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Exploiting Decentralized Channel State Information for Random Access

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Abstract

We study the use of channel state information for random access in fading channels. Traditionally, random access protocols have been designed by assuming simple models for the physical layer where all users are symmetric and there is no notion of channel state. We introduce a reception model that takes into account the channel states of various users. Under the assumption that each user has access to his channel state information (CSI), we propose a variant of Slotted ALOHA protocol for medium access control, where the transmission probability is allowed to be a function of the CSI. The function is called the *transmission control* scheme. Assuming the finite user infinite buffer model we derive expressions for the maximum stable throughput of the system. We introduce the notion of asymptotic stable throughput (AST) that is the maximum stable throughput as the number of users goes to infinity. We consider two types of transmission control namely population independent transmission control (PITC) where the transmission control is not a function of the size of the network and population dependent transmission control where the transmission control is a function of the size of the network. We obtain expressions for the AST achievable with PITC. For population dependent transmission control, we introduce a particular transmission control that can potentially lead to significant gains in AST. For both PITC and PDTC, we show that the effect of transmission control is equivalent to changing the probability distribution of the channel state. The theory is then applied to CDMA networks with Linear Minimum Mean Square Error (LMMSE) receivers and Matched Filters (MF) to illustrate the effectiveness of utilizing channel state. It is shown that through the use of channel state, with an arbitrarily small power, it is possible to achieve an AST that is lower bounded by the spreading gain of the network. This result has implications for the reachback problem in large sensor networks.

Keywords

Fading Channels, Channel State Information, Random Access, Slotted ALOHA, Maximum Stable Throughput, CDMA, Transmission Control, Reachback

I. Introduction

The rapid increase in the demand for data rate over wireless channels has led to a rethinking of the traditional network architecture and design principles. Cross layer design, where information is exchanged between layers is being explored as an alternative to the traditional design paradigm [1]. In this context, allowing interaction between MAC and PHY layers seems natural, especially for mobile wireless communication where the channel quality is changing with time. As illustrated in Figure 1, users might experience different channel conditions and this knowledge can be used to control the access to medium and improve the throughput of the network. The source of asymmetry between users might be due to various parameters like propagation channel gain, distance from the base station, and transmit power capabilities etc.

There is a recent line of work showing that the knowledge of channel can crucially change the resource allocation problem for multi-access fading channels [2], [3], [4], [5], [6]. In fact, it has been shown that the strategy that maximizes the information theoretic sum capacity is the one that allows at most the user with the best channel state to transmit [2], [3], [4]. This is currently being referred to as *opportunistic* communications. The strategy implies that the performance of the network is dictated by peak channel state rather than the average. Hence, the performance of the system improves with number of users. This effect was termed *multi-user diversity.* In fact, in a later work Viswanath *et al.* [7] have shown that it is

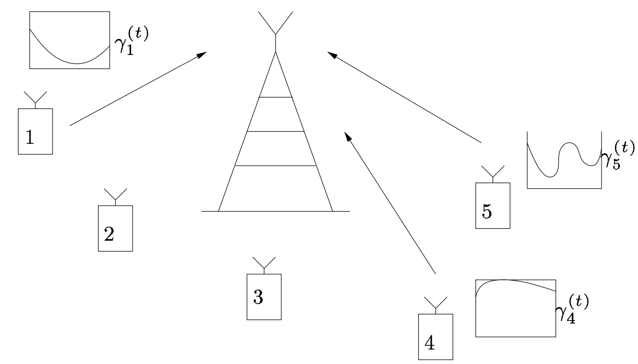

Fig. 1: Cellular Uplink

figure

beneficial to artificially induce fluctuations in the channel state in order to obtain multi-user diversity on the downlink. Further work on inducing fluctuations in the channel state in downlink has been considered in [8]. Throughput optimal scheduling for downlink over time-varying channels by a central controller has been considered in [9], [10] All these strategies require the knowledge of the channel states of all the users in the network. While this assumption might be reasonable for channel allocation on the downlink, a similar assumption on the uplink is not easy to justify.

Resource allocation on the uplink, specifically power control, with each user having access to his channel state alone was considered by Telatar and Shamai in [11]. It was shown that the loss due to the decentralized strategy in information theoretic capacity is minimal. Viswanath *et al.* [12] have shown the asymptotic optimality of a decentralized power control scheme for a multi-access fading channel that uses CDMA with an optimal receiver. The effect of decentralized power control on the sum capacity of CDMA with linear receivers and single user decoders was studied by Shamai and Verdu in [13].

Most papers that consider the effect of channel state information on centralized or decentralized resource allocation are information theoretic in nature. Information theoretic studies assume that data is always present at each user and this makes the analysis of network throughput, delay and buffer overflow with bursty sources difficult [14], [15]. The field of random access protocols is an approach to the multi-access problem that explicitly takes into account the burstiness of sources. In this paper, we will consider the use of decentralized channel state information for the design of random access protocols in the MAC layer.

As has been noted in [14], [15], the field of random access is built upon simplistic models for the physical layer. Random access protocols like traditional ALOHA, splitting algorithms and CSMA have all been developed assuming that the physical layer behaves like a collision channel. To conduct a meaningful study of the use CSI in random access, it is necessary to develop models that can first incorporate the channel states of the transmitting users and second abstract the increasing sophistication of the underlying signal processing algorithms. One such model is the multipacket reception (MPR) model introduced by Ghez *et al.* [16], [17]. It is possible to model the simultaneous reception of multiple packets using this model but the level of abstraction does not allow for the incorporation of the channel state information of the transmitting users. As a result, the version of ALOHA proposed in [16], [17] is symmetric with respect to the users. Random access protocols that are built upon the multi packet reception model have been proposed by Zhao and Tong [18], [19]. Again, there is no concept of channel state in these protocols. Random access for general reception models have also been considered in [20], [21]. However, both these papers do not consider the use of channel state information.

The contents and contributions of this paper can be broadly separated into two parts. In the first part, we focus on deriving a general theory of random access with channel state information. Our main contributions in this part can be summarized as follows :

- We introduce a model for the physical layer where the reception is allowed to depend on the *channel states* of the transmitting users and it is also possible to model the simultaneous reception of multiple packets. Any parameter that influences the reception could be chosen as channel state. Examples include propagation channel gain, position of the mobile with respect to the base station etc. This model can be considered as a generalization of the multi-packet reception model proposed by Ghez *et al.* in [16], [17]. Similar generalizations have also been considered in [20], [21].
- A variant to the classical Slotted ALOHA protocol where the knowledge of channel state is utilized to vary the transmission probability is used as the random access protocol. The function that maps the channel state information to the probability of transmission is termed the *transmission control* scheme.
- Maximum stable throughput [22] is used as a figure of merit to compare different transmission control schemes. We assume a network with finite number of users and infinite buffers and derive the expression of maximum stable throughput of the network as a function of the reception model, CSI distribution and the transmission control used. The notion of asymptotic stable throughput (AST) defined as the maximum stable throughput of the network as the number of users of go to infinity is introduced. This metric is important because it is relevant to large networks, and it is also amenable to derive “good” transmission control algorithms.
- Two types of transmission control schemes are studied namely population independent transmission control (PITC) and population dependent transmission control (PDTC). Population independent transmission control does not use the size of the network. Such a strategy is attractive when nodes are added and eliminated from the network from time to time because it is not necessary to keep track of the size of the network. We derive expressions for AST with population independent transmission and characterize what can be achieved by varying the transmission probability as a function of channel state but not the size of the network. In contrast, population dependent transmission control, as the name suggests, refers to transmission control schemes that are a function of the size of the network. We introduce a particular PDTC scheme, evaluate its AST and show that it can be used to obtain significant gains. For either type of control, the effect of using a transmission control sequence is shown to be equivalent to changing

the probability distribution of the channel state. Thus, the problem is one of identifying the good target distributions for various reception models.

In the second part we apply the results of the general theory to CDMA networks and demonstrate the effectiveness of the proposed strategies. We focus on the application of results to CDMA networks that use either a Linear Minimum Mean Square Error (LMMSE) multi-user receiver or a Matched Filter. This context provides us with two particular reception models for which the theory can be applied. For this application, we assume that the propagation channel gain is used as the channel state and it is assumed that the channel undergoes Rayleigh fading. Our main contributions in this part are as follows :

- We characterize the gain in AST through population independent transmission control. It is shown that the gain possible through this technique is quite limited.
- For population dependent transmission control, we identify the class of distributions that are good target distributions and construct transmission control schemes that can achieve this target distribution.
- We show that if we use an MMSE multi-user detector as the receiver, with an arbitrarily small power, it is possible to obtain an AST that is lower bounded by the spreading gain of the system.

The final comment above is important for the uplink of networks that have a large number of nodes but each is equipped with a small power. The regime of large number of nodes and small power is relevant to sensor networks. Thus the theory that we have derived finds an important application in the reachback problem in sensor networks. For us, reachback refers to the data gathering phase of the operation of sensor networks. Typically, hundreds and thousands of sensors, each with limited transmission power capabilities, are deployed in order to collect some information and this information has to be relayed back through some collecting agent like the airplane that is shown in Figure 2. Thus, our results for CDMA networks have an important implication on the design of the protocol stack for sensor networks.

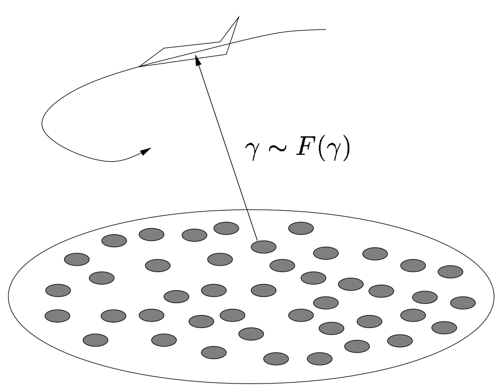

Fig. 2: Reachback in Sensor Networks

figure

Almost all other related work in collision resolution is in the analysis and design of the Slotted ALOHA protocol for the capture model, a specific model that can be represented with the proposed general reception model. The performance of Slotted ALOHA for uplink in fading channels both with and without capture has been previously explored in [23], [24], [25], [26] and the references there in. But these papers did not assume that the users have access to their channel state information. Design of retransmission probability was considered in [27], [28], [29]. An important concern in these papers was to make the protocol fair to all the users. In [30] Liu and Polydoros study the design of retransmission probabilities to maximize the throughput, but it was assumed that the design was done by a central controller who has access to channel state of all the users. In this paper, the retransmission probabilities are designed in a distributed way since each user has access only to his CSI. The Slotted ALOHA scheme where mobiles have knowledge of the uplink SNR was considered in [31], [32]. In [31], Qin and Berry used this knowledge to vary the power of transmission but the transmission probability was kept fixed. It was shown that with the choice considered, the throughput increases with the number of users. The reception model considered was a collision model. In [32], the design of transmission probability was chosen in a heuristic fashion and it was not optimized. In [33], Chockalingam *et al.* studied the design of slotted ALOHA for correlated Rayleigh fading channel. It was not assumed that the mobiles have access to the channel state but it was shown that the correlation in the fading channel can exploited to improve the throughput of ALOHA. Stability analysis for capture model was considered in [34] by Sant and Sharma. It was not assumed that the nodes have access to their channel state information. The retransmission probabilities of different users was therefore kept fixed. Characterization of stability region for Slotted ALOHA in networks with multiple-antennas (without the use of CSI) has been considered in [21].

The rest of the paper is organized as follows. In Section II, we describe the system model in detail. In Section III, we derive the expression for the maximum stable throughput of the system under consideration. In Section IV, we introduce the notion of asymptotic stable throughput (AST) and derive the expressions for AST for various types of transmission. In Section V, the theory is applied to CDMA networks. In Section VI, we list our concluding remarks and describe some interesting directions for related future research. The proofs of all the theorems and propositions have been included in the Appendix.

II. System Model

We consider a network where n users are communicating with a base station over a common channel. Each user has a buffer of infinite length for the incoming packets until they are sent successfully to the base station. Time is slotted into intervals equal to the time required to transmit a packet. We make the slot time equal to one time unit and slot t is assumed to occupy the time $[t, t+1)$. We denote by $X_m^{(t)}$ the number of incoming packets to user m during time slot t. The packet arrival process for different $X_m^{(t)}$ for $m = 1, \cdots, n$ and $t \in \mathbb{N}$ is assumed to be independent and identically distributed as well. The arrival process has a finite mean $\frac{\lambda}{n}$ (so that the cumulative input rate is λ) and finite variance. The above model for the arrival process is the same as that in [22] for a symmetric system.

The channel between the m^{th} user and the base station during slot t is parametrized by $\gamma_m^{(t)}$. It is assumed that the quantities $\gamma_m^{(t)}$ for $m = 1, \cdots, n$ and $t \in \mathbb{N}$ are independent and identically distributed with probability distribution $F(\gamma)$. Further, we assume that the user m has access to the uplink CSI $\gamma_m^{(t)}$ at time t.

We define a general reception model that is given by a set of n functions. The k^{th} function assigns probabilities to all the possible outcomes conditioned on the event that k users transmitted and that their channel states are given by $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_k)$. Assuming that k users transmitted, we let $\Theta_k = (\theta_k^{(1)}, \cdots, \theta_k^{(k)})$ be a binary k-tuple that represents the outcome of a slot. The bit $\theta_k^{(1)}$ equal to one represents the success of user 1 and so on. The k^{th} function $\Phi^{(k)}(\gamma_1, \cdots, \gamma_k; \Theta_k)$ is the probability of outcome Θ_k when k users whose CSI is given by $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_k)$ transmit. That is,

$$\Phi^{(k)}(\gamma_1, \cdots, \gamma_k; \Theta_k) = \Pr\{\Theta_k | k \text{ users transmit}, \boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_k)\}. \tag{1}$$

Define $\Psi^{(k)}(\gamma_1, \cdots, \gamma_k)$ as the expected number of packets successfully demodulated when the CSI of the transmitting users is $(\gamma_1, \cdots, \gamma_k)$, that is,

$$\Psi^{(k)}(\gamma_1, \cdots, \gamma_k) \triangleq \sum_{i=1}^{k} \mathrm{E}\{\theta_k^{(i)} | k \text{ users transmit}, \boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_k)\}. \tag{2}$$

Given a distribution function $F(\cdot)$, define $C_k(F(\cdot))$ as the expected number of packets received conditioned on k users transmit and their CSI is distributed i.i.d according to $F(\cdot)$. That is

$$C_k(F(\cdot)) \triangleq \sum_{i=1}^{k} \mathrm{E}\{\theta_k^{(i)} | k \text{ users tx}\}. \tag{3}$$

Note that this model allows the reception of multiple packets simultaneously. Special cases of this reception model are the classical collision model, capture model and Multi-packet reception model [16].

We impose some constraints on the reception model that hold for many practical scenarios. For each k, we assume that if we permute the CSI $(\gamma_1, \cdots, \gamma_k)$ and apply the same permutation to the bits of Θ_k, the value of $\Phi^{(k)}(\cdot)$ does not change. That is, we assume long term symmetry among the users. This condition has been relaxed in the reception model considered in [20], [21]. Further, we assume that for any given $(\gamma_1, \cdots, \gamma_k)$, adding an extra user decreases the probability of packets success for each of the k users. That is, for all $\gamma_1 \geq 0, \cdots, \gamma_{k+1} \geq 0$, for all $1 \leq i \leq k$,

$$\sum_{\Theta_k} \theta_k^{(i)} \Phi^{(k)}(\gamma_1, \cdots, \gamma_k; \Theta_k) \geq \sum_{\Theta_{k+1}} \theta_{k+1}^{(i)} \Phi^{(k+1)}(\gamma_1, \cdots, \gamma_{k+1}; \Theta_{k+1}). \tag{4}$$

The parameter $\gamma_m^{(t)}$ can be used to model various parameters that influence the reception. Examples include physical channel gain, position of the mobile etc. In some cases, it is possible to abstract the reception of an uplink using multiple antennas into the above model.

In the ALOHA protocol analyzed in [22], if the user m has a packet to transmit, he transmits it with a probability p_m. We consider a more general random access scheme where the probability of transmission for each user is allowed to be a function of his CSI $\gamma_m^{(t)}$. The function is called the *transmission control* scheme and is denoted by $s(\cdot)$. Thus we assume that in slot t, if user m has a packet then it is transmitted with probability equal to $s(\gamma_m^{(t)})$. At the end of slot t, the base station broadcasts the indexes of those users whose packets it was able to demodulate successfully. The type of ALOHA protocol considered in this paper, where the new arrivals are not transmitted immediately, is known as ALOHA with delayed first transmission. This is in contrast to ALOHA with immediate first transmission where new arrivals are transmitted in the slot immediately following their arrival.

