Stability of Queues in Slotted ALOHA with Multiple Antennas

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October 8, 2002

Abstract

We consider the problem of stability of slotted ALOHA for a system consisting of N users communicating with a common receiver, that is capable of receiving multiple transmissions simultaneuosly. We characterize the stability region of slotted ALOHA for the two user case explicitly. We show that the stability region undergoes a phase transition from a concave region to a convex region bounded by lines. Then, we derive some sufficient conditions for stability of slotted ALOHA for the $N > 2$ case.

1 Introduction

The recent surge of interest in multiple antenna wireless systems has once again brought into focus the ability of the physical layer to utilize spatial diversity for increasing capacity. Most of the effort in utilizing the spatial diversity at the Media Access Control (MAC) layer has been directed either towards modifying existing protocols like ALOHA, CSMA etc. [11] -[16] or design of new MAC protocols to exploit directional antennas and smart adaptive antenna arrays [17]-[21]. In systems with spatial diversity, the main issues that need to be addressed are the design of signal processing techniques for the multiple antenna receiver at the physical layer and keeping buffers of packets on the transmitter side under control at the MAC layer. One of the most significant characteristics of such systems is the possibility of receiving information from more than one user *simultaneously*. To date, the issues of multiple antenna receiver design and buffer stability have been looked at almost in isolation. Our main goal in this paper is to look at the aforementioned problems jointly *i.e.*, to see how signal processing techniques like beamforming affect buffer stability. In particular, we restrict ourselves to slotted ALOHA as the MAC layer protocol and examine the effect of Multipacket receptions on buffer stability.

*This work was supported in part by the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564 and the ARL CTA on Communications and Networks under Grant DAAD19-01-2-011.

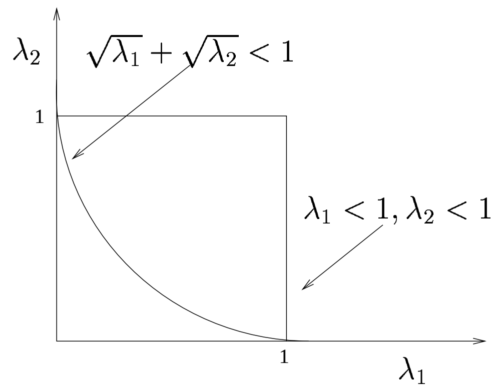

Figure 1: Two user stability region of slotted ALOHA for the Collision Channel and Orthogonal Channels

In Figure 1, we show the regions of buffer stability *i.e.*, the rate pairs for which queues are stable for slotted ALOHA with the collision channel and with perfectly orthogonal channels (no interference). For orthogonal channels we have a unit square, whereas for the collision channel we have a complex form. Our motivation was to look at the behavior of the stability region when the diversity we have lies in between these two extreme cases. We find that the stability region makes a smooth phase transition from concavity to convexity as we move from one extreme to another. In other words, as we allow multipacket receptions to become more likely there comes a point at which the stability region becomes convex. More surprisingly, we find that the stability region is bounded by lines as soon as it becomes convex.

In [8]-[10], a packet radio system is analyzed in which a multiple beam adaptive array is used at the base station to separate users signals. The authors characterize the performance of slotted ALOHA for such a system in terms of the throughput, assuming the users have single packet buffers. In [6], Ghez *et al.* consider the stability of ALOHA for an infinite user slotted channel with multipacket reception (MPR) capability. In such a channel, the number of packets successfully received in a slot is a random variable which depends only on the number of attempted transmissions in that slot. Thus, this model can capture the event of simultaneous packet successes although it is not sufficient to capture asymmetry among users since all users are treated equal by the model, which need not be true for a multiple antenna wireless system.

