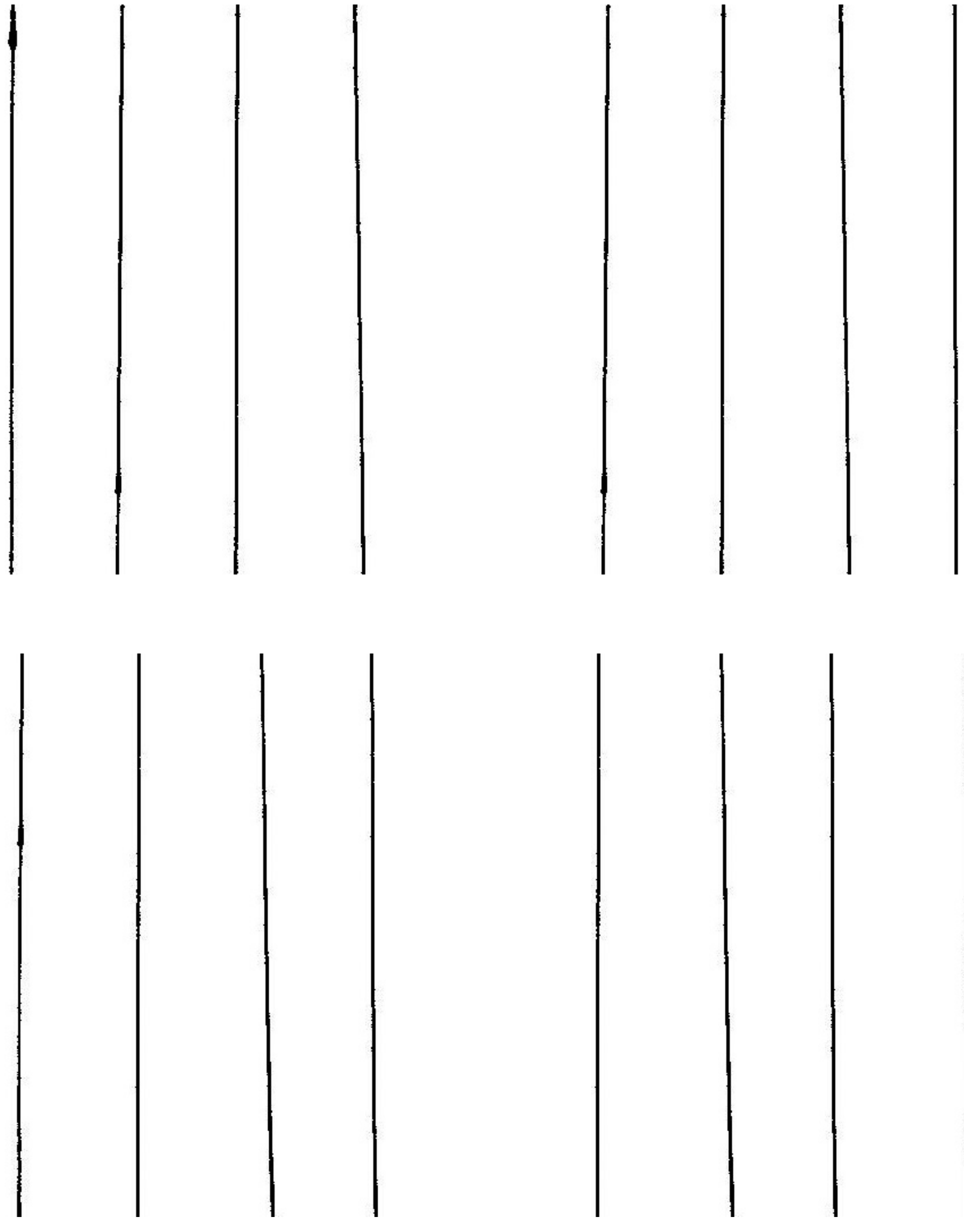

Figure 4: Extracted Chainspace Triplets from 2 Samples: upper left: A1 - first chain line quadruple in A; upper right: A2 - second chain line quadruple in A; bottom left: B1 -first chain line quadruple in B; bottom right: B2 - second chain line quadruple in B