III. Maximum Stable Throughput

In this section we derive the expression for maximum stable throughput as a function of the CSI distribution, reception model and the transmission control. The system is defined to be stable if for each node the queue size does not go to infinity . In other words, given a positive number $0 < \epsilon \leq 1$, there exists a buffer size such that the probability of buffer over flow is less than ϵ. It should be obvious that stability is one of the important requirements for a network. The requirement of stability can be said to impose a mild requirement on delay.

We now define the notion of maximum stable throughput in a formal manner. Let the n-tuple $\mathbf{N}^{(t)} = (N_1^{(t)}, N_2^{(t)}, \cdots, N_n^{(t)})$ be the length of the buffers at each node at the beginning of slot t. We say that the system is stable for a particular arrival process, if for $\mathbf{x} \in \mathbb{N}_+^{\aleph}$, there exists a $H(\mathbf{x})$ such that

$$\lim_{t\to\infty} \Pr\{\mathbf{N}^{(t)} < \mathbf{x}\} = H(\mathbf{x}) \quad \lim_{\mathbf{x}\to\infty} H(\mathbf{x}) = 1, \tag{5}$$

where $\mathbb{N}_+$ is the set of non-negative integers. This notion of stability is also used in [22]. We will see that the stability of the system can be characterized by λ, the cumulative mean of the arrival process alone. This will allow us to define maximum stable throughput as the supremum of all cumulative input rates λ for which the system is stable. The following theorem gives the expression for the maximum stable throughput of the system in terms of the transmission control, reception model and the underlying CSI distribution.

Theorem 1: Given $F(\gamma)$ the distribution function of the CSI, the transmission control $s(\gamma)$ and the reception functions $\{\Phi^{(k)}(\cdot)\}_{k=1}^n$, the maximum stable throughput is given by

$$\lambda_n(s(\cdot)) = \sum_{k=1}^{n} \binom{n}{k} \left(1 - \int_0^\infty s(\gamma)dF(\gamma)\right)^{n-k} \left(\int_0^\infty \cdots \int_0^\infty s(\gamma_1)\cdots s(\gamma_k) \Psi^{(k)}(\gamma_1, \cdots, \gamma_k) dF(\gamma_1)\cdots dF(\gamma_k)\right). \tag{6}$$

If $p_s \triangleq \int s dF \neq 0$, then

$$\lambda_n(s(\cdot)) = \sum_{k=1}^{n} \binom{n}{k} (1-p_s)^{n-k} p_s^k C_k(G_s(\cdot)), \tag{7}$$

where the distribution function $G_s(\cdot)$ is

$$G_s(\cdot) = \frac{\int_0^\gamma s dF(\gamma)}{p_s}. \tag{8}$$

Proof : Refer to the appendix

It should first be noted that p_s that is defined above is the unconditional probability of transmission and the distribution $G_s(\cdot)$ is the distribution of CSI conditioned on the event that a user transmits, that is it is the a posteriori distribution of the channel state. The expression in (7) can then be given the following intuitive interpretation. Suppose that all the users in the systems had backlogged (which is in some sense the worst scenario), then the expression for $\lambda_n(s(\cdot))$ gives the expected number of packets leaving the system. It is intuitively reasonable that the maximum stable throughput should depend on $F(\gamma)$ only through $G_s(\cdot)$ because this is the distribution of channel state that the base station "sees" ; the underlying distribution of CSI is not relevant. The power of using a transmission control is that it allows us to manipulate the a posteriori CSI distribution $G_s(\cdot)$. Thus, we would like to steer the underlying distribution to "good" a posteriori distributions by the use of the transmission control. The problem however is more complicated because the transmission control also affects the probability of transmission p_s. Thus, it is possible that transmission controls that lead to good a posteriori distributions might lead to an extremely low probability of transmission. It is this coupling that makes it difficult to find optimal transmission controls for various reception models. In the following section, we will consider the SNR threshold model as an example for which it is possible to obtain the optimal transmission control. The optimal transmission control for a simplified capture model was considered in [35]. Obtaining the optimal transmission control for the general capture model and other reception models is interesting and useful but is also hard.

A. An Example

In this section, we apply the results derived in the previous section for the SNR threshold model and obtain the maximum stable throughput. We then optimize the transmission control by maximizing this stable throughput. These results also shed light on how a transmission control can be used to increase the stable throughput of the system.

The SNR of the uplink is taken as the channel state parameter and the reception model is defined as follows. We assume that a user is successfully demodulated if no other user transmits, and if his SNR is larger than a given threshold γ_0. The reception model for this is given by

$$\Phi^{(1)}(\gamma;1) = \begin{cases} 1 & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases} . \tag{9}$$

The function $\Phi^{(1)}(\gamma;0)$ is of course equal to $1-\Phi^{1}(\gamma;1)$. For $k \geq 2, \Phi^{(k)}(\cdots)$ is identically equal to zero. This model is similar to the collision channel except that it also takes into account the channel state of the transmitting user.

Given a transmission control $s(\gamma)$, the maximum stable throughput is given by

$$\lambda_n(s(\cdot)) = n\left(1-\int_0^\infty s(\gamma)dF(\gamma)\right)^{n-1}\left(\int_{\gamma_0}^\infty s(\gamma)dF(\gamma)\right). \tag{10}$$

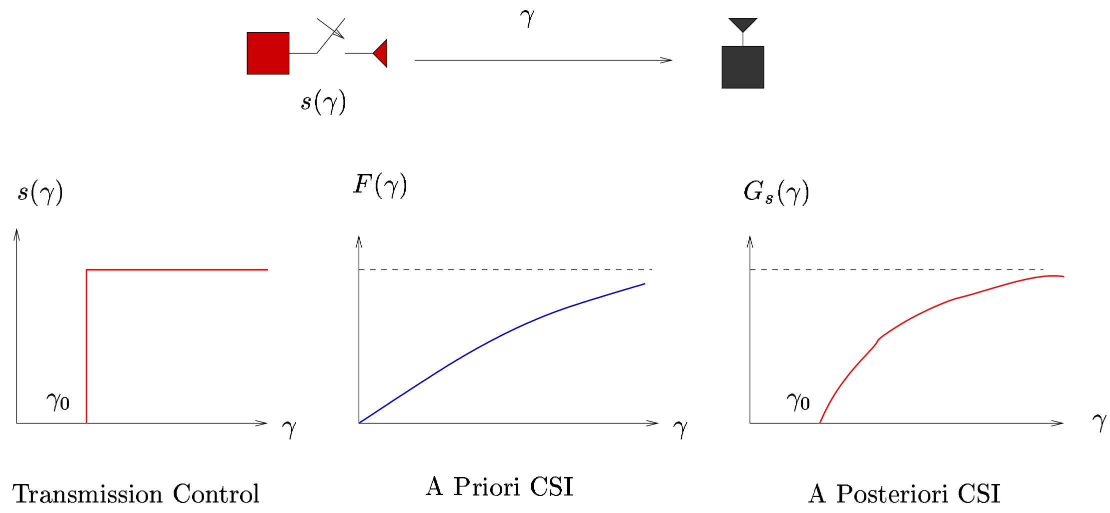

Fig. 3: Shaping of the A Priori Distribution

figure

The optimal transmission control is then obtained as

$$s^*(\cdot) = \arg\max_{s(\cdot)} \lambda_n(s(\cdot)). \tag{11}$$

We then have the following theorem.

Theorem 2: Denote $p_{\gamma_0} = P\{\gamma \geq \gamma_0\}$. The optimal transmission control is

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ \min\left(\frac{1}{np_{\gamma_0}}, 1\right) & \gamma \geq \gamma_0 \end{cases}, \tag{12}$$

and the corresponding maximum stable throughput is

$$\lambda_n(s^*(\cdot)) = n \min\left(\frac{1}{n}, p_{\gamma_0}\right)\left(1 - \min\left(\frac{1}{n}, p_{\gamma_0}\right)\right)^{n-1}. \tag{13}$$

Proof : Refer to the appendix.

The transmission control is a step function and we find that, as expected, if $\gamma < \gamma_0$, the mobiles do not transmit. If $\gamma \geq \gamma_0$, the probability of transmission is chosen such that the average number of transmitting users in each slot is equal to one. In order to understand the role of transmission control, we illustrate the a priori and a posteriori distributions of CSI in Fig. 3. We see that the a posteriori distribution starts from $\gamma = \gamma_0$ which means that the base station believes that the channel states below γ_0 do not occur. We would like to finally note that the optimal transmission control is not unique.

Figure 4 illustrates the variation of average delay with total input rate under the SNR threshold model. The threshold γ_0 was set at -5 dB. For the conventional transmission control, when a station has a packet to transmit it transmits the packet with a probability $\frac{1}{n}$. It can be seen that the optimal transmission control has a higher maximum stable throughput and also a lower delay at every load.

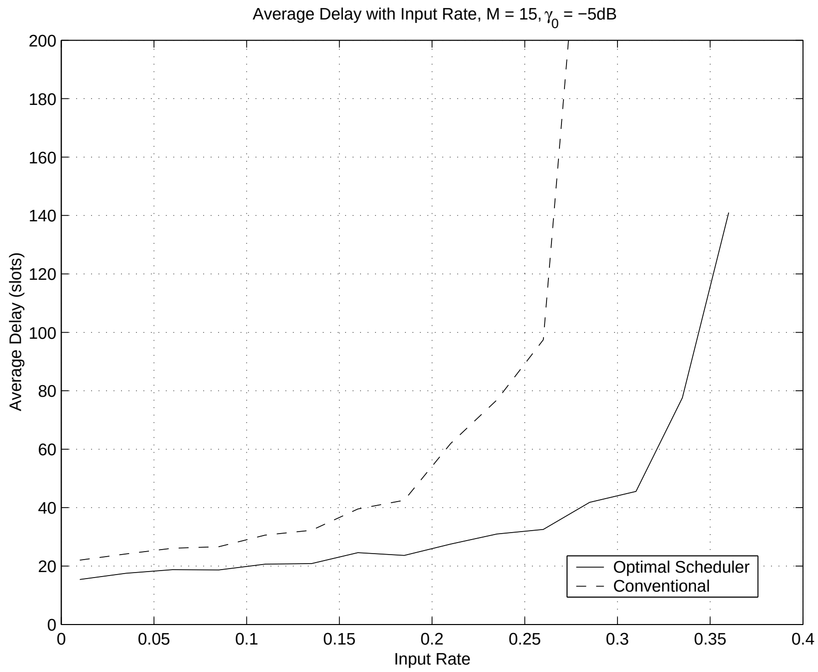

Fig. 4: Delay Vs Input Rate

figure

An interesting problem that we have not considered is the selection of γ_0 and how the various physical layer parameters and the signal processing algorithms influence its choice.

IV. Asymptotic Stable Throughput

In this section we define the notion of asymptotic stable throughput (AST) of a network and consider the problem of designing transmission controls that are optimal with respect to the asymptotic stable throughput. We consider two types of transmission control : the population dependent transmission control (PDTC) where the transmission control depends on the total number of users in the network, and the simpler population independent transmission control (PITC) where the transmission control is not allowed to be a function of total number of users in the network.

We know that given the number of users in the network n, reception model $\Phi^{(k)}(\cdot)$, the CSI distribution $F(\gamma)$, and the transmission control $s(\cdot)$ such that $p_s > 0$, the maximum stable throughput for the network is given by

$$\lambda_n^*(s(\cdot)) = \sum_{k=1}^{n} \binom{n}{k} (1-p_s)^{n-k} p_s^k C_k(G_s(\cdot)). \tag{14}$$

The asymptotic stable throughput (AST) is defined as the maximum stable throughput as the number of users in the network goes to infinity. Such a metric is of value for "large" networks, and it is possible to obtain transmission controls that are asymptotically good based on this metric. This enables us to design transmission controls for those reception models for which it is difficult to find the transmission controls that are optimal with respect to the maximum stable throughput. The formal definition of AST is as below.

Definition 1: Given the distribution function of CSI $F(\gamma)$, the transmission control sequence $s_n(\gamma)$ and the reception functions $\{\Phi^{(k)}(\cdot)\}_{k=1}^{\infty}$, the asymptotic stable throughput is defined as

$$\lambda_{\infty}(\{s_n(\cdot)\}) \triangleq \liminf_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_{s_n}(\cdot)). \tag{15}$$

For what follows, we impose the following technical restrictions on the kind of reception models $\{\Phi^{(k)}(\cdot)\}_{k=1}^{\infty}$ considered :

A1 For any distribution function $F(\gamma)$, $\lim_{k\to\infty} C_k(F(\cdot)) \triangleq C_{\infty}(F(\cdot))$ exists.

This restriction is quite mild and in fact holds for most reception models considered.

A. Population Independent Transmission Control

We first consider the scenario where the transmission control sequence is such that it does not depend on n. That is $s_n(\gamma) = s(\gamma), \forall n$. This kind of transmission control is termed Population Independent Transmission Control (PITC). Such transmission controls are interesting because they are simpler to implement and they can be expected to be robust to the size of the network. In cases where nodes may enter and leave the network it is easier to use a PITC because it is not necessary to keep track of the size of the network.

The asymptotic stable throughput with PITC becomes

$$\lambda_{\infty}(s(\cdot)) \triangleq \liminf_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k} (1-p_s)^{n-k} p_s^k C_k(G_s(\cdot)). \tag{16}$$

The asymptotic stable throughput can be given a simpler characterization as follows.

Proposition 1: Given the transmission control, $s(\cdot)$, the asymptotic stable throughput is given by

$$\lim_{k\to\infty} C_k(G_s(\cdot)). \tag{17}$$

Proof : The proof follows from [17].

In contrast, the AST for population independent transmission control that does *not* depend on the channel state is given by

$$\lim_{k\to\infty} C_k(F(\cdot)). \tag{18}$$

Thus the effect of the transmission control for PITC is equivalent to changing the underlying CSI distribution. It is therefore important to determine the set of probability distributions that can be reached through PITC from $F(\gamma)$. Given $F(\gamma)$, it is easy to see that the set of distributions that can be reached through PITC is given by

$$\Lambda_F \triangleq \left\{ G(\gamma) : \exists s(\cdot) \in \mathcal{S} \ s.t \int_0^{\infty} s(\gamma) dF(\gamma) > 0, G(\gamma) = \frac{\int_0^{\gamma} s(\gamma) dF(\gamma)}{\int_0^{\infty} s(\gamma) dF(\gamma)} \right\}, \tag{19}$$

where

$$\mathcal{S} \triangleq \{s(\gamma) : 0 \leq s(\gamma) \leq 1, \ \forall \ \gamma\}. \tag{20}$$

Thus we have the following proposition.

Proposition 2: The supremum of all possible stable throughput by optimizing the transmission control function is given by,

$$\sup_{s(\cdot)} \lambda_\infty(s(\cdot)) = \sup_{G(\cdot)\in\Lambda_F} \lim_{n\to\infty} C_n(G(\cdot)). \tag{21}$$

In what follows we derive the properties of the distributions in Λ_F and try to ascertain how large this set is. We first list some simple properties of the functions $G(\gamma) \in \Lambda_F$.

P1 $G(\gamma)$ is a distribution function.

P2 $\mu_F(A) = 0$ implies $\mu_G(A) = 0$ (Notation : $G \ll F$).

P3 There exist a positive constant $C < \infty$ such that the Radon-Nikodym derivation $\frac{dG}{dF} < C$ for all γ.

We now show that in fact the above three properties characterize the set Λ_F, namely if there exists a function $G(\cdot)$ satisfying the properties above then it belongs to Λ_F. Given $G(\cdot)$ and $F(\cdot)$ satisfying the properties given above, define the transmission control as

$$s(\gamma) = \frac{1}{C}\frac{dG}{dF}. \tag{22}$$

It is easy to see that the a posteriori CSI distribution with this transmission control is equal to $G(\gamma)$ and therefore $G(\cdot) \in \Lambda_F$.

Thus, if the underlying channel state distribution is $F(\gamma)$, it is possible to steer the conditional distribution of the channel state to any $G(\gamma)$ that satisfies the properties listed above by choosing the transmission control as

$$s(\gamma) = \frac{dG}{dF}\frac{1}{\sup \frac{dG}{dF}}. \tag{23}$$

It is important to determine how limiting the restriction to the set Λ_F is. As we shall see later, this restriction has an important bearing on the maximum achievable AST with PITC for many reception models and state distributions $F(\gamma)$.