Tsybakov and Mikhailov [1] initiated the study of the slotted ALOHA system in terms of the stability of queues at each of the terminals in the system. By stability we mean that all queues are finite with probability one. In such a buffered system, stability is not easy to establish because of the stochastic interdependance among the queues. Nonetheless, Tsybakov and Mikhailov found sufficient conditions for stability of the queues in the system using the principle of stochastic dominance. For the symmetric case (*viz.* equal arrival rates for all terminals), they found the maximum stable throughtput. They also found the stability region for the two-user case explicitly. Rao and Ephremides [2] explicitly used the principle of stochastic dominance to find inner bounds to the stability region for the $N > 2$ case. Szpankowski [3] found necessary and sufficient conditions for the stability of queues in a slotted ALOHA system for a fixed retransmission probability vector for the $N > 2$ case. Recently, Luo and Ephremides [4] introduced the concept of instability ranks in queues to obtain tight inner and outer bounds on the stability region for the $N > 2$ case. However, to date there is no closed form characterization of the stability region for the $N > 2$ case. The point to note is that the above results were derived assuming the collision channel model for packet success—an assumption that we relax.

The remainder of this paper is organized as follows. In Section 2, we specify the system model and define the notions of stability. In Section 3, we derive the stability region for the two user case. We also characterize some interesting properties of this region. In Section 4, we find sufficient conditions for stability for the $N > 2$ case. Finally, we conclude in Section 5.

2 System Model

The system consists of N users, each having an infinite buffer for storing arriving and backlogged packets, communicating with a common receiver. The receiver has multiple antennas used to implement beamforming for receiving multiple packets simultaneously. The channel is slotted in time and a slot duration equals the packet transmission time. Packets are assumed to be of equal length for all the users. The arrivals at the ith queue ($i \in \{1, 2, \ldots, N\}$) are independant and identically distributed Bernoulli random variables from slot to slot with mean λ_i. Arrival processes are assumed to be independant from user to user. If the ith users' buffer is nonempty, he transmits a packet with probability p_i in a slot.

Now we define a very general packet reception model to capture the event of multi-packet reception. Suppose that the set $\mathcal{S} \subseteq \{1, 2, \ldots, N\}$ of users transmit in a slot, then we define for $i \in \mathcal{S}$,

$$q_{i|\mathcal{S}} = \Pr\{i\text{th users' packet is successfully received} \mid \mathcal{S} \text{ transmits}\} \tag{1}$$

We assume that user i's packet is successfully received independantly from slot to slot. We further assume that the receiver gives an instantaneous feedback of all the packets that were successful in a slot at the end of the slot to all the users. The users remove successful packets from their buffers while unsuccessful packets are retained in the buffers. It should be clear that the conditional probabilities $q_{i|\mathcal{S}}$ are a function of the receiver front-end which will be employed by the receiver to "separate" users' signals. Before we proceed to derive some of the results of the next section, a few definitions are in order. We use the definition of stability used by Loynes [5].

Definition: A multidimensional stochastic process, $\mathbf{Q}^t = (Q_1^t, \ldots, Q_N^t)$ is *stable* if for $\mathbf{x} \in \mathbb{N}^N$ the following holds

$$\lim_{t\to\infty} \Pr\{\mathbf{Q}^t < \mathbf{x}\} = F(\mathbf{x}) \text{ and } \lim_{\mathbf{x}\to\infty} F(\mathbf{x}) = 1. \tag{2}$$

If a weaker condition holds *viz.*,

$$\lim_{\mathbf{x}\to\infty} \liminf_{t\to\infty} \Pr\{\mathbf{Q}^t < \mathbf{x}\} = 1 \tag{3}$$

then the process is called substable. Further, the process is said to be unstable if it is not substable.

It can be easily shown that stability implies substability. For the slotted ALOHA system we described in Section 2, the stochastic process under consideration is the queue length at the N buffers. Thus, Q_i^t represents the queue length at ith buffer at time t.

Because of the special arrival and departure statistics in this system, the N dimensional queue evolution is an aperiodic and irreducible Markov chain. The notion of stability in this system is then equivalent to the positive recurrence of the Markov chain. Intuitively this means that the buffers in the system are not growing to infinity.

Definition: For an N user slotted ALOHA buffer system, the stability region is defined as the set of arrival rates $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_N]$ for which there exists a retransmission probability vector $\mathbf{p} = [p_1, p_2, \ldots, p_N]$ such that the buffers in the system are stable. The stability region clearly depends on the underlying packet reception model.