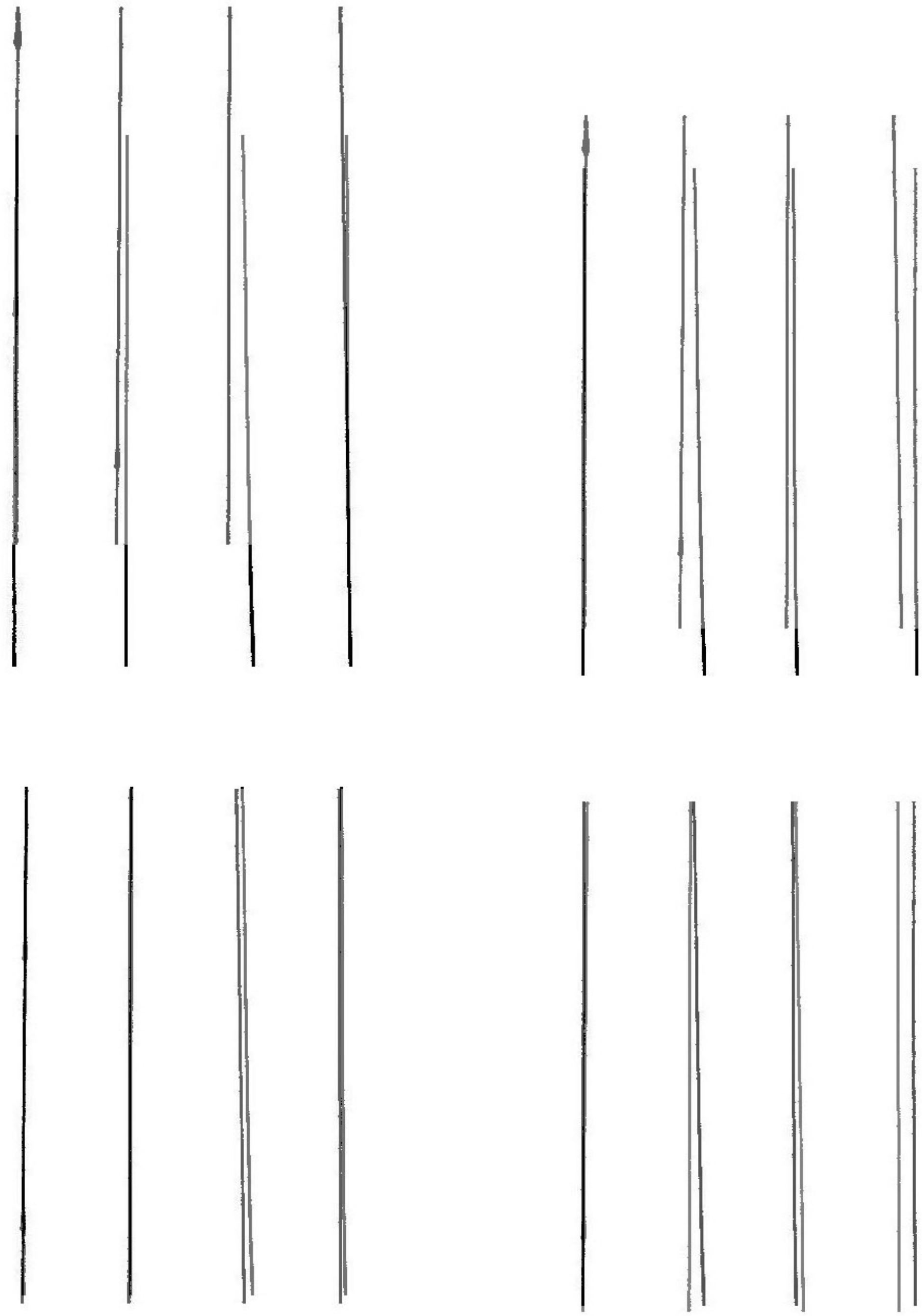

Figure 5: Chainspace Triplet Comparisons: upper left: A1 and B1; upper right: A1 and B2; lower left: A2 and B1; lower right: A2 and B2