B. Population Dependent Transmission Control

We now consider the more general case when the transmission control is allowed to be a function of number of users in the network. As discussed previously, given a sequence of transmission controls $s_n(\gamma)$, the asymptotic stable throughput is defined as

$$\lambda_\infty(\{s_n(\cdot)\}) = \liminf_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k}(1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_{s_n}(\cdot)). \tag{24}$$

We will first derive the AST for transmission control sequences that do not use channel state information and then introduce a simple population dependent transmission control sequence that can improve significantly over this AST. The results in [17] can be directly used to show that if we use a transmission control $s_n(\gamma) = \min\left(\frac{x}{n}, 1\right)$, we achieve an AST equal to

$$e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)) \triangleq f(x, F). \tag{25}$$

The following proposition says that in fact the control above achieves all possible AST, and it is not possible to do better using a more complicated transmission control.

Proposition 3: If the sequence of transmission control $s_n(\gamma)$ is chosen to be independent of γ but as a function of n alone, then the maximum possible AST is given by $\sup_x f(x, F)$, where $F(\gamma)$ is the distribution of γ.

Proof : The proof follows directly from [17].

It is possible to construct a simple sequence of transmission controls that improves significantly upon the AST obtained above. Let $T(\cdot)$ be a distribution function such that $T(\cdot) \ll F(\cdot)$. From the Radon-Nikodym theorem, there exists a non-negative function $\frac{dT}{dF}$ such that

$$\mu_T(A) = \int_A \frac{dT}{dF} dF. \tag{26}$$

The sequence of transmission controls is chosen as

$$s_n(\gamma) = \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right). \tag{27}$$

The following proposition characterizes the achievable throughput.

Proposition 4: With the sequence of transmission controls chosen as

$$s_n(\gamma) = \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right), \tag{28}$$

the asymptotic stable throughput is given by

$$\lambda_\infty(\{s_n(\cdot)\}) = e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)) = f(x, T). \tag{29}$$

Proof : Refer to the Appendix.

By comparing Proposition 3 with Proposition 4, it can be seen that the effect of the chosen transmission control sequence is to effectively change the CSI distribution from $F(\gamma)$ to $T(\gamma)$. From Propositions 4 and 1, we can see that the advantage of using population dependent transmission control is two-fold. Firstly, the set of distributions that can be reached is larger than the set Λ_F because we do not need the target distribution to obey P3 listed above. Secondly, the performance is no longer limited by $C_\infty(T(\cdot))$ but is given by $\sup_x f(x, T)$. It can be shown using techniques in [17] that the function $f(x, T)$ has the property that

$$\lim_{x \to \infty} f(x, T) = C_\infty(T(\cdot)). \tag{30}$$

This implies that $\sup_x f(x, T) \geq C_\infty(T(\cdot))$.

The intuition behind choosing the particular sequence of transmission controls is that firstly we have $p_{s_n} \to 0$ while $np_{s_n} \to x$ and secondly we have the a posteriori distribution

$$G_{s_n}(\gamma) \quad \to \quad T(\gamma) \qquad \text{point wise.} \tag{31}$$

The first condition ensures that the number of transmission attempts in any given slot converges to a Poisson random variable with mean x, and the second condition ensures that the a posteriori CSI distribution converges to $T(\gamma)$.

It can be seen that through a judicious choice of transmission control sequence, it is possible to achieve an AST of

$$\lambda_c^* = \sup_{x, T(\cdot) \ll F(\cdot)} e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)) = \sup_{x, T(\cdot) \ll F(\cdot)} f(x, T). \tag{32}$$

The quantity λ_c^* is in some sense the capacity associated with the reception model, CSI distribution $F(\cdot)$, and the protocol proposed. For a given reception model, it is important to characterize λ_c^* and find distributions $T(\cdot)$ that achieve an AST that is close to λ_c^*.

For a given reception model and CSI distribution $F(\gamma)$, choosing a target distribution that guarantees improvement is in general not easy. This is due to the reason that the AST is equal to $\sup_x f(x, F)$ which in turn depends on all of $C_k(F(\cdot))$. However, it is at times easy to characterize the value of $f(x, F)$ as $x \to \infty$ and this offers us a way of comparing different distributions. As mentioned previously, for any $F(\cdot)$ [17]

$$\lim_{x \to \infty} f(x, F) = C_\infty(F(\cdot)). \tag{33}$$

Hence, if for a distribution function $F(\cdot)$, we find a distribution $T(\cdot)$ such that $C_\infty(T(\cdot)) > \sup_x f(x, F)$, then improvement is guaranteed by using a transmission control that changes the distribution to $T(\cdot)$. However, it is important to note that $C_\infty(T(\cdot)) > C_\infty(F(\cdot))$ does not in general guarantee improvement.

V. Application to CDMA Networks

In this section, we apply the results derived in the previous section to the uplink of CDMA networks. This application illustrates the theory and also demonstrates the magnitude of gains possible through the use of CSI.

In order to apply the theory, we need to first select the parameter that will used as the channel state. The choice of the channel state parameter might be influenced by issues like potential gain and ease of estimation. Once the channel state parameter is fixed, the distribution of the channel state information should be determined. Then a reception model as described in Section II should be developed for the physical layer processing.

For the purposes of the current application, we will choose the propagation channel gain as the channel state information. The possible models for the CSI and the distributions that arise due to these models are delineated as part of the section below on channel model. We analyze the CDMA network under two receiver structures ; one where the receiver uses a Matched Filter and the other where the receiver uses a linear MMSE multi-user receiver. The two structures give rise to two different reception models. The results for the linear MMSE multi-user receiver are presented in considerable detail and the corresponding results for the matched filter are stated in brief because they are conceptually similar to the ones for the LMMSE multi-user receiver. For each reception model, the program is to first analyze the performance possible without transmission control. Since the use of transmission control essentially changes the underlying CSI distribution, the objective then is to find distributions that improve over the existing CSI distributions. In this connection, we will show that distributions with roll-off (q.v. (36)) form "good" target distributions for population dependent transmission control and that it is possible to obtain large gains by using transmission controls that steer the underlying CSI distribution to this distribution.

A. Channel Model

The propagation channel gain from each user to the base station is selected as the channel state. Since we require that each user has access to his channel state, we imagine a time division duplex (TDD) system where the base station is transmitting a pilot tone.

If the received power is modeled as

$$P_R = KR^2 P_T, \tag{34}$$

where K is a constant, R is Rayleigh distributed and the channel state is given by $\gamma = P_R$, then the underlying CSI distribution is exponential. This corresponds to the case when a slow power control is being employed. This model is also reasonable for modeling the propagation channel gain in the reachback problem because all the nodes are typically at the same distance from the collecting station and undergo the same propagation loss and shadow fading. Thus the underlying CSI distribution for the reachback problem can be assumed to be exponential.

Another possible model for received power at the base station is

$$P_R = R^2 K_s e^{\xi} K r^{-\alpha} P_T, \tag{35}$$

where R is Rician or Rayleigh distributed, ξ is Gaussian distributed, with zero mean and standard deviation γ_s and P_T is the constant transmitted power, r is the distance from base station and α is the propagation constant that typically lies between 2 and 6. In this case the CSI distribution is a complicated function of the distribution of r, the distance from the base station. A particular property of this distribution that turns out to be very crucial is the way in which the tail of the distribution rolls off. Given a distribution function $G(\cdot)$, define δ to be the roll-off of $G(\cdot)$, if there exists a c such that $0 < c < \infty$ and [36]

$$\lim_{\gamma \to \infty} (1 - G(\gamma))\gamma^{\delta} = c. \tag{36}$$

If there exists a positive constant a such that the CDF of the distance r of a station satisfies

$$\lim_{x \to 0} F_r(x) x^{-a} = c_r, \tag{37}$$

where c_r is a positive constant. Then it can then be shown that [36] the distribution of the received power P_R above has a roll-off $\delta = \frac{a}{\alpha}$. This model corresponds to the case when there is no power control. Different possible distributions for r are the so called uniform distribution where

$$\mathrm{P}\{r < x\} = x^2 \quad 0 \leq x \leq 1, \tag{38}$$

the quasi-uniform distribution for which the density of r is given by

$$f_r(x) = xe^{-\frac{\pi x^4}{4}} \quad 0 \leq x < \infty, \tag{39}$$

and bell-shaped distribution.

B. Linear MMSE Multi-user Receiver

In this section, we study the case when the receiver uses a LMMSE multi-user receiver. We start by describing the reception model to be used and then apply the results to the this reception model.

We assume that each user is assigned a particular signature waveform that is used to modulate the data. Each packet starts with sufficient training symbols that the receiver can use to form an equalizer. The packet is assumed to be successfully demodulated if the signal to interference ratio after the LMMSE multi-user receiver is greater than β. (The parameter β is a function of modulation, code, and quality of service required for the application.) For the LMMSE receiver structure, the signal to interference ration (SIR) for each user is a complicated function of the received power and signature sequences of the transmitting users. However, if the signature sequences are random, the size of the network and the spreading gain are large, the SIR can be approximated as a simple function of the received powers [37]. Due to the heuristics in [37], we will use the following reception model for the LMMSE multi-user detector. Given that K users transmit, P_i is the power received from user i, user i goes through if

$$\frac{\gamma_i}{\sigma^2 + \frac{1}{N}\sum_{k=1,k\neq i}^{K} \frac{\gamma_i \gamma_k}{\beta\gamma_k + \gamma_i}} > \beta. \tag{40}$$

This condition can be used to completely specify the reception model as defined in Section II. The above condition can be rewritten as

$$\frac{\beta N \sigma^2}{\gamma_i} + \sum_{k=1,k\neq i}^{K} \frac{\beta\gamma_k}{\gamma_i + \beta\gamma_k} < N. \tag{41}$$

This shows that the effective interference from other users is limited to at most 1. This is the advantage of using an MMSE multi-user detector over a matched filter. In deriving this condition, it is assumed in [37] that the receiver employs a true MMSE filter or equivalently that the receivers knows the spreading sequences of the transmitting users. This assumption is not a contradiction to the fact that we are considering a random access protocol because we assume that each packet starts with training and these training symbols are used to obtain a least squares equalizer and if we have sufficient number of training symbols present we can ensure that the least squares equalizer converges to the true LMMSE equalizer derived under the assumption that the receiver knows exactly who the transmitting users are.

B.1 Population Independent Transmission Control

For population independent transmission control, the AST is without the use of CSI is $C_\infty(F(\cdot))$ and the AST with CSI is $C_\infty(G(\cdot))$ where $G(\cdot) \in \Lambda_F$. We first assume that the underlying CSI distribution $F(\cdot)$ is exponential and we evaluate C_∞ for the exponential distribution.

Proposition 5: Let $F(\cdot) = 1 - e^{-\frac{\gamma}{P_T}}$ and the noise variance be equal to σ^2, then

$$\lim_{k\to\infty} C_k(F(\cdot)) = 0. \tag{42}$$

Proof : Refer to the appendix

Thus the AST for exponential distribution is equal to zero. The following proposition gives the AST for the set of distributions that can be reached from the exponential distribution.

Proposition 6: Let $F(\cdot) = 1 - e^{-\frac{\gamma}{P_T}}$ and the noise variance be equal to σ^2, and $G(\cdot) \in \Lambda_F$, then

$$\lim_{k\to\infty} C_k(G(\cdot)) = 0. \tag{43}$$

Proof : Refer to the Appendix.

This proposition implies that it is not possible to improve the asymptotic throughput with population independent transmission control if the underlying distribution is exponential. Hence the set Λ_F is not "large enough" to improve the throughput.

We now consider the case when the distribution of the received power has a roll-off δ. This corresponds to the case when there is no power control.

Proposition 7: If $F(\cdot)$ has a roll-off δ, then

$$\lim_{k\to\infty} C_k(G(\cdot)) = \begin{cases} \frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta} + e(N,\delta) & 0 < \delta \le 1 \\ 0 & \delta > 1, \end{cases} \tag{44}$$

where $e(N,\delta)$ satisfies

$$\lim_{N\to\infty} e(N,\delta) = \frac{\sin\pi\delta}{\pi\delta}\frac{1-\delta}{2\beta^\delta}. \tag{45}$$

Proof : Refer to the appendix.

Thus for large N, we can neglect the quantity $e(N,\delta)$ and assume that AST is $\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}$. We conjecture that for any finite N

$$\max_\delta e(N,\delta) = e(N,0) = \frac{1}{2}. \tag{46}$$

For all the arguments that follow, we assume that the AST is given by $\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}$.

When the distribution of the received power has a roll-off, the asymptotic throughput is therefore not equal to zero. In order to determine if the use of CSI can increase the asymptotic stable throughput, we consider the asymptotic stable throughput of the distributions in the set Λ_F.

Proposition 8: Let $F(\gamma)$ be a distribution function with roll-off δ, $G(\cdot) \in \Lambda_F$ and $G(\cdot)$ has a roll-off δ' then $\delta' \ge \delta$. Further, for all $\delta' > \delta$, there exists a $G(\cdot) \in \Lambda_F$ such that the roll-off of $G(\cdot)$ is δ'.

Proof : Refer to the appendix.

If $\frac{1}{\beta} \le 1$ (which is typical), then the asymptotic throughput is a decreasing function of δ and if $\frac{1}{\beta} > 1$

the asymptotic throughput reaches a maximum for some value of δ that lies between 0 and 1. This fact together with Proposition 8 has the following implications on possible improvements in AST. If $\frac{1}{\beta} \leq 1$ the asymptotic stable throughput cannot be improved by steering to distributions with a roll-off. However if $\frac{1}{\beta} \geq 1$, it can be shown quite easily that for a δ, the asymptotic stable throughput can be improved by population independent transmission control if

$$\left(1 - \frac{\pi\delta}{\tan \pi\delta} \frac{1}{\delta}\right) < \log \frac{1}{\beta}. \tag{47}$$

To illustrate for example, if $\frac{1}{\beta} > e^2$ and $\delta \leq 0.5$ then improvement is possible.

Thus, for the reception model under consideration, if the transmission control is not allowed to use the size of the network, improvement in AST is not possible for most cases. As shown below, this will change quite significantly when the transmission control is allowed to use the size of the network.

B.2 Population Dependent Transmission Control

We now consider the use of channel state information for PDTC when the underlying distribution is exponential. As shown in Proposition 3, the AST obtained without the use of CSI is given by

$$f(x, F) = e^{-x} \sum_{k=0}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)). \tag{48}$$

Figure 5 illustrates the AST for LMMSE when the underlying CSI distribution is exponential and CSI is not used for transmission control. The transmit power is 4 dB over noise, spreading gain $N = 16$ and $\beta = 4dB$. The x-axis is the design variable x, the average number of transmissions in each slot. We see that it is possible to achieve an AST of approximately 2.6 packet per slot without using CSI by setting x to be approximately equal to 15 transmissions per slot. We should now find distributions $T(\cdot)$ such that $T \ll F$ and $f(x, T) > 2.6$ for some x. If $T(\cdot)$ is a distribution with a roll-off, Proposition 7 gives the value of $f(\infty, T)$, and we see that there are many distributions for which $f(\infty, T) > 2.6$. This implies that for these distributions, there do exist y such that $f(y, T)$ improves over $\sup_x f(x, F)$. We select distributions with roll-off 0.5 and 0.3 as the target distributions and Fig 6 plots $f(x, T)$ for each of them. The solid line in Figure 6 illustrates the AST when the underlying channel state is distributed exponentially with mean 4 dB. The dotted lines are the AST for distributions with a roll-off. We see that it is in fact possible to obtain significant gains over the maximum AST that can obtained without the use of CSI. From Proposition 4, a transmission control that can be used to steer the underlying exponential distribution to a distribution with roll-off is given by

$$s_n(\gamma) = \min\left(\frac{e^{\gamma} P_T}{\gamma^{1+\delta}} \frac{x}{n}, 1\right) 1_{\gamma > \gamma_0}, \tag{49}$$

where γ_0 is any fixed constant.