3 Stability region for the two user case

Since the stability region for the collision channel is unknown for the $N > 2$ case, we first try to find the stability region for the $N = 2$ case for the general reception model given by (1). For notational convenience, we define the probabilities of packet success in the two user case as

$$q_i^{(1)} = \Pr\{\text{user } i \text{ is successful} \mid \text{only user } i \text{ transmits}\} \tag{4}$$
$$q_i^{(2)} = \Pr\{\textit{only} \text{ user } i \text{ is successful} \mid \text{both users transmit}\} \tag{5}$$
$$q^{(2)} = \Pr\{\text{both users are successful} \mid \text{both users transmit}\} \tag{6}$$

Further, we define $Q_1 \triangleq q_1^{(1)} - q_1^{(2)} - q^{(2)}$ and $Q_2 \triangleq q_2^{(1)} - q_2^{(2)} - q^{(2)}$. Thus, Q_1 and Q_2 denote the difference between the (conditional) probability of success in the absence of interference and the (conditional) probability of success in the presence of interference for the users. Note that the above probabilities can capture not only all possible packet reception events but also correlations among those events and user asymmetry. To find the stability region, we first need to find the stability region of the system for a *fixed* retransmission probability vector $\mathbf{p}(= [p_1, p_2])$. The following lemma gives us exactly that.

Lemma 1 *If $Q_1 \geq 0$ and $Q_2 \geq 0$, the stability region of slotted ALOHA for the general packet reception model for a given $[p_1, p_2]$ is given by*

$$\lambda_1 < p_1 q_1^{(1)} - \frac{p_1 p_2 \lambda_2 Q_1}{\lambda_2^*}, \quad \textit{for } \lambda_2 < \lambda_2^* \tag{7}$$

and

$$\lambda_2 < p_2 q_2^{(1)} - \frac{p_1 p_2 \lambda_1 Q_2}{\lambda_1^*}, \quad \textit{for } \lambda_1 < \lambda_1^* \tag{8}$$

where,

$$\lambda_1^* = p_1 q_1^{(1)} - p_1 p_2 Q_1 \;\; \textit{and} \;\; \lambda_2^* = p_2 q_2^{(1)} - p_1 p_2 Q_2$$

Proof: We use the idea of stochastic dominance and use an argument similar to that by Rao and Ephremides [2]. □

Figure 2 shows us the stability region as given by Lemma 1. The conditions $Q_1 \geq 0$ and $Q_2 \geq 0$ are needed for the stochastic dominance of the associated dominant systems. In fact, these conditions are equivalent to the probability of success of any user in the presence of interference (from the other user) be no greater than the probability of success in the absence of interference—a reasonable and practical assumption.

We now give a key result of this paper in the form of this theorem.

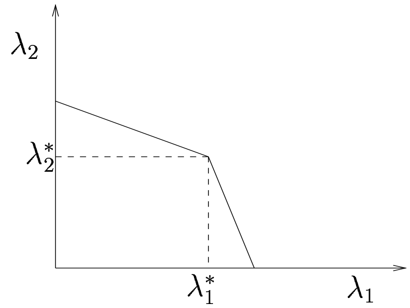

Figure 2: Stability region for a fixed retransmission probability vector $[p_1, p_2]$

Theorem 1 *If $Q_1 \geq 0$ and $Q_2 \geq 0$, then the stability region of slotted ALOHA for the general reception model is given by $\mathcal{R}_1 \cap \mathcal{R}_2$ where*

$$\mathcal{R}_1 \triangleq \{(\lambda_1, \lambda_2) : (\lambda_1, \lambda_2) \geq (0,0),\ (\lambda_1, \lambda_2) \textit{ lies below the curve } \lambda_2 = f(\lambda_1, q_1^{(1)}, q_2^{(1)}, Q_1, Q_2)\} \tag{9}$$

and

$$\mathcal{R}_2 \triangleq \{(\lambda_1, \lambda_2) : (\lambda_1, \lambda_2) \geq (0,0),\ (\lambda_1, \lambda_2) \textit{ lies below the curve } \lambda_1 = f(\lambda_2, q_2^{(1)}, q_1^{(1)}, Q_2, Q_1)\} \tag{10}$$

where

$$f(\lambda, \alpha, \beta, \gamma, \delta) = \begin{cases} \beta - \frac{\lambda\delta}{\alpha-\gamma}, & \lambda \in \mathcal{I}_1 \\ \left(\frac{\alpha\beta - \sqrt{\lambda\delta\alpha\beta}}{\gamma}\right)\left(1 - \sqrt{\frac{\lambda\delta}{\alpha\beta}}\right), & \lambda \in \mathcal{I}_2 \end{cases} \tag{11}$$

where

$$\mathcal{I}_1 = [0, \frac{\beta(\alpha-\gamma)^2}{\alpha\delta}] \textit{ and } \mathcal{I}_2 = (\frac{\beta(\alpha-\gamma)^2}{\alpha\delta}, \frac{\alpha\beta}{\delta}]. \tag{12}$$

