The current primary objective of the WIRE Project is the creation of an interrogatory, computer-assisted decision tree for identification of the watermarks in Rembrandt’s prints according to the taxonomy in [Hinterding, 2006]. This document uses a small scale example employing watermark images of the Paschal Lamb type in [Hinterding, 2006] to illustrate issues encountered in the selection of distinguishing features and decision tree construction.

Selecting Distinguishing Features

Consider the 4 watermarks labelled I, II, III, and IV in Figure 1 of the Paschal Lamb watermark in the laid paper supports of four Rembrandt etchings. (In [Hinterding, 2006], I is labelled as E-a-a, II as E-a-b, III as E-b-a, and IV as A-a-a.) In each of these images the lamb is facing left.

First consider the 2 images I and II in Figure 1 and select a feature that distinguishes them. This is difficult as I and II are quite similar. Indeed in [Hinterding, 2006], they are designated as twins [Stephenson, 1951-2]. According to their classification in [Hinterding, 2006], I and II are more similar to each other than they are to either III or IV. Furthermore, III is more similar to either I or II than it is to IV. Careful looking reveals a clear-cut distinction between I and II in the pole crossing the lamb’s shoulder which is seen to be a cross at the
Figure 1: Paschal Lamb Watermarks: Top - Left I, Right II; Bottom - Left III, Right IV
upper end. The lower end pointing down and to the left extends to a chain line in I but ends short of the chain line in II.

Now select features that distinguish the pair I and II, for which we have a distinguishing feature, from its most similar among the remainder, i.e. III. For example, in both I and II the bottom of the shield is pointed downward. In III the bottom of the shield is flat. Alternatively, consider the band at the bottom of the crown which has 3 round jewels in all 3 images. In I and II the leftmost jewel is bisected by a vertical chain line, while in III the rightmost is similarly bisected. Either of these features could be used to distinguish III from I and II.

Figure 2: A Decision Tree for the Four Watermarks in I, II, III, IV

In selecting a feature that distinguishes IV from I, II, and III one possibility is the center of the cross which is to the right of the closest
chain line in IV, but to the left in I, II, and III. Thus, a decision tree with 3 questions, as diagrammed in Figure 2 could be posed to identify the 4 watermarks.

- **Question 1:** With the lamb facing left, is the cross bar in the cross completely to the right of the chain line crossing through the back hip of the lamb? YES: IV; NO: I, II, III

- **Question 2:** Is the bottom of the shield flat? YES: III; NO: I and II

- **Question 3:** Is the lower end of the cross touching a chain line? YES: I; NO: II

In Figure 2 the images remaining to be classified after answering a question are indicted in parentheses along the branch to the next question. So the branch from question 1 to question 2 is labelled with (I,II,III). Question 2 separates I and II from III. Question 3 separates I and II.

**Decision Tree Construction**

The three questions in the decision tree in Figure 2 were determined by a strategy of finding a distinguishing feature for the most similar pair and then progressing by finding a distinguishing feature between that pair and the most similar watermark among the remaining images. This is repeated with the designation of a feature distinguishing the accumulated group of identified watermarks from the most similar image in the (temporarily) unidentified remainder from the original library. The associated decision tree structure in Figure 2 is not the only possible decision tree that will uniquely separate, and thereby identify, the four images, even for the same three questions.
To help construct other successful trees, we compose a listing of the answers for each question for each image as a feature table in Table 1. Examining the columns of Table 1 reveals that Question 3, if asked first, like Question 1 would divide the 4 images into 2 groups, one with one member and the other with 3. Only Question 2 divides the 4 images into two equal-sized groups. Using Question 2 first splits the 4 images into (a) I and II and (b) II and IV. Question 1 can be used to separate II and IV, but not I and II. Similarly, question 3 can be used to separate I and II, but not II and IV. This produces the decision tree in Figure 3.

An advantage of the decision tree in Figure 3 over the one in Figure 2 is that the decision tree in Figure 3 never requires more than two questions to provide the unique answer while the decision tree in Figure 2 requires three questions if the image is I or II. Conversely, the decision tree in Figure 2 could be preferred if image IV were to be encountered for identification far more frequently than any of the other images. This is because the very first question asked in the decision tree in Figure 2 isolates (and identifies) the watermark in image IV.

Two other successful decision trees appear in Figures 4 and 5. The decision trees in Figures 4 and 5 both have the same structure as the
Figure 3: Another Decision Tree for the Four Watermarks in I, II, III, IV

decision tree in Figure 2.

**Expanding the Library**

All of the decision trees in Figures 2 - 5 use all three questions for a library of four watermarks. The number of questions with a binary (e.g. yes or no) answer used is $N - 1$ to provide unique identification of a library of size $N$ for $N = 4$. This relationship continues for larger $N$. Thus, if we now add a 5th image displayed in Figure 6 to our library of distinct watermarks, we will need $N - 1 = 4$ questions. We will attempt to add a new question to the three we already have.

First we will compose the feature presence table in Table 2 by adding image V to Table 1. This reveals that the feature vector $(N,Y,Y)$ of image V matches that of image III. This suggests that the new (4th) question needss to distinguish III and V. We noted ear-
Figure 4: Yet Another Decision Tree for the Four Watermarks in I, II, III, IV

Figure 5: Still Yet Another Decision Tree for the Four Watermarks in I, II, III, IV
lier that for I, II, III, and IV one of the circles in the band below the crown was bisected by a vertical chain line. This is not the case with image V.

The new question is

- Question 4: Are any of the circles in the band below the crown bisected by a chain line? YES: I, II, III, IV; NO: V

This additional question expands the feature presence table of Table 2 to that of Table 3.

For any of the previous decision trees in Figures 2 - 5, the endpoint for III will be the endpoint for V because their feature vectors are the same: (N, Y, Y). Thus, the endpoint for III in any of the decision trees in Figures 2 - 5 is the point where question 4 should be added. This is done to the decision tree in Figure 2 to produce the decision tree in
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<th>Question 3</th>
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</tr>
<tr>
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</tr>
<tr>
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<td>Yes</td>
<td>Yes</td>
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<tr>
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Table 2: Features: Three Questions and Five Images

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<tr>
<td>V</td>
<td>No</td>
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Table 3: Features: Four Questions and Five Images
Figure 7: A Decision Tree for the Five Watermarks in I, II, III, IV, and V

Figure 7 for the five watermarks.
A full branch identifying all “5-pointed foolscap” watermarks in [Hinterding, 2006] appears in Figure 8.

References


Figure 8: A Decision Tree for Five-Pointed Foolscap Watermarks in [Hinterding, 2006]