From Figure 6, it can also be seen that for a given AST, the mean number of transmissions required is smaller for the roll-off distributions compared to those required for the exponential distribution. This

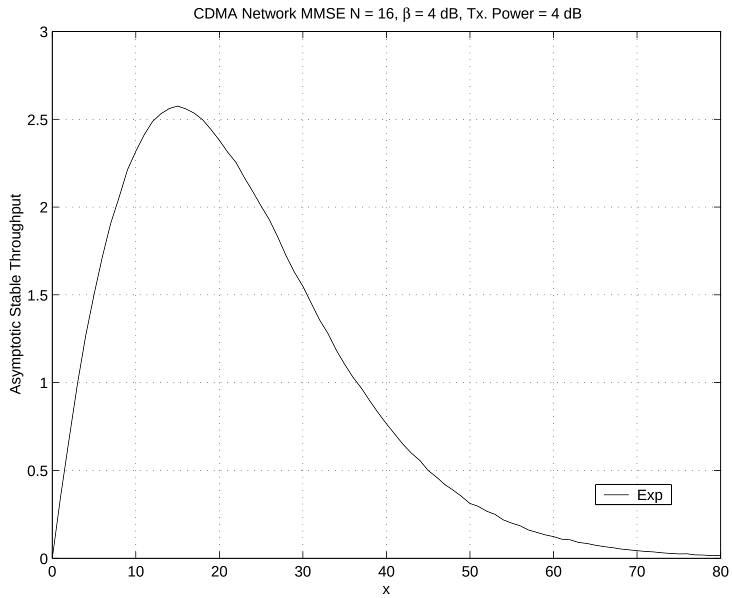

Fig. 5: AST with PDTC that does not use CSI

figure

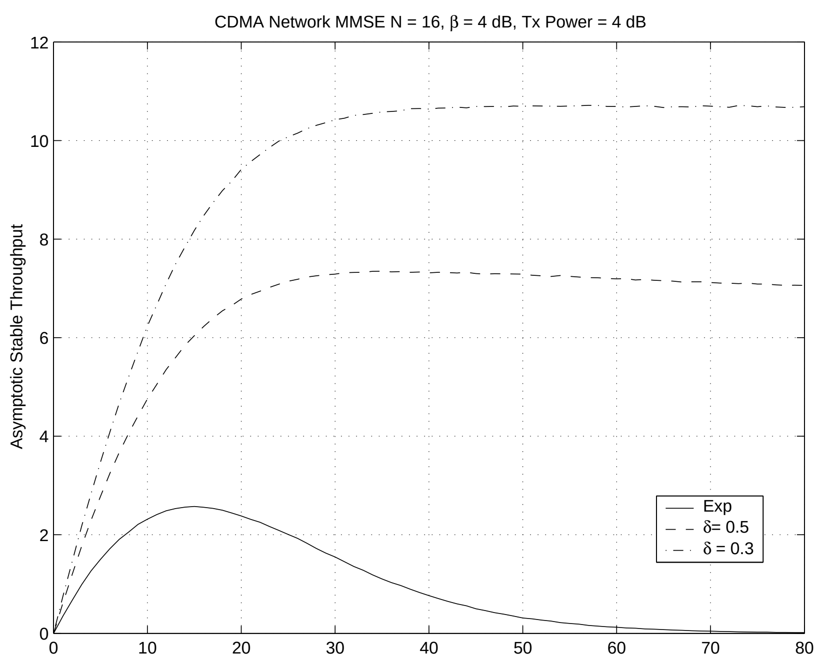

Fig. 6: AST with and without CSI for PDTC

figure

implies that utilizing a transmission control that uses CSI decreases the required average number of transmissions in a slot. This has implications on network wide power savings.

We now consider the importance of the use of CSI at low transmit power. Even for arbitrarily small power P_T, the distributions with roll-off are dominated by the underlying exponential CSI distribution. This implies that it is possible to steer to the roll-off distributions from an exponential distribution with an arbitrarily small mean. From Proposition 7, when $\beta > 1$ (typical) it is possible to achieve an AST of N using distributions with a roll-off (corresponds to $\delta \to 0$). Thus, even if the CSI is exponential with an arbitrarily small mean it is possible to achieve an AST of N using CSI. However, without the use of CSI the maximum achievable AST goes to zero. The following theorem summarizes the importance of CSI for the reception model under consideration at small powers.

Theorem 3: Assume $\beta > 1$ and $F(\gamma) = 1 - e^{-\frac{\gamma}{P_T}}$, then

$$\lim_{P_T \to 0} f(x, F) = 0. \tag{50}$$

However, for any given P_T, the maximum achievable AST with CSI satisfies

$$N \leq \lambda_c^* \leq N + \frac{N}{\beta}. \tag{51}$$

Proof : Refer to the appendix.

The theorem above implies that channel state information can be used to achieve large asymptotic throughputs even in cases where each node is equipped with small power. This result is of relevance for the reachback problem because sensors are typically deployed in large numbers but each is capable of transmitting at a low power. However, with the use of the CSI, it is possible for the nodes to employ transmission control and achieve a large throughput.

As a final note, we would like to point out that the distributions that can be used to improve the AST beyond N and achieve the capacity λ_c^* are not known.

C. Matched Filter

In this section, we list the results that correspond to the matched filter reception model. We do not give detailed comments in this part, because the results are conceptually similar the ones obtained for the LMMSE reception model.

The reception model is as follows : given that K users transmit, P_i is the power received from user i, user i goes through if and only if the corresponding SINR is greater than β that is,

$$\frac{P_i}{\sigma^2 + \sum_{k=1, k \neq i}^{K} P_k} > \beta. \tag{52}$$

This criterion follows from the heuristics [37] for networks with large N. It can be seen that criterion is quite similar to the capture model and is most popular one for CDMA networks with matched filter.

C.1 Population Independent Transmission Control

We will now characterize the AST with PITC with and without using CSI when the underlying distribution is exponential.

Proposition 9: If $F(\gamma)$ is the distribution function of an exponential random variable with mean P_T, then

$$\lim_{k\to\infty} C_k(F(\cdot)) = 0 \tag{53}$$

Proof : Refer to the appendix

Proposition 10: If $F(\gamma)$ is the distribution function of an exponential random variable with mean α, and $G(\cdot) \in \Lambda_F$ then

$$\lim_{k\to\infty} C_k(G(\cdot)) = 0 \tag{54}$$

Proof : Refer to the appendix.

The above propositions implies that if the received power is distributed exponentially, then PITC does not improve the AST.

We now consider the case when the received power has a distribution with a roll-off. The following proposition follows from a straight forward application of the result in [36].

Proposition 11: If $G(\cdot)$ has a roll-off δ, then

$$\lim_{k\to\infty} C_k(G(\cdot)) = \begin{cases} \left(\frac{N}{\beta}\right)^{\delta} \frac{\sin \pi\delta}{\pi\delta} & 0 < \delta \leq 1 \\ 0 & \delta > 1. \end{cases} \tag{55}$$

We see that in this case it is possible to obtain non-zero asymptotic throughput with constant transmission control. In order to determine if the use of CSI can increase the asymptotic stable throughput, we consider the asymptotic stable throughput of the distributions in the set Λ_F. From Proposition 8, we have that if we start with a distribution with a roll-off, we can go to distributions that have a larger roll-off but we cannot go to distributions with a smaller roll-off. This fact has the following implications on the possible improvements in AST. If $\frac{N}{\beta} \leq 1$, then the asymptotic throughput is a decreasing function of δ, therefore the asymptotic stable throughput cannot be improved if by steering to distributions with a roll-off. However, if $\frac{N}{\beta} > 1$ (typical) the asymptotic throughput reaches a maximum for some value of δ that lies between 0 and 1. Hence it is possible that decreasing δ increases the throughput. It can be shown quite easily that for a δ the asymptotic stable throughput can be improved by population independent transmission control if

$$\left(1 - \frac{\pi\delta}{\tan \pi\delta}\frac{1}{\delta}\right) < \log \frac{N}{\beta}. \tag{56}$$

To illustrate for example, if $\frac{N}{\beta} > e^2$ and $\delta \leq 0.5$ then improvement is possible. Figure 7 shows the variation of asymptotic throughput with δ for $N = 20$ and $\beta = 4dB$. It can be seen that significant gains are possible if we start with a δ is less than 0.5.

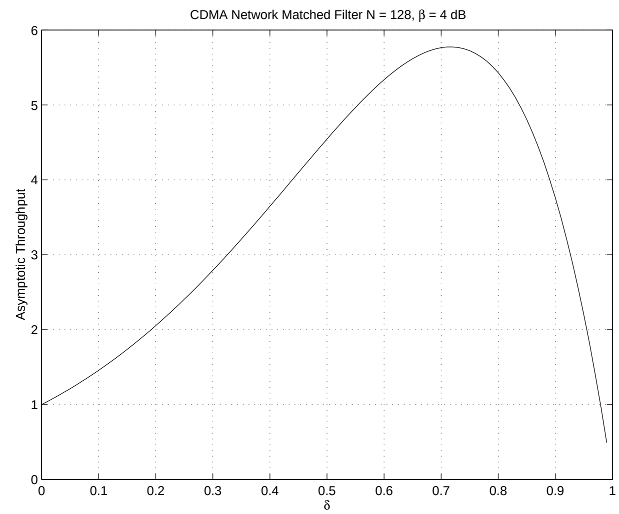

Fig. 7: Asymptotic Throughput Vs Delta

figure

C.2 Population Dependent Transmission Control

We now consider the use of channel state information for PDTC. As shown in Proposition 3, the AST obtained without the use of CSI is given by

$$f(x, F) = e^{-x} \sum_{k=0}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)). \tag{57}$$

Figure 8 illustrates the AST for matched filter when the underlying CSI distribution is exponential. The transmit power is 4 dB over noise, spreading gain $N = 16$ and $\beta = 4dB$. The x-axis is the design variable x. We see that it is possible to achieve an AST of approximately 1 packet per slot without using CSI by setting x to be approximately equal to 7 transmissions per slot. We would like to find if there exist distributions $T(\cdot)$ such that, for some x, $f(x, T) > 1$ and $T \ll F$. If $T(\cdot)$ is a distribution with a roll-off, Proposition 11 gives the value of $f(\infty, T)$, and we see that there exists a distribution with a roll-off for which $f(\infty, T) > 1$. Fig 9 plots $f(x, T)$ for distributions with a roll-off 0.5 and 0.3. We see that it is in fact possible to improve over the AST that was possible without CSI. From Proposition 4, we know that a transmission control that can be used to steer to a distribution with roll-off is given by

$$s_n(\gamma) = \min\left(\frac{e^{\gamma} P_T}{\gamma^{1+\delta}} \frac{x}{n}, 1\right) 1_{\gamma > \gamma_0}, \tag{58}$$

where γ_0 is any fixed constant.

VI. Conclusions and Future Directions

In this paper, we studied the use of decentralized channel state information for random access. To perform this study, we first proposed a reception model for the physical layer that takes into account the

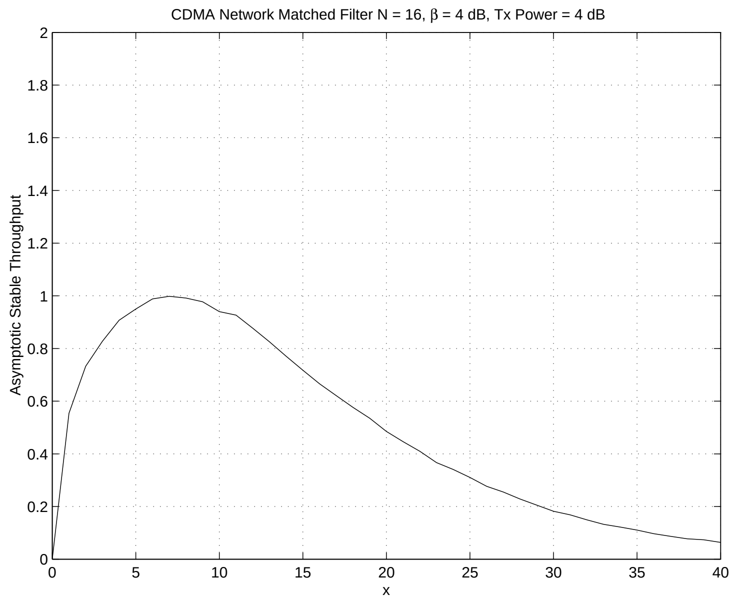

Fig. 8: AST with PDTC that does not use CSI

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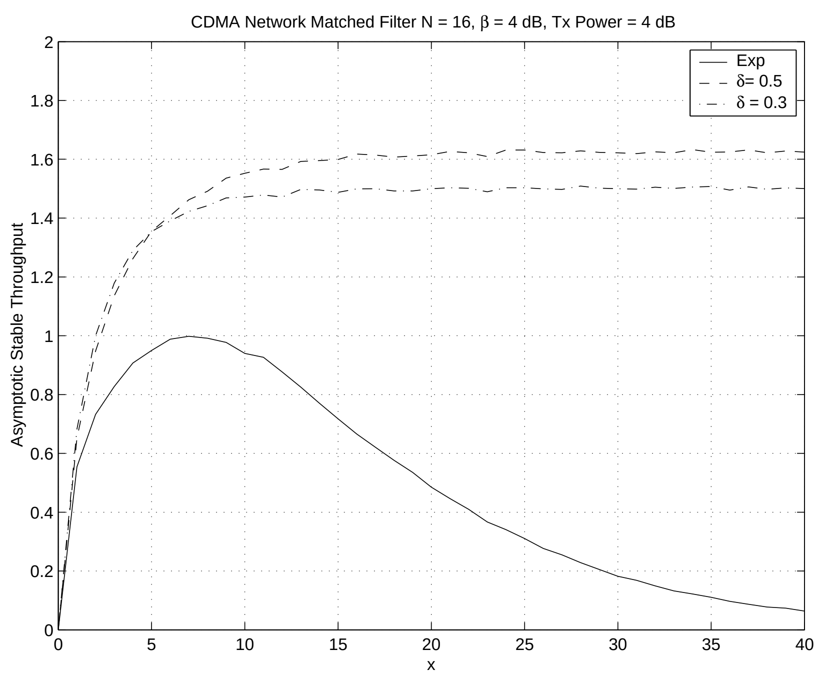

Fig. 9: AST with PDTC that uses CSI

figure

channel states of the transmitting users. A variant of Slotted ALOHA where the transmit probability is a function of the channel state was used for random access. We then obtained expressions for the maximum stable throughput of the network as a function of the transmission control used and the reception model. Determining optimal transmission controls for a reception model is in general a hard problem.

We then considered the regime of large networks and introduced the notion of asymptotic stable throughput (AST). Asymptotic stable throughput is the maximum stable throughput of the network as the number of users goes to infinity. Population independent transmission control (transmission control is not a function of the size of the network) was considered and the AST was derived for it. It was shown that the effect of transmission control is to effectively change the underlying CSI distribution and the set of distributions that can be reached through PITC was characterized. Population dependent transmission control (transmission control is a function of the size of the network) was then studied. If transmission control is not used, then the maximum possible AST is given by

$$\sup_x e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)), \tag{59}$$

where $F(\cdot)$ is the CSI distribution. We showed that if the transmission control sequence is chosen as

$$s_n(\gamma) = \min\left(\frac{dT}{dF}\frac{x}{n}, 1\right), \tag{60}$$

where F is the underlying distribution, T is a target distribution that is dominated by F, n is the size of the network, and x is a design variable which is equal to the average number of attempts per slot then the AST is given by

$$e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)). \tag{61}$$

The problem then is one of identifying the right target distributions to use for a given reception model.

The theory was then applied to the uplink of CDMA networks with LMMSE multi-user detectors and Matched Filter receivers. In either case, propagation channel gain was used as the channel state. Two different models leading to two different distributions were considered for the propagation channel gain. It was shown that if the channel state distribution is exponential, there is no gain to be got from PITC. However, with PDTC if the target distribution is chosen as a distribution with a roll-off, it is possible to obtain significant gains. For the LMMSE receiver, it was shown that if the nodes do not use CSI then the AST tends goes to zero as transmit power decreases but with the use of CSI the achievable AST is lower bounded by the spreading gain of the network. This outcome has important implications for the reachback problem in sensor networks where the number of nodes is large but each is equipped with small transmit power capabilities.

We now discuss some possible further research directions that arise from this study. The theory can be applied to a variety of reception models with different channel state parameters. In this paper, we have primarily considered the case when the propagation channel gain is chosen as the channel state.

Other possibilities include position of the mobile etc. This leads to interesting problems in development of reception models for different signal processing and physical layer architectures assuming different channel state parameters. Once the reception model has been developed, then it is important to determine good target distributions and then evaluate the possible gains from transmission control.

The results presented in this paper are mostly asymptotic in nature and there are different transmission control algorithms that give the same AST. But, these different choices might have different performance in terms of convergence to the asymptotic value. We feel that convergence will depend on how "different" the target distribution is from the current distribution. Hence, more work needs to be performed to characterize the rate of convergence. We suspect that this will have a bearing on the delay of the network.

For the LMMSE and Matched Filter reception model, we have only characterized the AST for two types of probability distributions (exponential and roll-off distributions). An interesting direction is to determine the AST for other distributions and the related problem of the capacity of both reception models is open.