If either Q_1 or Q_2 equals zero, then we assume $\frac{1}{0} = \infty$ and our result still holds.

Proof: We use Lemma 1. Since we know the stability region for a fixed retransmission probability vector $\mathbf{p}$, we need to find the union of all the stability regions as the parameter $\mathbf{p}$ varies over $[0,1]^2$. One way of doing this is to setup a corresponding constrained optimization problem *i.e.* for a fixed λ_1, maximize λ_2 as $\mathbf{p}$ varies over $[0,1]^2$, where λ_1 and λ_2 are related by (7) and (8). This is the method which we used in our proof [22]. □

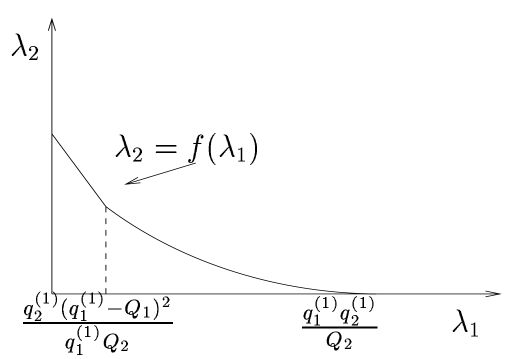

Figure 3: The appearance of $f(\lambda_1)$ as a function of λ_1.

We note a few interesting things about the stability region. First, the function f characterizing the stability region in (11) is linear for some part of the domain and is strictly concave in the remainder of its domain as illustrated in Figure 3. The stability region for the two user collision channel can be found as a special case of our model with $q_1^{(1)} = 1$, $q_2^{(1)} = 1$, $q_1^{(2)} = 0$, $q_2^{(2)} = 0$ and $q^{(2)} = 0$ and is bounded by the curve $\sqrt{\lambda_1} + \sqrt{\lambda_2} = 1$, which is strictly concave everywhere. In fact, it is easy to see from Figure 3 that the interval where f is linear has non-zero Lebesgue measure as soon as there is a nonzero probability of success in the presence of interference *i.e.* $q_1^{(1)} - Q_1 > 0$. Thus, there is a characteristic *change* in the structure of the stability region as soon as we have multipacket reception. Second, we see that there is a symmetry in the way in the two regions $\mathcal{R}_1$ and $\mathcal{R}_2$ are defined in terms of the function f. Third, the stability region is entirely characterized by $q_1^{(1)}$, $q_2^{(1)}$, Q_1 and Q_2. In turn, Q_1 and Q_2 depend only on the *marginal* probabilities of success in the presence and absence of interference, which is not surprising since the two users are not collaborating their packet transmissions.

Further, we observe the following three important properties of the stability region:

Property 1 (P1): Assume that $q_1^{(1)} > 0$, $q_2^{(1)} > 0$, $q_1^{(1)} - Q_1 > 0$ and $q_2^{(1)} - Q_2 > 0$ *i.e.* non-zero probability of success in the presence and absence of interference. Then, the stability region is bounded by lines iff

$$\frac{Q_1}{q_1^{(1)}} + \frac{Q_2}{q_2^{(1)}} \leq 1 \tag{13}$$

Property 2 (P2): Assume that $q_1^{(1)} > 0$, $q_2^{(1)} > 0$, $q_1^{(1)} - Q_1 > 0$ and $q_2^{(1)} - Q_2 > 0$. Then, the stability region is bounded by lines iff the stability region is convex.

Property 3 (P3): If the stability region is not bounded by lines, then the strictly concave parts of f which bound the regions $\mathcal{R}_1$ and $\mathcal{R}_2$ coincide on the boundary of the stability region.