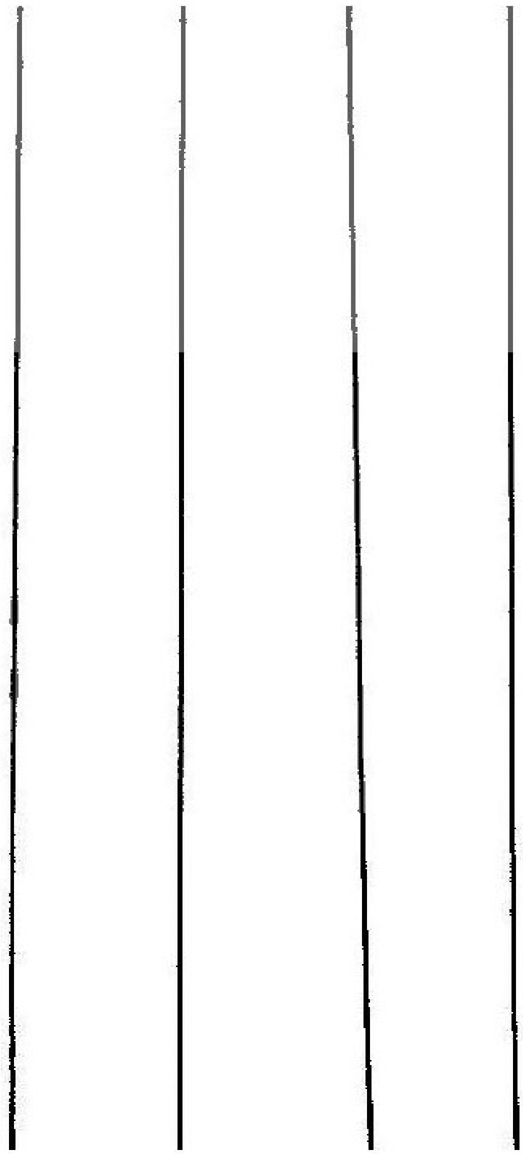

Figure 6: Shifted Best Triplet Match (actually lines 2, 3, 4, 5 numbered from left of chain line pattern in Figure 2)