For the case of CDMA networks, it is interesting to compare the strategy of transmission control with the strategy of power control. Both of them require only decentralized channel state information. The comparison between the two strategies is currently under investigation. It should be noted that transmission control is in general easier to implement than power control because power control might require a large dynamic range for the power amplifier.

In this paper, we have assumed that the distribution of the channel state is the same across users and that the reception model is invariant to permutation of channel states. The reception model considered in this paper thus cannot capture long run asymmetry in the users of the network. Addressing the problem after relaxing these assumptions is definitely interesting. We have assumed that the channel state is independent from slot and slot and we have restricted ourselves to stationary policies. Other important models which we believe might lead to interesting results are when the channel state is independent from user to user but correlated in time and we are allowed to use non-stationary policies. The model where the channel state is correlated between users is also quite interesting and might lead to different solutions. The results in this paper are quite surprising because we have demonstrated that channel state information can be used to improve the performance of the network even when it is i.i.d and the users are restricted to stationary policies. However, we conjecture that our results go through if the channel states are independent from user to user and ergodic and the user is restricted to stationary policies. In this case the proofs might be more involved because the theory of Markov Chains cannot be used to analyze the queue lengths.

Appendix

I. Proof of Theorem 1

The time evolution of the random variable $\mathbf{N}^{(t)}$ is given by

$$N_j^{(t+1)} = (N_j^{(t)} - Y_j^{(t)})^+ + X_j^{(t)}, \tag{62}$$

where $Y_j^{(t)}$ is equal to one if node j successfully transmits a packet during slot t and is equal to zero otherwise and $X_j^{(t)}$ is the number of newly arrived packets in slot t. Since the channel is independent from slot to slot and the transmission probability depends only on the current channel state, the n-dimensional process $\mathbf{N}^{(t)}$ is a Markov chain. We assume that the arrival process and the reception model are such that the Markov chain is aperiodic and irreducible. This is a mild requirement that is satisfied for most non trivial arrival processes and reception models. The stability of the system, which is equivalent to the existence of a limiting distribution for the Markov Chain is therefore also equivalent to the ergodicity of the Markov chain.

In order to show the stability of this Markov chain we borrow the techniques that were used in [22]. We state a key lemma from [22] that will used to obtain a sufficient condition for stability.

Lemma 1: Assume that $\xi(t)$ and $\chi(t)$, $t \in \mathbb{N}$ are two random sequences with values from the set $\{0, 1, 2, \cdots\}$, while $\mathcal{A}$ is some event associated with them. If for any x, t, k, s

$$\begin{aligned} \Pr\{\chi(t) > x | \chi(t) = s, \chi(0) = k\} &\leq \Pr\{\chi(t) > x | \chi(t) = s+1, \chi(0) = k\} &(63)\\ \Pr\{\xi(t) > x | \xi(t) = s, \xi(0) = k, \mathcal{A}\} &\leq \Pr\{\chi(t) > x | \chi(t) = s, \chi(0) = k\} &(64) \end{aligned}$$

then

$$\Pr\{\xi(t) > x | \xi(0) = k, \mathcal{A}\} \leq \Pr\{\chi(t) > x | \chi(0) = k\}. \tag{65}$$

This lemma says that the stability of $\chi(t)$ implies the stability of $\xi(t)$. The properties listed in the lemma are commonly notated as $\chi(t)$ stochastically dominates $\xi(t)$ [38], [39], [40]. Given the random sequence $N_i(t)$, the key is to identify a sequence that stochastically dominates $N_i(t)$ and whose stability is easy to analyze. As in [22], we define a one-dimensional Markov chain $Q_j^{(t)}$ which is the fully loaded version of $N_j^{(t)}$. That is, $Q_j^{(t)}$ is a Markov chain and

$$\Pr\{Q_j^{(t+1)} = k | Q_j^{(t)} = s\} = \Pr\{N_j^{(t+1)} = k | N_j^{(t)} = s, N_i^{(t)} > 0, i = 1, \cdots, n, i \neq j\}. \tag{66}$$

In order that we use stochastic dominance to analyze $\mathbf{N}^{(t)}$, we need to first show that the random sequence defined above satisfies the properties listed in Lemma 1.

Lemma 2: The Markov Chain $Q_j(t)$ stochastically dominates $N_j(t)$.

Proof : Refer to the appendix.

For the fully loaded system an application of Pakes' Lemma[41], which gives a sufficient condition on drift $D_i \triangleq \mathrm{E}\{Q_j(t+1) - Q_j(t) | Q_j(t) = i\}$, can be used to obtain a sufficient condition for stability. For the sake of completeness, we state Pakes' lemma below.

Lemma 3: Suppose that the drift $D_i < \infty$ for all i, and that for some scalar $\delta > 0$ and integer $\bar{i} \geq 0$ we have $D_i \leq -\delta$, for all $i > \bar{i}$. Then the Markov chain has a stationary distribution.

It is easy to see that the drift D_i for the fully loaded system is independent of i and is given by

$$\begin{aligned} D_i &= \frac{\lambda}{n} - \sum_{k=0}^{n-1} \binom{n-1}{k} \left(1 - \int_0^\infty s(\gamma) dF(\gamma)\right)^{n-1-k} \Bigg(\int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{k+1}) \\ &\qquad \sum_{\Theta_{k+1}} \theta_{k+1}^{(1)} \Phi^{(k+1)}(\gamma_1, \cdots, \gamma_{k+1}; \Theta_{k+1}) dF(\gamma_1) \cdots dF(\gamma_{k+1}) \Bigg) \qquad (67) \\ &= \frac{1}{n} \Bigg(\lambda - \sum_{k=1}^{n} \binom{n}{k} \left(1 - \int_0^\infty s(\gamma) dF(\gamma)\right)^{n-k} \Bigg(\int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_k) \\ &\qquad \Psi^{(k)}(\gamma_1, \cdots, \gamma_k) dF(\gamma_1) \cdots dF(\gamma_k) \Bigg) \Bigg). \qquad (68) \end{aligned}$$

The second equality follows from the symmetry of the reception model that was assumed in Section II. The above equation gives a sufficient condition for the stability of $Q_j(t)$, which due to Lemma 2 is also a sufficient condition for the stability of $\mathbf{N}(t)$.

We obtain necessary conditions for stability in a straight forward way by following the arguments in [22]. We now state a key result that is proved in [22] in a slightly more general form.

Lemma 4: Let Markov chain $\mathbf{N}(t)$ defined over $\mathbb{Z}_+^{\ltimes}$ possess the following property of bounded homogeneity with respect to its states : for any $\mathbf{s} \in \mathbb{Z}_+^{\ltimes}$ and $\mathbf{r} \in \mathbb{Z}_+^{\ltimes}$, such that for every i, either $s_i = r_i = 0$ or $s_i > 0, r_i > 0$ and for any $\mathbf{k} \in \mathbb{Z}_+^{\ltimes}$, we have

$$\Pr\{\mathbf{N}(t+1) = \mathbf{s} + \mathbf{k} | \mathbf{N}(t) = \mathbf{s}\} = \Pr\{\mathbf{N}(t+1) = \mathbf{r} + \mathbf{k} | \mathbf{N}(t) = \mathbf{r}\}. \qquad (69)$$

Then for $j = 1, \cdots, n$, $\mathrm{E}\{Q_j(t+1) - Q_j(t) | Q_j(t) > 0\} > 0$ implies that $N_j(t) \to \infty$ as $t \to \infty$ with probability 1 for all j.

It is easy to see that the bounded homogeneity property holds for the Markov chain under consideration. Thus the lemma above implies that the following condition is necessary for stability :

$$\begin{aligned} \lambda \leq \sum_{k=1}^{n} \binom{n}{k} \left(1 - \int_0^\infty s(\gamma) dF(\gamma)\right)^{n-k} \Bigg(\int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_k) \\ \Psi^{(k)}(\gamma_1, \cdots, \gamma_k) dF(\gamma_1) \cdots dF(\gamma_k) \Bigg). \qquad (70) \end{aligned}$$

Thus the theorem about maximum stable throughput follows. □

II. Proof of Lemma 2

We need that for all t, s, k, x,

$$\Pr\{Q_j^{(t+1)} > x | Q_j^{(t)} = s, Q_j^{(0)} = k\} \quad \leq \quad \Pr\{Q_j^{(t+1)} > x | Q_j^{(t)} = s+1, Q_j^{(0)} = k\}.$$

In other words, the probability that the buffer goes above a certain level in slot $(t+1)$ is larger if the queue has more packets in slot t. It is obvious that this is indeed the case. The other property to be shown is

$$\Pr\{N_j^{(t+1)} > x | N_j^{(t)} = s, \mathbf{N}^{(0)} = \mathbf{k}\} \quad \leq \quad \Pr\{Q_j^{(t+1)} > x | Q_j^{(t)} = s, Q_j^{(0)} = k_j\},$$

where $\mathbf{k} = (k_1, \cdots, k_n)$. In other words, the tendency of the buffer of the fully loaded system to exceed a level x is higher than that of the original system. In order to show this, we first observe that the evolution of the j^{th} buffer in the original system and the fully loaded system is given by

$$\begin{aligned} N_j^{(t+1)} &= (s - Y_j^{(t)})^+ + X_j^{(t)} \\ Q_j^{(t+1)} &= (s - Z_j^{(t)})^+ + X_j^{(t)}. \end{aligned} \tag{71}$$

Hence, in order to show (71), it is only necessary that we show that the probability of success is higher in the original system,or

$$\Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k\} \quad \geq \quad \Pr\{Z_j^{(t)} = 1 | Q_j^{(t)} = s, Q_j^{(0)} = k\}. \tag{72}$$

If $U_j^{(t)}$ is the number of nodes competing with node j to send packets in time slot t (the nodes with non-empty queues), we note that

$$\begin{aligned} \Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k\} &= \sum_{k=0}^{n-1} \Pr\{U_j^{(t)} = k\} \\ &\quad \underbrace{\Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k, U_j^{(t)} = k\}}_{f_k}, \end{aligned} \tag{73}$$

where as

$$\Pr\{Z_j^{(t)} = 1 | Q_j^{(t)} = s, Q_j^{(0)} = k\} = \underbrace{\Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k, U_j^{(t)} = n-1\}}_{f_{n-1}}. \tag{74}$$

We show that the probability of success f_k is a decreasing function of k which will then imply (72) because of (73) and (74). We have the following formula for f_k :

$$\begin{aligned} f_k &= \sum_{l=0}^{k} \binom{k}{l} (1-p)^{k-l} \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{l+1}) \sum_{\Theta_{l+1}} \theta_{l+1}^{(1)} \\ &\qquad \Phi^{(l+1)}(\gamma_1, \cdots, \gamma_{l+1}; \Theta_{l+1}) dF(\gamma_1) \cdots dF(\gamma_{l+1}), \end{aligned}$$

where

$$p = \int_0^\infty s(\gamma) dF(\gamma). \tag{75}$$

Equivalently, f_k is the coefficient of x^k in

$$(1 + (1-p)x)^k \, h(x), \tag{76}$$

where $h(x) = h_0 + h_1 x + h_2 x^2 + \cdots$ and

$$h_l = \int_0^\infty \cdots \int_0^\infty s(\gamma_1)\cdots s(\gamma_{l+1}) \sum_{\Theta_{l+1}} \theta_{l+1}^{(i)} \Phi^{(l+1)}(\gamma_1,\cdots,\gamma_{l+1};\Theta_{l+1}) dF(\gamma_1)\cdots dF(\gamma_{l+1}). \tag{77}$$

Therefore, $f_k - f_{k+1}$ is the coefficient of x^{k+1} in

$$(1+(1-p)x)^k (xh(x)) - (1+(1-p)x)^{k+1} h(x) \tag{78}$$

$$= (1+(1-p)x)^k (pxh(x) - h(x)). \tag{79}$$

The difference $f_k - f_{k+1}$ is a function of the coefficients of $x, \cdots, x^{k+1}$ in $(pxh(x) - h(x)), which we will show are all positive.$ The coefficient of $x^j, j = 1, \cdots, (k+1)$ is given by

$$p\int_0^\infty \cdots \int_0^\infty s(\gamma_1)\cdots s(\gamma_j) \sum_{\Theta_j} \theta_j^{(1)} \Phi^{(j)}(\gamma_1,\cdots,\gamma_j;\Theta_j) dF(\gamma_1)\cdots dF(\gamma_j) - \int_0^\infty \cdots \int_0^\infty s(\gamma_1)\cdots s(\gamma_{j+1}) \sum_{\Theta_{j+1}} \theta_{j+1}^{(1)} \Phi^{(j+1)}(\gamma_1,\cdots,\gamma_{j+1};\Theta_{j+1}) dF(\gamma_1)\cdots dF(\gamma_{j+1}).$$

Due to the condition (4) on the reception functions $\Phi^{(k)}(\cdot)$, the coefficients of x^j for $j = 1, \cdots, (k+1)$ are greater than zero which implies that $f_k \geq f_{k+1}$. Hence the Markov chain $Q_j(t)$ stochastically dominates $N_j(t)$. $\square$

III. Proof of Theorem 2

Let $\Lambda(u)$ be defined as

$$\Lambda(u) = \left\{ s(\gamma) : 0 \leq s(\gamma) \leq 1, \int_0^\infty s(\gamma) dF(\gamma) = u \right\}. \tag{80}$$

For $s(\gamma) \in \Lambda(u)$, we have

$$\lambda_n(s(\cdot)) = n(1-u)^{n-1} \left(\int_{\gamma_0}^\infty s(\gamma) dF(\gamma) \right). \tag{81}$$

We note that $\int_{\gamma_0}^\infty s(\gamma) dF(\gamma) \leq p_{\gamma_0}$. This leads to the following upper bound on the maximum stable throughput.

$$\lambda_n(s(\cdot)) \leq \begin{cases} n(1-u)^{n-1} u. & u \leq p_{\gamma_0} \\ n(1-u)^{n-1} p_{\gamma_0} & u > p_{\gamma_0} \end{cases} \tag{82}$$

If $p_{\gamma_0} \geq \frac{1}{n}$, maximizing the upper bound by varying u between 0 and 1, we find that for all $s(\cdot)$,

$$\lambda_n(s(\cdot)) \leq (1 - \frac{1}{n})^{n-1}. \tag{83}$$

Hence choosing the transmission control as

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ \frac{1}{np_{\gamma_0}} & \gamma \geq \gamma_0 \end{cases} \tag{84}$$

achieves the maximum and is hence optimal. If $p_{\gamma_0} < \frac{1}{n}$, then the above choice is not valid since $\frac{1}{np\gamma_0} > 1$. If $p_{\gamma_0} < \frac{1}{n}$, we find that the upper bound is maximized at $u = p_{\gamma_0}$ and

$$\lambda_n(s(\cdot)) \leq n(1 - p_{\gamma_0})^{n-1} p_{\gamma_0}. \tag{85}$$

Hence for $p_{\gamma_0} < \frac{1}{n}$, choosing the optimal choice for the transmission control is given by

$$s^*(\gamma) \quad = \quad \begin{cases} 0 & \gamma < \gamma_0 \\ 1 & \gamma \geq \gamma_0 \end{cases} . \tag{86}$$

□

IV. Proof of Proposition 4

For convenience, we define the function $H_n(s_n(\cdot))$ as

$$H_n(s_n(\cdot)) \triangleq \sum_{k=1}^{n} \binom{n}{k} \left(1 - \int s_n dF\right)^{n-k} \int \cdots \int s_n(\gamma_1) \cdots s_n(\gamma_k) \Psi^{(k)}(\gamma_1, \cdots, \gamma_k) \\ dF(\gamma_1) \cdots dF(\gamma_k). \tag{87}$$

We assume that given a distribution function $T(\cdot)$ that is dominated by $F(\cdot)$, the sequence of transmission controls is chosen as $t_n(T, x, \gamma)$, where

$$t_n(T, x, \gamma) = \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right). \tag{88}$$

We claim that for this choice of transmission control sequence,

$$\liminf_{n\to\infty} H_n(t_n(\cdots)) = e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)). \tag{89}$$

Due to the assumption A1, we have

$$e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)) < \infty, \tag{90}$$

which implies that for all $\epsilon > 0$, $\exists M$ such that

$$\sum_{k=M+1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)) < \epsilon. \tag{91}$$

Therefore for $n > M$, we have

$$\begin{aligned} & \sum_{k=M+1}^{n} \binom{n}{k} \left(1 - \int t_n dF\right)^{n-k} \int \cdots \int t_n(x, T, \gamma_1) \cdots t_n(x, T, \gamma_k) \\ & \qquad \Psi^{(k)}(\gamma_1, \cdots, \gamma_k) dF(\gamma_1) \cdots dF(\gamma_k) && (92) \\ \leq & \sum_{k=M+1}^{n} \frac{n!}{(n-k)!k!} \frac{x^k}{n^k} C_k(T(\cdot)) && (93) \\ \leq & \sum_{k=M+1}^{n} \frac{x^k}{k!} C_k(T(\cdot)) && (94) \\ < & \ \epsilon. && (95) \end{aligned}$$