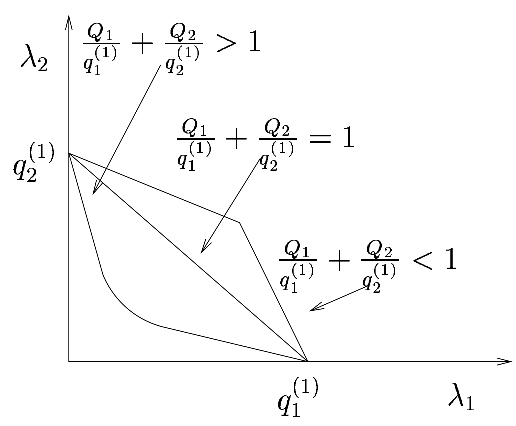

Figure 4: The shape of the stability region for different $\mathbf{q}$ vectors with $q_1^{(1)}$ and $q_2^{(1)}$ fixed.

Figure 4 shows the stability regions characterized by the $\mathbf{q}$ vector based on the above properties. P1 has a nice interpretation; there is a critical point for the $\mathbf{q}$ vector at which the behavior of the stability region makes a phase transition from a very simple form (bounded by lines) to a much more complex form. Further, this critical point depends only on the sum of the ratios of probability of success of users in the presence of interference to that in the absence of interference. P2 tells us that the two simple properties (convexity and being bounded by lines) of a region are equivalent for the stability region. The condition of the stability region being bounded by lines and being convex corresponds to a regime in which when one user increases his rate, the other users' maximum supportable rate decreases only linearly, and that too at a rate which is low till a certain point and then suddenly increases. Another interpretation is that when the stability region is convex then higher sum rates can be achieved. In addition, when the stability region is convex we know that if two rate pairs are stable then any rate pair lying on the line segment joining those two rate pairs is also stable. This is an important point because the stability region for the two user collision channel is not convex. When equality holds in equation (13), the stability region is a triangle as shown in Figure 4. All the rate pairs in this region can be stabilized by TDMA schemes (in a collision channel). Thus, the condition $\frac{Q_1}{q_1^{(1)}} + \frac{Q_2}{q_2^{(1)}} < 1$ gives us the regime in which a distributed strategy like slotted ALOHA can do better than a centralized TDMA scheme. P3 also has an interesting interpretation; it tells us that if the stability region has a complex form then there are certain regions where the behavior of the channel is similar to a collision channel. This is also the regime in which the stability region is not convex. In this regime, when one user increases his rate the other user has to reduce his rate drastically for stability. In [7], we apply the results of this section to a two user cellular setup and compare the performance of three different front-ends *viz.*, MMSE, Decorrelating and Matched Filter in terms of the stability region.

4 A sufficient condition for the $N > 2$ case

Deriving stability conditions for the $N > 2$ case is quite hard even for the collision channel model. Nonetheless, for a fixed retransmission probability vector $(\mathbf{p})$, Szpankowski [3] gave a sufficient and necessary condition for stability of the ALOHA system with the collision channel model for the $N > 2$ case. In this section we also restrict ourselves to finding sufficient conditions for stability for the general reception model for a fixed retransmission probability vector $(\mathbf{p})$. The main ideas involved here are those of stochastic dominance and of constructing suitable dominating systems for which stability conditions are easier to determine. The way to construct such dominant systems is to assume that some of the queues in the system continue to transmit interfering dummy packets even when they are empty. For the collision channel, such systems are known to stochastically dominate the original ALOHA system [1]. However, for a general packet reception model this may not be the case.

Let $\mathcal{P} = (\mathcal{V}, \mathcal{U})$ be a partition of $\mathcal{M} \triangleq \{1, 2, \cdots, N\}$ such that users in $\mathcal{V} \neq \mathcal{M}$ behave just like those in the original ALOHA system while those in $\mathcal{U}$ continue to transmit dummy packets even when their queues are empty. We call users in $\mathcal{U}$ persistent and those in $\mathcal{V}$ nonpersistent. For a partition $\mathcal{P}$ defined above, let $\Theta^{\mathcal{P}}$ denote the ALOHA system where users behave as specified by $\mathcal{P}$. Further, let $\mathbf{Q}^t_{\mathcal{P}} = (\mathbf{Q}^t_{\mathcal{V}}, \mathbf{Q}^t_{\mathcal{U}})$ denote the queue lengths in $\Theta^{\mathcal{P}}$.