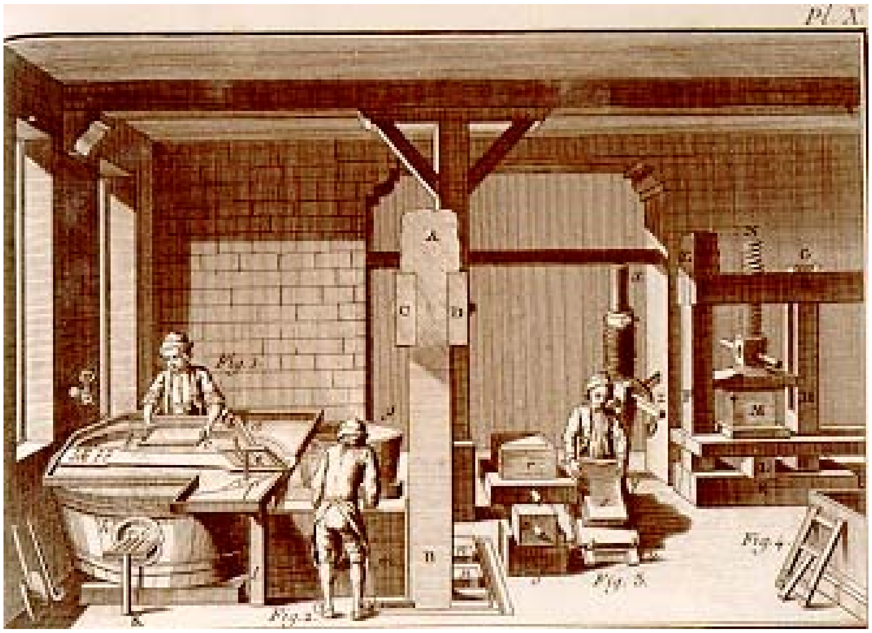

Figure 7: The vatman on the left lifts a mold onto which a freshly made sheet of paper has formed. The workmen to the right are transferring the sodden sheet from the mold and stacking them between woolen felts onto a growing pile of papers. Heavy presses used to remove the remaining water from the paper appear in the background. This print is from (Diderot, *Encyclopedie*, Plate X).