The second inequality follows because for all γ

$$t_n(x, T, \gamma) \leq \frac{x}{n}\frac{dT}{dF}. \tag{96}$$

Hence

$$\begin{aligned}\liminf_{n\to\infty} H_n(t_n(x,T,\gamma)) \quad &\leq \quad \liminf_{n\to\infty} \sum_{k=1}^{M} \binom{n}{k} \left(1 - \int t_n dF\right)^{n-k} \int \cdots \int t_n(x,T,\gamma_1) \\ &\qquad \cdots t_n(x,T,\gamma_k)\Psi^{(k)}(\gamma_1,\cdots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) + \epsilon.\end{aligned} \tag{97}$$

For each k we have

$$\begin{aligned}&\binom{n}{k}\left(1 - \int t_n dF\right)^{n-k} \int \cdots \int t_n(x,T,\gamma_1)\cdots t_n(x,T,\gamma_k) \\ &\qquad \Psi^{(k)}(\gamma_1,\cdots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) \\ &= \frac{1}{k!}\left(1 - \frac{1}{n}\int nt_n dF\right)^{n-k} \frac{n!}{(n-k)!n^k} \int \cdots \int n^k t_n(x,T,\gamma_1)\cdots t_n(x,T,\gamma_k) \\ &\qquad \Psi^{(k)}(\gamma_1,\cdots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k)\end{aligned} \tag{98}$$

Since $nt_n(x,T,\gamma) \uparrow s\frac{dT}{dF}(\gamma)$ for all γ, we have using monotone convergence theorem

$$\lim_{n\to\infty} \int nt_n(x,T,\gamma)dF \quad = \quad \int x\frac{dT}{dF}dF \tag{99}$$

$$= \quad x. \tag{100}$$

Similarly, we also have

$$\begin{aligned}&\lim_{n\to\infty} \int n^k t_n(x,T,\gamma_1)\cdots t_n(x,T,\gamma_k)\Psi^{(k)}(\gamma_1,\cdots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) \\ &= x^k \int \cdots \int \Psi^{(k)}(\gamma_1,\cdots,\gamma_k)dT(\gamma_1)\cdots dT(\gamma_k) \\ &= x^k C_k(T(\cdot)).\end{aligned} \tag{101}$$

Define $f_n(y)$ as

$$f_n(y) \triangleq \left(1 - \frac{y}{n}\right)^{(n-k)}. \tag{102}$$

We know that

$$\lim_{n\to\infty} f_n(y) = e^{-y}. \tag{103}$$

In fact for any $A > 0$, the sequence of functions $f_n(y)$ converges uniformly to e^{-y} over the range $[0, A]$. This implies that if the sequence $x_n \to x$, where $x < A$, then

$$\lim_{n\to\infty} f_n(x_n) = e^{-x}. \tag{104}$$

Hence, taking the limit of (98), we have

$$\lim_{n\to\infty} \frac{x^k}{k!}\left(1 - \frac{x}{n}\right)^{n-k} C_k(T(\cdot)) \prod_{i=0}^{k-1}\left(1 - \frac{i}{n}\right) = e^{-x}\frac{x^k}{k!}C_k(T(\cdot)). \tag{105}$$

Therefore

$$\liminf_{n\to\infty} H_n(t_n(\cdots)) = e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(T(\cdot)). \tag{106}$$

□

V. Proof of Proposition 5

If $G(\cdot) = 1 - e^{-\frac{\gamma}{P_T}}$, it is easy to see that

$$\begin{aligned} C_{k+1}(G(\cdot)) &= \frac{k+1}{P_T}\int_0^\infty \Pr\left\{\frac{\beta N\sigma^2}{\gamma} + \sum_{i=1}^{k}\frac{\beta\gamma_i}{\beta\gamma_i+\gamma} < N\right\} e^{-\frac{\gamma}{P_T}}\,d\gamma & (107)\\ &\leq \frac{k+1}{P_T}\int_0^\infty \Pr\left\{\sum_{i=1}^{k}\frac{\beta\gamma_i}{\beta\gamma_i+\gamma} < N\right\} e^{-\frac{\gamma}{P_T}}\,d\gamma. & (108)\end{aligned}$$

We obtain an upper bound on the inner probability using Chernoff's bound as follows. Given a $h > 0$, we have

$$\begin{aligned} \Pr\left\{\sum_{i=1}^{k}\frac{\beta\gamma_i}{\beta\gamma_i+\gamma} < N\right\} &\leq \mathrm{E}\left\{\exp\left(hN - \sum_{i=1}^{k}\frac{h\beta\gamma_i}{\beta\gamma_i+\gamma}\right)\right\} \\ &= e^{hN}\mathrm{E}^k\left\{\exp\left(-\frac{h\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\} & (109)\end{aligned}$$

We let $h = 1$ and define $\mu(\gamma)$ as the characteristic function

$$\mathrm{E}\left\{\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\} = \frac{1}{P_T}\int_0^\infty \exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\exp(-\frac{\gamma_1}{P_T})d\gamma_1. \tag{110}$$

Therefore,

$$C_{k+1}(G(\cdot)) \leq \frac{(k+1)e^N}{P_T}\int_0^\infty \mu^k(\gamma)e^{-\frac{\gamma}{P_T}}\,d\gamma. \tag{111}$$

We now show that there exists $c > 0$ such that

$$\lim_{\gamma\to\infty}\gamma(1-\mu(\gamma)) > c, \tag{112}$$

which will imply that there exists γ^* such that $\gamma > \gamma^*$ implies

$$\mu(\gamma) \leq 1 - \frac{c}{\gamma}. \tag{113}$$

We have

$$\begin{aligned} \gamma(1-\mu(\gamma)) &= \frac{1}{P_T}\int_0^\infty \gamma\left(1 - \exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right)\exp(-\frac{\gamma_1}{P_T})d\gamma_1 \\ &\geq \frac{1}{2P_T}\int_0^\infty \frac{\beta\gamma\gamma_1}{\gamma+\beta\gamma_1}\exp(-\frac{\gamma_1}{P_T})d\gamma_1.\end{aligned}$$

The inequality follows because

$$1 - e^{-x} \geq \frac{x}{2} \qquad 0 \leq x \leq 1. \tag{114}$$

Using monotone convergence theorem, we have

$$\begin{aligned}\lim_{\gamma\to\infty}\gamma(1-\mu(\gamma)) &\geq \frac{1}{2P_T}\int_0^\infty \lim_{\gamma\to\infty}\frac{\gamma\gamma_1}{\gamma+\gamma_1}\exp(-\frac{\gamma_1}{P_T})d\gamma_1\\ &= \frac{\beta P_T}{2}.\end{aligned} \tag{115}$$

We therefore have

$$\begin{aligned}\frac{1}{e^N}C_{k+1}(G(\cdot)) &\leq \frac{k+1}{P_T}\int_0^\infty \mu^k(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma\\ &\leq \frac{1}{P_T}\int_0^{\gamma^*}(k+1)\mu^k(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma+\frac{1}{P_T}\int_{\gamma^*}^\infty(k+1)\left(1-\frac{c}{\gamma}\right)^k e^{-\frac{\gamma}{P_T}}d\gamma.\end{aligned} \tag{116}$$

The first integral goes to zero as $k\to\infty$, since $\mu(\gamma)\uparrow 1$ as $\gamma\to\infty$ which implies that there exists an $r<1$ such that for $0\leq\gamma\leq\gamma^*$ implies $\mu(\gamma)<r$. The second integral can be shown to go to zero by dominated convergence theorem since

$$(k+1)\left(1-\frac{c}{\gamma}\right)^k \leq \gamma \quad \forall k \tag{117}$$

and

$$\int_{\gamma^*}^\infty \gamma e^{-\frac{\gamma}{P_T}}d\gamma<\infty. \tag{118}$$

We therefore have that

$$\lim_{k\to\infty}C_{k+1}(G(\cdot))=0. \tag{119}$$

□

VI. Proof of Proposition 6

The proof of this proposition is similar to the previous one. Let $F(\gamma)=1-e^{-\frac{\gamma}{P_T}}$ and $G(\cdot)\in\Lambda_F$. If $s(\gamma)$ is the transmission control used then

$$C_{k+1}(G(\cdot)) = \frac{k+1}{p_sP_T}\int_0^\infty \Pr\left\{\frac{\beta N\sigma^2}{\gamma}+\sum_{i=1}^k\frac{\beta\gamma_i}{\beta\gamma_i+\gamma}<N\right\}s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma \tag{120}$$

$$\leq \frac{k+1}{p_sP_T}\int_0^\infty \Pr\left\{\sum_{i=1}^k\frac{\beta\gamma_i}{\beta\gamma_i+\gamma}<N\right\}s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma. \tag{121}$$

We obtain an upper bound on the inner probability using Chernoff's bound as follows. Given a $h>0$, we have

$$\begin{aligned}\Pr\left\{\sum_{i=1}^k\frac{\beta\gamma_i}{\beta\gamma_i+\gamma}<N\right\} &\leq \mathrm{E}\left\{\exp\left(hN-\sum_{i=1}^k\frac{h\beta\gamma_i}{\beta\gamma_i+\gamma}\right)\right\}\\ &= e^{hN}\mathrm{E}^k\left\{\exp\left(-\frac{h\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\}\end{aligned} \tag{122}$$

We let $h=1$ and define $\mu(\gamma)$ as the characteristic function

$$\mathrm{E}\left\{\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\}=\frac{1}{p_sP_T}\int_0^\infty\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)s(\gamma_1)\exp(-\frac{\gamma_1}{P_T})d\gamma_1. \tag{123}$$

As before, we show below that there exists $c > 0$ such that

$$\lim_{\gamma \to \infty} \gamma(1 - \mu(\gamma)) > c, \tag{124}$$

which will imply that there exists γ^* such that $\gamma > \gamma^*$ implies

$$\mu(\gamma) \leq 1 - \frac{c}{\gamma}. \tag{125}$$

We have

$$\begin{aligned} \gamma(1-\mu(\gamma)) &= \frac{1}{p_s P_T} \int_0^\infty \gamma \left(1 - \exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1 + \gamma}\right)\right) s(\gamma_1) \exp(-\frac{\gamma_1}{P_T}) d\gamma_1 \\ &\geq \frac{1}{2p_s P_T} \int_0^\infty \frac{\beta\gamma\gamma_1}{\gamma + \beta\gamma_1} s(\gamma_1) \exp(-\frac{\gamma_1}{P_T}) d\gamma_1. \end{aligned}$$

The inequality follows because

$$1 - e^{-x} \geq \frac{x}{2} \qquad 0 \leq x \leq 1. \tag{126}$$

Using monotone convergence theorem, we have

$$\begin{aligned} \lim_{\gamma \to \infty} &\geq \frac{1}{2p_s P_T} \int_0^\infty \lim_{\gamma\to\infty} \frac{\beta\gamma\gamma_1}{\gamma + \beta\gamma_1} s(\gamma_1) \exp(-\frac{\gamma_1}{P_T}) d\gamma_1 \\ &= \frac{\beta}{2} \mathrm{E}\{\gamma_1\}. \end{aligned} \tag{127}$$

It is easy to see that $0 < \mathrm{E}\{\gamma_1\} < \infty$.

We therefore have

$$\begin{aligned} \frac{1}{e^N} C_{k+1}(G(\cdot)) &\leq \frac{1}{p_s P_T} \int_0^\infty \mu^k(\gamma) s(\gamma) e^{-\frac{\gamma}{P_T}} d\gamma \\ &\leq \frac{1}{p_s P_T} \int_0^{\gamma^*} (k+1)\mu^k(\gamma) s(\gamma) e^{-\gamma} d\gamma + \frac{1}{p_s P_T} \int_{\gamma^*}^\infty (k+1) \left(1 - \frac{c}{\gamma}\right)^k s(\gamma) e^{-\gamma} d\gamma. \end{aligned} \tag{128}$$

The first integral goes to zero as $k \to \infty$, since $\mu(\gamma) \uparrow 1$ as $\gamma \to \infty$ which implies that there exists an $r < 1$ such that for $0 \leq \gamma \leq \gamma^*$ implies $\mu(\gamma) < r$. The second integral can be shown to go to zero by dominated convergence theorem since

$$(k+1)\left(1 - \frac{c}{\gamma}\right)^k \leq \gamma \quad \forall k \tag{129}$$

and

$$\int_{\gamma^*}^\infty \gamma e^{-\frac{\gamma}{P_T}} d\gamma < \infty. \tag{130}$$

We therefore have that

$$\lim_{k \to \infty} C_{k+1}(G(\cdot)) = 0. \tag{131}$$

□

VII. Proof of Proposition 7

From an extension of the arguments in [36], the asymptotic MMSE throughput for a distribution of roll-off δ is given by

$$C(\beta, N, \delta) = \int_0^\infty \mathrm{P}\left[\sum_{i=1}^\infty \frac{\beta S_i^{-\frac{1}{\delta}}}{\beta S_i^{-\frac{1}{\delta}} + s^{-\frac{1}{\delta}}} < N\right] ds, \tag{132}$$

where $\{S_i\}_{i=1}^\infty$ are points of a homogeneous Poisson process of rate 1. We then have

$$\begin{aligned} C(\beta, N, \delta) &= \int_0^\infty \mathrm{P}\left[\sum_{i=1}^\infty \frac{1}{N + \frac{N}{\beta} S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right] ds \\ &= \left(\frac{N}{\beta}\right)^\delta \int_0^\infty \mathrm{P}\left[\sum_{i=1}^\infty \frac{1}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right] ds \end{aligned} \tag{133}$$

The second equality follows from a simple substitution. Note that

$$\mathrm{P}\left[\sum_{i=1}^\infty \frac{1}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right] = \mathrm{P}\left[-1 < \sum_{i=1}^\infty \frac{1}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right]. \tag{134}$$

We also have [42](page 346) that

$$\mathrm{P}\left[-1 < \sum_{i=1}^\infty \frac{1}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right] = \lim_{M\to\infty} \int_{-M}^{M} \frac{e^{j\omega} - e^{-j\omega}}{j\omega} \mu^\infty(\omega) d\omega, \tag{135}$$

where

$$\mu^\infty(\omega) \triangleq \mathrm{E}\left[\exp\left(\sum_{i=1}^\infty \frac{j\omega}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}\right)\right]. \tag{136}$$

Therefore

$$C(\beta, N, \delta) = \int_0^\infty \lim_{M\to\infty} \int_{-M}^{M} \frac{e^{j\omega} - e^{-j\omega}}{j\omega} \mu^\infty(\omega) d\omega ds. \tag{137}$$

It is easy to see that

$$\begin{aligned} \mu^\infty(\omega) &= \mathrm{E}\left[\lim_{T\to\infty} \exp\left(\sum_{i=1}^{N(T)} \frac{j\omega}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}\right)\right] && (138) \\ &= \lim_{T\to\infty} \mathrm{E}\left[\exp\left(\sum_{i=1}^{N(T)} \frac{j\omega}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}\right)\right] && (139) \\ &= \lim_{T\to\infty} \sum_{k=1}^\infty \frac{e^{-T}T^k}{k!} \mathrm{E}\left[\exp\left(\sum_{i=1}^{k} \frac{j\omega}{N + U_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}\right)\right] && (140) \\ &= \lim_{T\to\infty} \sum_{k=1}^\infty \frac{e^{-T}T^k}{k!} \mu^k(\omega, T) && (141) \\ &= \lim_{T\to\infty} \exp\left(T\mu(\omega, T) - T\right) && (142) \end{aligned}$$

where $N(T)$ is the number of Poisson points in $[0, T)$, $\{U_i\}_{i=1}^\infty$ are independent and uniformly distributed between 0 and T and $\mu(\omega, T)$ is the characteristic function of $\frac{1}{N + U_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}$. The first and third equalities follow due to the properties of the Poisson process. The second equality follows due to bounded convergence theorem. We have

$$\mu(\omega, T) = \frac{1}{T}\int_0^T \exp\left(\frac{j\omega}{N + u^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}\right) du. \tag{143}$$

Therefore,

$$\begin{aligned}
\mu^{\infty}(\omega) &= \lim_{T\to\infty} \exp\left(\int_0^T \left(\exp\left(\frac{j\omega}{N + u^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}}\right) - 1\right) du.\right) && (144)\\
&= \exp\left(s\int_0^\infty \left(\exp\left(\frac{j\omega}{N + p^{\frac{1}{\delta}}}\right) - 1\right) dp.\right). && (145)
\end{aligned}$$