We note that the reception probabilities given by (1) are not enough to characterize the probability transition matrix for $\mathbf{Q}^t$. However, we find that the marginal probabilities given by (1) are enough to find sufficient conditions for stability of $\mathbf{Q}^t$ even for $N > 2$. We conjecture that the marginal probabilities of success are sufficient to completely characterize the stability region for a fixed retransmission probability. Assume that the reception probabilities $(\mathbf{q}^{\mathcal{M}} = \{q^{\mathcal{M}}_{i|\mathcal{S}} \triangleq q_{i|\mathcal{S}}, i \in \mathcal{S}, \mathcal{S} \subseteq \mathcal{M}\})$ satisfy $\forall\ \mathcal{V} \subset \mathcal{M}$ and for all

$j \in \mathcal{M}$,

$$\max_{\mathcal{A} \subseteq \mathcal{V} \backslash \{j\}} p_j \left[\sum_{\mathcal{S} \subseteq (\mathcal{A} \cup \mathcal{U}) \backslash \{j\}} \left(\prod_{i \in S} p_i \prod_{k \in (\mathcal{A} \cup \mathcal{U}) \backslash (\mathcal{S} \cup \{j\})} \overline{p}_k q^{\mathcal{M}}_{j|j \cup \mathcal{S}} \right) \right] \leq \tag{14}$$

$$\min_{\mathcal{A} \subseteq \mathcal{M} \backslash \{j\}} p_j \left[\sum_{\mathcal{S} \subseteq \mathcal{A}} \left(\prod_{i \in S} p_i \prod_{k \in \mathcal{A} \backslash \mathcal{S}} \overline{p}_k q^{\mathcal{M}}_{j|j \cup \mathcal{S}} \right) \right] \tag{15}$$

These conditions are sufficient to ensure that for all $\mathcal{P}$, $\mathbf{Q}^t_{\mathcal{P}}$ stochastically dominates the original queue length process $\mathbf{Q}^t$ provided both systems begin with the same initial conditions. The point to note is that the $|\mathcal{V}| < N$ dimensional process $\mathbf{Q}^t_{\mathcal{V}}$ is also a Markov Chain which mimics the original ALOHA system [3] except with *modified* reception probabilities ($\mathbf{q}^{\mathcal{V}} = \{q^{\mathcal{V}}_{i|\mathcal{S}}, i \in \mathcal{S}, \mathcal{S} \subseteq \mathcal{V}\}$) and so we can use induction arguments to establish its stability. Note that here we use our result from Section 3 that the stability region for the two user case depends only on the marginal probabilities of success. More precisely, for any $\mathcal{V}' \subseteq \mathcal{V}$ and $i \in \mathcal{V}'$, the modified reception probabilities for the smaller ALOHA system consisting of the stand alone persistent set $\mathcal{V}$ become

$$q^{\mathcal{V}}_{i|\mathcal{V}'} = \sum_{\mathcal{T} \subseteq \mathcal{U}} \left(\prod_{j \in \mathcal{T}} p_j \prod_{k \in \mathcal{U} \backslash \mathcal{T}} \overline{p}_k \right) q^{\mathcal{M}}_{i|\mathcal{V}' \cup \mathcal{T}} \tag{16}$$

Now, suppose that the Markov Chain $\mathbf{Q}^t_{\mathcal{V}}$ is stationary and ergodic. We denote the stationary version of queue lengths in the nonpersistent set by $\overline{\mathbf{Q}}_{\mathcal{V}}$. If we initialize $\mathbf{Q}^t_{\mathcal{V}}$ with its stationary distribution, the departure process from jth users' queue in $\Theta^{\mathcal{P}}$ is also stationary. Let $\mathcal{V} = \{v_1, v_2, \cdots, v_{|\mathcal{V}|}\}$ and define $\mathcal{X}(\mathbf{Q}) = (I(Q_1 > 0), I(Q_2 > 0), \cdots, I(Q_{|\mathbf{Q}|} > 0))$, where $I(\cdot)$ is the indicator function. Also, for $\mathbf{z}_{\mathcal{V}} = (z_1, z_2, \cdots, z_{|\mathcal{V}|}) \in \{0,1\}^{|\mathcal{V}|}$, define $\mathcal{C}(\mathcal{V}, \mathbf{z}_{\mathcal{V}}) = \{v_i : z_i = 1, i \leq |\mathcal{V}|\}$. Let $P^j_{\Theta^{\mathcal{P}}}$ be the probability of success of the jth user in $\Theta^{\mathcal{P}}$ in the stationary version constructed above. Then, by [3] we have,