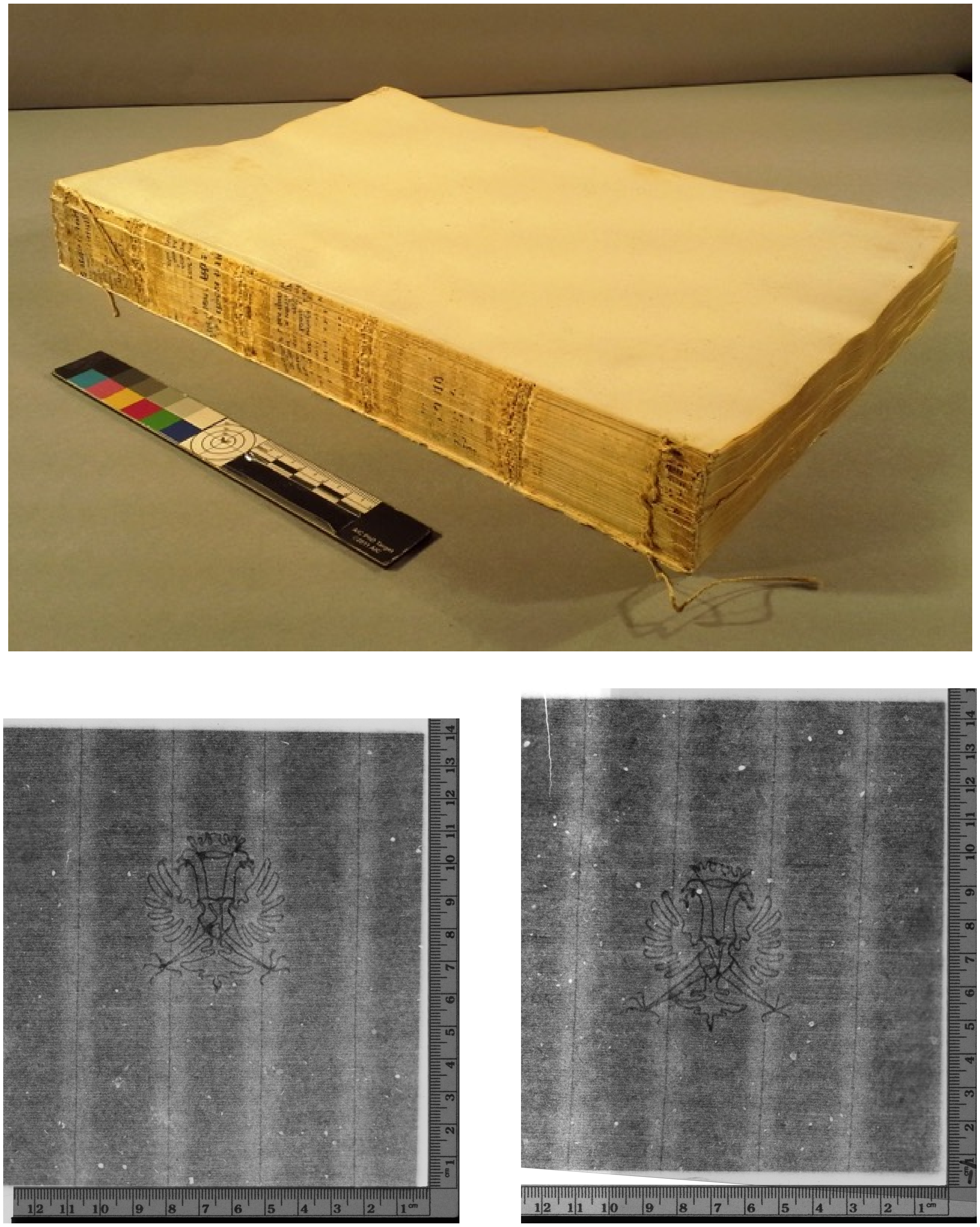

Figure 8: A sixteenth-century blank ledger book contains papers (top) from only two moulds, A (bottom left) and B (bottom right), which can be differentiated by their watermarks and chain line patterns. (Beta-radiographs courtesy S. Haqqi).

Figure 9: Pupil of Rembrandt, *The Artist's Mother in Widow's Dress and Black Gloves* (B344) is shown on the left. A beta-radiograph of the area near the watermark is shown on the right. Thanks are extended to Reba Snyder for providing the beta-radiograph of B344. (Etching Photo Credit: The Morgan Library & Museum, New York. RvR 459. Photography by Graham S. Haber, 2014.)

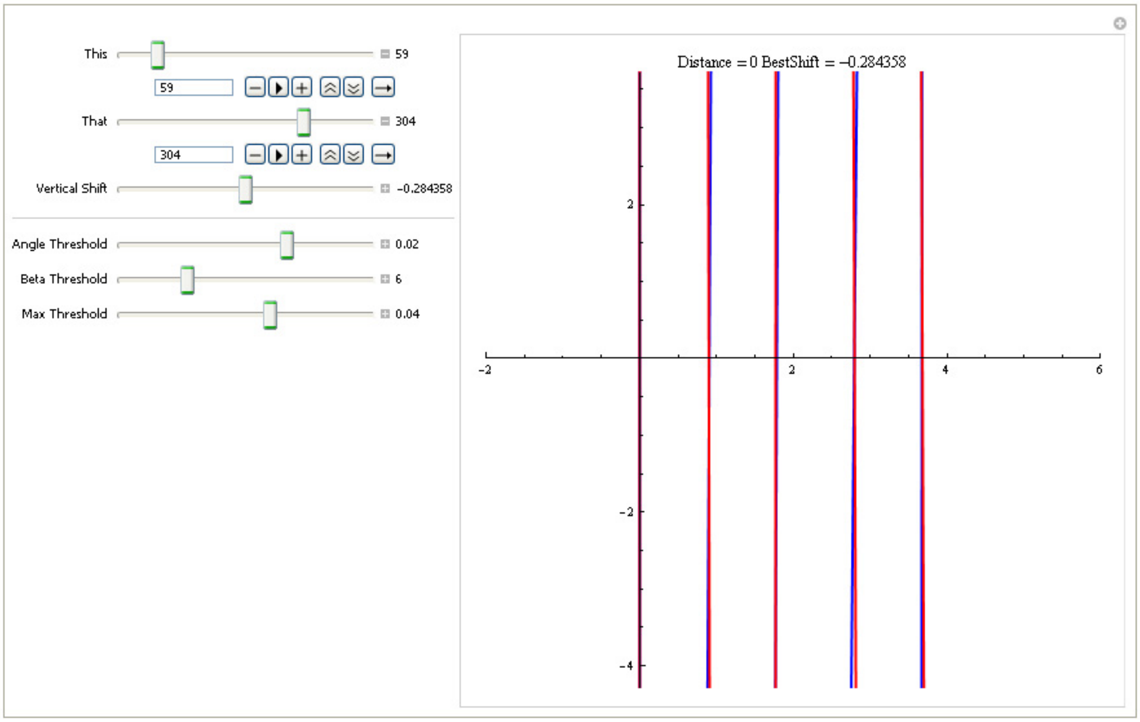

Figure 10: Extracted chain line "optimum" overlay for B112iv (quadruplet #59) and B344 (quadruplet #304).