Consider the inner integral. It turns out that

$$\int_0^\infty \left(\exp\left(\frac{j\omega}{N + p^{\frac{1}{\delta}}}\right) - 1\right) dp = \frac{j\omega}{N} N^\delta \frac{\pi\delta}{\sin \pi\delta} F_1(1-\delta, 2; \frac{j\omega}{N}), \tag{146}$$

where $F_1(a, b; z)$ is a confluent hyper-geometric function [43]. Therefore, the original integral can be written as

$$C(\beta, N, \delta) = \left(\frac{N}{\beta}\right)^\delta \frac{1}{2\pi}\int_0^\infty \lim_{M\to\infty}\int_{-M}^M \frac{2\sin\omega}{\omega} \exp\left(s\frac{j\omega}{N} N^\delta \frac{\pi\delta}{\sin\pi\delta} F_1(1-\delta, 2; \frac{j\omega}{N})\right) d\omega ds \tag{147}$$

After a simple change of variables, this becomes

$$C(\beta, N, \delta) = \frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}\frac{1}{\pi}\int_0^\infty \lim_{M\to\infty}\int_{-M}^M \underbrace{\frac{\sin\omega}{\omega}\exp\left(sj\omega F_1(1-\delta, 2; \frac{j\omega}{N})\right)}_{F(\omega,s)} d\omega ds. \tag{148}$$

We now consider some properties of the function

$$j\omega F_1(1-\delta, 2; \frac{j\omega}{N}) \triangleq F_R(\omega) + jF_I(\omega). \tag{149}$$

that are crucial for the evaluation of the integral. The above function can also be written as [43]

$$\int_0^1 j\omega \exp\left(\frac{j\omega t}{N}\right)\left(\frac{1}{t} - 1\right)^\delta dt. \tag{150}$$

We therefore have

$$\begin{aligned}
F_R(\omega) &= -\int_0^1 \omega \sin\left(\frac{\omega t}{N}\right)\left(\frac{1}{t} - 1\right)^\delta dt && (151)\\
&\geq -\frac{\omega^2}{N}\int_0^1 t^{1-\delta}(1-t)^\delta dt && (152)\\
&= -k\omega^2, && (153)
\end{aligned}$$

where k is positive constant. The second inequality follows because $\sin(\omega t) \leq \omega t$. It is also easy to see that the function $F_R(\omega) \leq 0$ for all ω. Further the Taylor series expansion of $F_R(\omega)$ is given by

$$F_R(\omega) = -\frac{\omega^2}{N}\frac{1-\delta}{2} + \frac{\omega^4}{4!N^3}\frac{3-\delta}{3}\frac{2-\delta}{2}\frac{1-\delta}{1} - \cdots. \tag{154}$$

It follows that the function $F_R(\omega)$ is even. Similarly the Taylor series expansion of $F_I(\omega)$ is given by

$$F_I(\omega) = \omega - \frac{\omega^3}{3!N^2}\frac{2-\delta}{2}\frac{1-\delta}{1} + \frac{\omega^5}{5!N^4}\frac{4-\delta}{4}\frac{3-\delta}{3}\frac{2-\delta}{2}\frac{1-\delta}{1} - \cdots. \tag{155}$$

More importantly, we have $F_I(\omega) = \omega + O(\omega^3)$ as $\omega \to 0$. It can also be seen that the function $F_I(\omega)$ is odd.

Let $\epsilon > 0$. In order to find $C(\beta, N, \delta)$, we need to evaluate the following integral :

$$\int_0^\infty \lim_{M\to\infty} \left(\int_{-M}^{-\epsilon} F(\omega, s) d\omega + \int_{-\epsilon}^{\epsilon} F(\omega, s) d\omega + \int_0^\infty \int_\epsilon^M F(\omega, s) d\omega \right) ds \tag{156}$$

Even though these integrals are difficult to evaluate, it turns out that the middle integral contains most of the mass. So let ϵ tend to zero and consider the middle integral which can be written as

$$\begin{aligned} \lim_{\epsilon\to 0} \int_0^\infty \int_{-\epsilon}^{\epsilon} F(\omega, s) d\omega ds &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} F(\omega, s) d\omega ds \\ &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} \mathrm{Re}\{F(\omega, s)\} d\omega ds \\ &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} \frac{\sin\omega}{\omega} \exp(sF_R(\omega)) \cos(sF_I(\omega)) d\omega ds \\ &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} \exp(sF_R(\omega)) \cos(sF_I(\omega)) d\omega ds. \end{aligned} \tag{157}$$

The last equality follows because ϵ can be made as small as possible and hence $\frac{\sin\omega}{\omega}$ can be made as close as possible to 1. Since $F_R(\omega) \leq 0$, an upper bound on the above integral is

$$\lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} \cos(sF_I(\omega)) d\omega ds. \tag{158}$$

Since $F_R(\omega) \geq -k\omega^2$, where $k \geq 0$, a lower bound on the integral is

$$\lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} \exp(-ks\omega^2) \cos(sF_I(\omega)) d\omega ds. \tag{159}$$

due to the property of $F_R(\omega)$ given in (153). We will show that both these integrals are equal to π and thus the required integral would have been evaluated. Consider

$$\begin{aligned} \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_0^t \int_{-\epsilon}^{\epsilon} \cos(sF_I(\omega)) d\omega ds &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_{-\epsilon}^{\epsilon} \int_0^t \cos(sF_I(\omega)) ds d\omega \\ &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_{-\epsilon}^{\epsilon} \frac{\sin tF_I(\omega)}{F_I(\omega)} d\omega \qquad (160) \\ &= \lim_{\epsilon\to 0} \lim_{t\to\infty} \int_{F_I(-\epsilon)t}^{F_I(\epsilon)t} \frac{\sin\theta}{\theta} \frac{1}{g(\frac{\theta}{t})} d\theta, \qquad (161) \end{aligned}$$

where

$$g(\omega) \triangleq F_I'(F_I^{-1}(\omega)). \tag{162}$$

The last inequality follows after the substitution $F_I(\omega)t = \theta$. For convenience we define the function $f(\omega)$ as

$$f(\omega) \triangleq F_I^{-1}(\omega). \tag{163}$$

Since $F_I'(\omega) = 1 + O(\omega^2)$, we can choose ϵ small enough such that the inverse function of $F_I(\omega)$ is defined. This also implies, from inverse function theorem [44], that the function F_I^{-1} is continuously differentiable.

It is therefore simple to show that the function $F_I^{-1}(\omega) = O(\omega)$ as $\omega \to 0$. It is also easy to show that the function $F_I'(F_I^{-1}(\omega)) = 1 + O(\omega^2)$ as $\omega \to 0$. Thus $f(\omega) = O(\omega)$ and $g(\omega) = 1 + O(\omega^2)$. For simplicity, we define $h(\omega) = \frac{1}{g(\omega)}$. It is easy to see that $h(\omega) = 1 + O(\omega^2)$. This implies that the above integral is in fact equal to π.

We now consider the lower bound which after interchange of integrals becomes

$$\begin{aligned}
\lim_{\epsilon \to 0} \lim_{t \to \infty} \int_{\epsilon}^{\epsilon} & \exp(-kt\omega^2) \frac{\sin F_I(\omega) t}{F_I(\omega)} \frac{F_I^2(\omega)}{F_I^2(\omega) + k^2\omega^4} d\omega \\
&= \lim_{\epsilon \to 0} \lim_{t \to \infty} \int_{\epsilon}^{\epsilon} \exp(-kt\omega^2) \frac{\sin F_I(\omega) t}{F_I(\omega)} d\omega \\
&= \lim_{\epsilon \to 0} \lim_{t \to \infty} \int_{F_I(-\epsilon)t}^{F_I(\epsilon)t} \exp\left(-tf^2\left(\frac{\theta}{t}\right)\right) \frac{\sin\theta}{\theta} \frac{1}{g(\frac{\theta}{t})} d\theta. \qquad (164)
\end{aligned}$$

The last inequality follows after the substitution $F_I(\omega)t = \theta$ and the function $f(\omega)$ and $g(\omega)$ are as defined previously. We know that

$$\lim_{\epsilon \to 0} \lim_{t \to \infty} \int_{F_I(-\epsilon)t}^{F_I(\epsilon)t} \frac{\sin\theta}{\theta} d\theta = \pi, \qquad (165)$$

and hence we consider the difference between the two integrals and bound the difference. Set $F_I(\epsilon) = \delta$ and note that by making ϵ small δ can be made as small as possible. Now

$$\begin{aligned}
&\left| \int_{-\delta t}^{\delta t} \frac{\sin\theta}{\theta} \exp\left(-tf^2\left(\frac{\theta}{t}\right)\right) h(\frac{\theta}{t}) d\theta - \int_{-\delta t}^{\delta t} \frac{\sin\theta}{\theta} d\theta \right| \\
&= \left| \int_{-\delta t}^{\delta t} \frac{\sin\theta}{\theta} \left(\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right) h(\frac{\theta}{t}) - 1 \right) \right| \\
&= \left| -\cos\theta \frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t}) - 1}{\theta} \right|_{-\delta t}^{\delta t} - \int_{-\delta t}^{\delta t} -\cos\theta \\
&\quad \left\{ \frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t}) - 1}{-\theta^2} + \frac{1}{\theta} \left(\frac{h'(\frac{\theta}{t}) \exp(-tf^2(\frac{\theta}{t}))}{t} + \right. \right. \\
&\quad \left. \left. \exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t}) \frac{-2tf(\frac{\theta}{t})f'(\frac{\theta}{t})}{t} \right) \right\} d\theta \Bigg| \\
&\leq \frac{c_1}{\delta t} + \left| \int_{-\delta t}^{\delta t} \frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t}) - 1}{\theta^2} d\theta \right| + \left| \frac{1}{t} \int_{-\delta t}^{\delta t} \frac{h'(\frac{\theta}{t}) \exp(-tf^2(\frac{\theta}{t}))}{\theta} d\theta \right| \\
&\quad + \left| 2 \int_{-\delta t}^{\delta t} \frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t})f(\frac{\theta}{t})f'(\frac{\theta}{t})}{\theta} d\theta \right| \\
&\leq \frac{c_1}{\delta t} + \int_{-\delta t}^{\delta t} \left| \frac{h'(\frac{\theta}{t})}{\theta} \right| d\theta + 2 \int_{-\delta t}^{\delta t} \left| \frac{h(\frac{\theta}{t})f(\frac{\theta}{t})f'(\frac{\theta}{t})}{\theta} \right| d\theta \\
&\quad + \int_{-\delta t}^{\delta t} \left| \frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t}) - 1}{\theta^2} \right| d\theta \\
&\leq \frac{c_1}{\delta t} + \frac{c_2 \delta}{t} + c_3 \delta + 4 \int_0^{\sqrt{\delta t}} \left| \frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t}) - 1}{\theta^2} \right| d\theta \qquad (166)
\end{aligned}$$

$$+ 4\int_{\sqrt{\delta t}}^{\delta t}\left|\frac{\exp(-tf^2(\frac{\theta}{t}))h(\frac{\theta}{t})-1}{\theta^2}\right|d\theta$$
$$\leq \frac{c_1}{\delta t}+\frac{c_2\delta}{t}+c_3\delta+c_4\sqrt{\frac{\delta}{t}}+\frac{c_5}{\sqrt{\delta t}}. \tag{167}$$

The second equality follows from integration by parts. The first inequality follows because $h(omega) = 1 + O(\omega^2)$ and $\exp\left(-tf^2(\frac{\theta}{t})\right) \leq 1$.The second inequality follows because $\exp\left(-tf^2(\frac{\theta}{t})\right) \leq 1$. The final inequality follows because in 0 to $\sqrt{\delta t}$, $tf^2(\frac{\theta}{t})$ is small and hence $\exp(tf^2(\frac{\theta}{t}))$ is close to $1-tf^2(\frac{\theta}{t}))$.

The rest of the integral is given by

$$\lim_{\epsilon\to 0^+}\int_0^\infty \lim_{M\to\infty}\left(\int_{-M}^{-\epsilon}F(\omega,s)dw+\int_\epsilon^M F(\omega,s)dw\right)ds \tag{168}$$

It is easy to see that this is equal to

$$\lim_{\epsilon\to 0^+}\int_0^\infty \lim_{M\to\infty}\int_\epsilon^M 2\mathrm{Re}\left(F(\omega,s)\right)d\omega ds \tag{169}$$

We now show that the integrals can be interchanged due to Fubini's theorem. Consider

$$\begin{aligned}\int_0^\infty\int_\epsilon^\infty \left|\mathrm{Re}\left(F(\omega,s)\right)\right|d\omega ds &\leq \int_0^\infty\int_\epsilon^\infty \exp(sF_R(\omega))d\omega ds &(170)\\ &= \int_\epsilon^\infty -\frac{1}{F_R(\omega)}d\omega &(171)\\ &= \int_\epsilon^\infty \frac{1}{\omega\int_0^1 \sin\left(\frac{\omega t}{N}\right)\left(\frac{1}{t}-1\right)^\delta dt}d\omega &(172)\\ &< \infty &(173)\end{aligned}$$

The last inequality follows because for ω large the integral

$$\int_0^1 \sin\left(\frac{\omega t}{N}\right)\left(\frac{1}{t}-1\right)^\delta dt \tag{174}$$

increases faster than ω^δ. Hence, interchange of integrals is justified. Due to this, the rest of the integral can now be written as

$$C(\beta,N,\delta)=\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}+\left(\frac{N}{\beta}\right)^\delta\frac{2}{\pi}\left(\lim_{\epsilon\to 0}\int_\epsilon^\infty\frac{\sin\omega}{\omega}\mathrm{Re}\left\{\frac{-1}{F_R(\omega)+jF_I(\omega)}\right\}d\omega\right) \tag{175}$$

Therefore, we need to evaluate

$$\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}\frac{2}{\pi}\left(\mathrm{Re}\left\{\int_0^\infty\frac{\sin\omega}{\omega}\frac{1}{j\omega F_1(1-\delta,2;\frac{j\omega}{N})}d\omega\right\}\right) \tag{176}$$

This integral can be rewritten with a change of variables as

$$\frac{1}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}\frac{2}{\pi}\left(\mathrm{Re}\left\{\int_0^\infty\frac{\sin N\omega}{\omega}\frac{1}{j\omega F_1(1-\delta,2;j\omega)}d\omega\right\}\right) \tag{177}$$

It is difficult to evaluate the inner integral and it is bounded as follows. Due to the properties of delta functions, the limit of the integral as $N\to\infty$ is given by

$$\frac{\pi}{2}\lim_{\omega\to 0}\mathrm{Re}\left\{\frac{1}{j\omega F_1(1-\delta,2;j\omega)}d\omega\right\} \tag{178}$$

which is equal to $\frac{\pi(1-\delta)}{4}$. Therefore, we have shown that

$$C(\beta, N, \delta) = \frac{N}{\beta^\delta}\frac{\sin \pi\delta}{\pi\delta} + e(N, \delta), \tag{179}$$

where

$$\lim_{N\to\infty} e(N, \delta) = \frac{1-\delta}{2\beta^\delta}\frac{\sin \pi\delta}{\pi\delta}. \tag{180}$$

We conjecture that for any finite N

$$\max_\delta e(N, \delta) = e(N, 0) = \frac{1}{2}. \tag{181}$$

We take the value of hyper-geometric function as $\delta = 0$. We have $F_1(1, 2, j\omega) = \frac{e^{j\omega}-1}{j\omega}$. Making this substitution, we have

$$\begin{aligned}
&\frac{2}{\pi}\mathrm{Re}\left\{\lim_{\epsilon\to 0^+}\int_{[\epsilon,\infty)}\frac{\sin N\omega}{\omega}\frac{1}{e^{j\omega}-1}d\omega\right\} \\
&= \frac{2}{\pi}\left(\lim_{\epsilon\to 0^+}\int_{[\epsilon,\infty)}\frac{1}{2}\frac{\sin N\omega}{\omega}d\omega\right) \\
&= \frac{1}{2}
\end{aligned} \tag{182}$$

□

VIII. Proof of Proposition 8

Since $F(\gamma)$ has roll-off δ, we have

$$\lim_{\gamma\to\infty} F^c(\gamma)\gamma^\delta = c, \tag{183}$$

where $0 < c < \infty$. Let $G(\cdot) \in \Lambda_F$ and if $G(\cdot)$ has a roll-off smaller than δ, this implies

$$\lim_{\gamma\to\infty} G^c(\gamma)\gamma^\delta = \infty. \tag{184}$$

Therefore,

$$\lim_{\gamma\to\infty}\frac{G^c(\gamma)}{F^c(\gamma)} = \infty. \tag{185}$$

But, we also have that

$$\begin{aligned}
\frac{G^c(\gamma)}{F^c\gamma} &= \frac{\int_\gamma^\infty \frac{dG}{dF}dF}{\int_\gamma^\infty dF} && (186)\\
&\leq C\frac{\int_\gamma^\infty dF}{\int_\gamma^\infty dF} && (187)\\
&= C. && (188)
\end{aligned}$$

The second inequality follows because $G(\cdot) \in \Lambda_F$ and P3. The equation (188) is clearly in contradiction with (185) which implies that the roll-off of $G(\cdot)$ cannot be smaller than δ.