$$P^j_{\Theta^{\mathcal{P}}} = p_j \left[\sum_{\mathbf{z}_{\mathcal{V}} \in \{0,1\}^{|\mathcal{V}|}} \Pr\{\mathcal{X}(\overline{\mathbf{Q}}_{\mathcal{V}}) = \mathbf{z}_{\mathcal{V}}\} \left(\sum_{\mathcal{B} \subseteq \mathcal{U} \cup \mathcal{C}(\mathcal{V}, \mathbf{z}_{\mathcal{V}}) \backslash j} \prod_{k \in \mathcal{B}} p_k \prod_{l \in \mathcal{U} \cup \mathcal{C}(\mathcal{V}, \mathbf{z}_{\mathcal{V}}) \backslash (j \cup \mathcal{B})} \overline{p}_l q_{j|j \cup \mathcal{B}} \right) \right] \tag{17}$$

Now define a region $\mathcal{R}(\mathbf{q}^{\mathcal{M}})$ recursively as,

$$\mathcal{R}(\mathbf{q}^{\mathcal{M}}) = \bigcup_{\mathcal{P}} \{\boldsymbol{\lambda}_{\mathcal{M}} = \{\lambda_i\}_{i \in \mathcal{M}} : \lambda_k < P^k_{\Theta^{\mathcal{P}}} \ \forall \ k \in \mathcal{U}, \text{ and } \boldsymbol{\lambda}_{\mathcal{V}} \in \mathcal{R}(\mathbf{q}^{\mathcal{V}})\} \tag{18}$$

with $\mathcal{R}(\mathbf{q}^{\{i\}}) \triangleq \{\lambda_i < p_i q^{\{i\}}_{i|i}\}$.

Now, we claim the sufficient condition for stability in the form of this theorem.

Theorem 2 *Under conditions of stochastic dominance of $\Theta^{\mathcal{P}}$ over the original ALOHA system, if $\boldsymbol{\lambda}_{\mathcal{M}} \in \mathcal{R}(\mathbf{q}^{\mathcal{M}})$ then the ALOHA system is stable.*

Proof: We use Loynes results [5] and [22]. □

We believe that the region defined by (18) is actually the *exact* stability region. The reasoning behind why $\boldsymbol{\lambda}_{\mathcal{M}} \in \mathcal{R}(\mathbf{q}^{\mathcal{M}})$ is sufficient for stability is as follows: For a particular partition $\mathcal{P}$, $\boldsymbol{\lambda}_{\mathcal{V}} \in \mathcal{R}(\mathbf{q}^{\mathcal{V}})$ is sufficient for stability (by induction arguments) of the Markov Chain consisting of the non-persistent set and this makes the departure process for queues in the persistent set stationary. Then, $\lambda_k < P^k_{\Theta^{\mathcal{P}}}\ \forall\ k \in \mathcal{U}$ is sufficient for stability of persistent queues by Loynes theorem. Thus, $\Theta^{\mathcal{P}}$ is stable and by virtue of stochastic dominance, the original system is stable.

5 Conclusions

In this paper we considered the problem of stability of slotted ALOHA for a general reception model intended to capture the behavior of a multiple antenna wireless system. We characterized the stability region of slotted ALOHA for the two user case. We found that in some cases the two user stability region has a simple structure as given by P1 and P2. We conjecture that the stability region for the $N > 2$ user case is bounded by hyperplanes in some multipacket reception regime. In fact, it would be interesting to see if we get some properties analogous to P1, P2 and P3 for the $N > 2$ stability region. Future work would be directed towards finding stronger results for the $N > 2$ case.

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