If $\delta' > \delta$, we show that there exists a transmission control $s(\gamma)$ such that the roll-off of $G_s(\gamma)$ is equal to δ'. Since the roll-off of $F(\gamma)$ is equal to δ, for all $\epsilon > 0$ there exists a γ^* such that $\gamma > \gamma^*$ implies that

$$\frac{c-\epsilon}{\gamma^{\delta}} \leq F^c(\gamma) \leq \frac{c+\epsilon}{\gamma^{\delta}}. \tag{189}$$

Choose $s(\gamma)$ as

$$s(\gamma) = \begin{cases} 0 & 0 \leq \gamma < \gamma^* \\ \min\left(1, \frac{1}{\gamma^{\delta'-\delta}}\right) & \gamma \geq \gamma^* \end{cases} \tag{190}$$

For γ large enough, we have

$$\begin{aligned} G^c(\gamma) &= \frac{1}{p_s}\int_{\gamma}^{\infty} s(x)dF(x) && (191)\\ &= \frac{1}{p_s}s(x)F(x)\Big|_{\gamma}^{\infty} - \frac{1}{p_s}\int_{\gamma}^{\infty} s'(x)F(x)dx && (192)\\ &= \frac{s(\gamma)F(\gamma)}{p_s} + \frac{\delta'-\delta}{p_s}\int_{\gamma}^{\infty}\frac{F(x)}{x^{\delta'-\delta+1}}dx && (193)\end{aligned}$$

Using (189), the second term can be bounded as

$$\int_{\gamma}^{\infty}\frac{c-\epsilon}{x^{\delta'+1}}dx \leq \int_{\gamma}^{\infty}\frac{F(x)}{x^{\delta'-\delta+1}}dx \leq \int_{\gamma}^{\infty}\frac{c+\epsilon}{x^{\delta'+1}} \tag{194}$$

Therefore, we have

$$G^c(\gamma) \leq \frac{c+\epsilon}{p_s}\left(\frac{1}{\gamma^{\delta'}} + \frac{\delta'-\delta}{\delta'}\frac{1}{\gamma^{\delta'}}\right) \tag{195}$$

and

$$G^c(\gamma) \geq \frac{c-\epsilon}{p_s}\left(\frac{1}{\gamma^{\delta'}} + \frac{\delta'-\delta}{\delta'}\frac{1}{\gamma^{\delta'}}\right). \tag{196}$$

Hence, the roll-off of $G(\gamma)$ is δ'. □

IX. Proof of Proposition 9

It is easy to see the asymptotic throughput is given by

$$\begin{aligned} \lim_{k\to\infty} C_k(F(\cdot)) &= \lim_{k\to\infty} k\int\cdots\int F^c\Big(\frac{\beta}{N}\sum_{i=1}^{k-1}\gamma_i + \beta\sigma^2\Big)dF(\gamma_1)\cdots dF(\gamma_{k-1}) \\ &= \lim_{k\to\infty}\frac{k}{P_T^{k-1}}e^{-\frac{\beta\sigma^2}{P_T}}\int e^{-\frac{\beta\gamma_1}{NP_T}}e^{-\frac{\gamma_1}{P_T}}d\gamma_1\cdots\int e^{-\frac{\beta\gamma_{k-1}}{NP_T}}e^{-\frac{\gamma_{k-1}}{P_T}}d\gamma_{k-1} \\ &= e^{-\frac{\beta\sigma^2}{P_T}}\lim_{k\to\infty}\frac{k}{(1+\frac{\beta}{N})^{k-1}} \\ &= 0. && (197)\end{aligned}$$

□

X. Proof of Proposition 10

Let $G(\cdot) \in \Lambda_F$, then there exists a transmission control $s(\cdot)$ such that $\int sdF > 0$ and $G(\gamma) = \frac{\int_0^\gamma s(x)dF(x)}{\int sdF}$. We therefore have

$$\begin{aligned}
\lim_{k\to\infty} C_k(G(\cdot)) &= \lim_{k\to\infty} \frac{k}{(\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma)^k} \int s(\gamma_1)e^{-\frac{\gamma_1}{P_T}}d\gamma_1 \cdots \int s(\gamma_{k-1})e^{-\frac{\gamma_{k-1}}{P_T}}d\gamma_{k-1} \\
&\quad \int s(\gamma)e^{-\frac{\gamma}{P_T}} I\left(\gamma > (\frac{\beta}{N}\sum_{i=1}^{k-1}\gamma_i + \beta\sigma^2)\right)d\gamma \\
&\leq \lim_{k\to\infty} \frac{ke^{-\frac{\beta\sigma^2}{P_T}}}{(\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma)^k} \int s(\gamma_1)e^{-\frac{\gamma_1}{P_T}}e^{-\frac{\beta\gamma_1}{NP_T}}d\gamma_1 \cdots \\
&\quad \int s(\gamma_{k-1})e^{-\frac{\gamma_{k-1}}{P_T}}e^{-\frac{\beta\gamma_{k-1}}{NP_T}}d\gamma_{k-1} \\
&= \frac{e^{-\frac{\beta\sigma^2}{P_T}}}{\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma} \lim_{k\to\infty} k \left(\frac{\int s(\gamma)e^{-\frac{(1+\frac{\beta}{N})\gamma}{P_T}}d\gamma}{\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma}\right)^{k-1} \\
&= 0. \qquad (198)
\end{aligned}$$

The second inequality follows because $s(\gamma) \leq 1$ and the last equality follows because

$$\frac{\int s(\gamma)e^{-\frac{(1+\frac{\beta}{N})\gamma}{P_T}}d\gamma}{\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma} < 1. \qquad (199)$$

□

XI. Proof of Theorem 3

If $F(\gamma) = 1 - e^{-\frac{\gamma}{P_T}}$, we have

$$\begin{aligned}
C_{k+1}(F(\cdot)) &= \frac{k+1}{P_T}\int_0^\infty \Pr\left\{\frac{\beta N\sigma^2}{\gamma} + \sum_{i=1}^k \frac{\beta\gamma_i}{\beta\gamma_i+\gamma} < N\right\} e^{-\frac{\gamma}{P_T}}d\gamma \qquad (200) \\
&\leq \frac{k+1}{P_T}\int_0^\infty \Pr\left\{\frac{\beta N\sigma^2}{\gamma} < N\right\} e^{-\frac{\gamma}{P_T}}d\gamma \\
&= (k+1)e^{-\frac{\beta\sigma^2}{P_T}}. \qquad (201)
\end{aligned}$$

From (110) and (119), we have that for every $\epsilon > 0$ there exists a k^* such that $k > k^*$ implies that

$$C_{k+1}(F(\cdot)) < \epsilon. \qquad (202)$$

It is also easy to see that this k^* does not depend on P_T. This is because of the interference limited nature of the system. For $k < k^*$, we use the upper bound on $C_k(F(\cdot))$ due to the noise limited nature of the system and for larger k we use the upper bound due to the interference limited nature of the system. Therefore,

$$f(x,F) = e^{-x}\sum_{k=1}^\infty \frac{x^k}{k!}C_k(F(\cdot))$$

$$\leq \quad e^{-x}\sum_{k=1}^{k^*}\frac{x^k}{k!}ke^{-\frac{\beta\sigma^2}{P_T}}+\epsilon e^{-x}\sum_{k=k^*}^{\infty}\frac{x^k}{k!} \tag{203}$$

$$\leq \quad e^{-\frac{\beta\sigma^2}{P_T}}k^*+\epsilon. \tag{204}$$

Hence, we have

$$\lim_{P_T\to 0}\sup_x f(x,F)=0. \tag{205}$$

We have a lower bound of N on λ_c^* because for any given P_T, it is possible to use a control that changes the CSI distribution to one with a roll-off as close to 0 as possible and achieve an AST of at least N. The upper bound of $N+\frac{N}{\beta}$ is easily obtained due to the reception model for MMSE. □

References

[1] L. Tong, Q. Zhao, and G. Mergen, "Multipacket reception in random access wireless networks: from signal processing to optimal medium access control," *IEEE Communication Magazine*, vol. 39, pp. 108–112, Nov 2001. *Special issue on Design Methodologies for Adaptive and Multimedia Networks.*

[2] R.Knopp and P.A.Humblet, " Information capacity and power control in single cell multi-user communications ," in *Proc. Intl Conf. Comm.*, (Seattle, WA), pp. 331–335, June 1995.

[3] R.Cheng, "Optimal transmit power management on a fading multiaccess channel," in *Proceedings of Information Theory Workshop*, (Haifa, Israel), June 1996.

[4] D.N.C.Tse and S.V.Hanly, "Multiaccess Fading Channels : I Polymatroid Structure, Optimal Resource Allocation and Throughput Capacities ," *IEEE Trans. Information Theory*, vol. 44, pp. 2796–2815, November 1998.

[5] S.V.Hanly and D.N.C.Tse, "Multiaccess Fading Channels: Part II : Delay Limited Capacities ," *IEEE Trans. Information Theory*, vol. 44, pp. 2816–2831, November 1998.

[6] D.N.C.Tse, "Optimal Power Allocation over Parallal Gaussian Broadcast Channels ." `http://degas.eecs.berkeley.edu/~dtse/opp_comm.html.`

[7] P.Viswanath, D.N.C.Tse, and R.Laroia, "Opportunistic Beamforming using Dumb Antennas," *IEEE Trans. Information Theory*, vol. 48, pp. 1277–94, June 2002.

[8] S.Sanghavi and B.Hajek, "Adaptive Induced Fluctuations for Multiuser Diversity," in *Proc. Intl. Symp. Information Theory*, (Lausanne, Switzerland), pp. 450–450, July 2002.

[9] S. Shakkottai and A. Stolyar, "Scheduling for multiple flows sharing a time-varying channel: The exponential rule." `http://www.ece.utexas.edu/~shakkott`, 2001.

[10] A. E. L. Tassiulas, "Dynamic server allocation to parallel queues with randomly varying connectivity," *IEEE Trans. Info. Theory*, vol. 39, pp. 466 – 478, March 1993.

[11] I.E.Telatar and S.Shamai, " Some information theoretic aspects of decentralized power control in multiple access fading channels," in *Proc. Info. Theory and Networking Workshop*, (Piscataway, NJ), pp. 23–23, 1999.

[12] P.Viswanath, D.N.C.Tse, and V.Anantharam, "Asymptotically Optimal Waterfilling in Vector Multiple Access Channels," *IEEE Trans. Information Theory*, vol. 47, pp. 241–267, January 2001.

[13] S.Shamai and S.Verdu, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. Info Theory*, vol. 47, pp. 1302–1327, September 2001.

[14] R. Gallager, " A Perspective on Multiaccess Channels ," *IEEE Trans. Information Theory*, vol. IT-31, pp. 124–142, March 1985.

[15] A. Ephremides and B. Hajek, "Information Theory and Communication Networks: An Unconsummated Union," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2416–2434, October 1998.

[16] S. Ghez, S. Verdú, and S. Schwartz, "Stability Properties of Slotted Aloha with Multipacket Reception Capability," *IEEE Trans. Automatic Control*, vol. 33, pp. 640–649, July 1988.

[17] S. Ghez, S. Verdú, and S. Schwartz, "Optimal Decentralized Control in the Random Access Multipacket Channel," *IEEE Trans. Automatic Control*, vol. 34, pp. 1153–1163, Nov. 1989.

[18] Q. Zhao and L. Tong, " A multi-queue service room MAC protocol for wireless networks with multipacket reception." To appear in *IEEE/ACM Trans. on Networking*, Feb., 2003,`http://www.ee.cornell.edu/~ltong`.

[19] Q. Zhao and L. Tong, "A dynamic queue protocol for multiaccess wireless networks with multipacket reception." submitted to *IEEE/ACM Trans. on Networking*, June, 2001, `http://www.ee.cornell.edu/~ltong`.

[20] G.Mergen and L.Tong, "Stability and Capacity of Wireless Networks," *To be submitted to IEEE Trans. Information Theory*, October 2002.

[21] V.Naware and L. Tong, "Stability of Queues in Slotted Aloha with Multiple Antennas," in *Proceedings of the 30th Allerton Conf. on Communications, Control, and Computing*, (Monticello, IL), October 2002.

[22] B. Tsybakov and V. Mikhailov, "Ergodicity of Slotted Aloha System," *Problemy Peredachi Informatsii*, vol. 15, pp. 73–87, Oct-Dec 1979.

[23] M.Zorzi and R.Rao, "Capture and Retransmission Control in Mobile Radio," *IEEE JSAC*, vol. 12, pp. 1289–1298, October 1994.

[24] R. LaMaire, A. Krishna, and M. Zorzi, "On the Use of Transmitter Power Variations to Increase Throughput in Multiple Access Radio Systems," *Wireless Networks*, vol. 4, pp. 263–277, June 1998.

[25] M.Zorzi, "Mobile Radio Slotted ALOHA with Capture, Diversity and Retransmission Control in the Presence of Shadowing," *Wireless Networks*, vol. 4, pp. 379–388, August 1998.

[26] M.W.Subbarao and B.L.Hughes, "Optimal transmission ranges and code rates for frequency-hop packet radio networks," *IEEE Trans. on Communications*, vol. 48, pp. 670–8, April 2000.

[27] T. K.Mori, T.Kobayashi and A.Ogawa, "Adaptive transmit permission probability control in CDMA cellular packet communications with site diversity," *IEICE Transactions on Communications*, vol. E83-B, pp. 1445–53, July 2000.

[28] B.Wu and Q.Wang, "Maximization of the channel utilization in wireless heterogeneous multiaccess networks," *IEEE Transactions on Vehicular Technology*, vol. 46, pp. 437–44, May 1997.

[29] J. W.Ren and M.Sweeting, "Capture effect and its enhancement in LEO satellite channel ," in *Proc. EUROCOMM'00*, pp. 184–188, May 2000.

[30] J. T.-K.Liu and A.Polydoros, "Retransmission control and fairness issue in mobile slotted ALOHA networks with fading and near-far effect," *Mobile Networks and Applications*, vol. 2, no. 1, pp. 101–10, 1997.

[31] X.Qin and R.Berry, "Exploiting Multiuser Diversity in Wireless ALOHA Networks," in *Proc. Allerton Conf. on Communication, Control and Computing*, (Allerton, IL), October 2001.

[32] A.Jahn and A.Bottcher, "Improvement of slotted Aloha for land-mobile satellite communications, using channel state information," in *IEEE Vehicular Technology Conference*, pp. 339–42, May 1993.

[33] A.Chockalingam, M.Zorzi, L.B.Milstein, and P.Venkataram, "Performance of a wireless access protocol on correlated Rayleigh-fading channels with capture," *IEEE Transactions on Communications*, vol. 46, pp. 644–655, May 1998.

[34] J.Sant and V.Sharma, "Performance analysis of a slotted-ALOHA protocol on a capture channel with fading," *Queueing Systems, Theory and Applications*, vol. 34, no. 1, pp. 1–35, 2000.

[35] S.Adireddy and L.Tong, " Optimal Transmission Probabilities for Slotted ALOHA in Fading Channels ," in *Proc. CISS'02*, (Princeton, NJ), March 2002.

[36] B.Hajek, A.Krishna, and R.O.LaMaire, "On the Capture Probability for a Large Number of Stations," *IEEE Trans. Communications*, vol. 45, pp. 254–260, February 1997.

[37] D.N.C.Tse and S.V.Hanly, " Linear Multiuser Receivers : Effective Interference, Effective Bandwidth and User Capacity ," *IEEE Trans. Information Theory*, vol. 45, pp. 641–657, March 1999.

[38] R.Rao and A.Ephremides, "On the stability of interacting queues in a multi-access system," *IEEE Trans. Inf. Theory*, vol. 34, pp. 918–930, September 1988.

[39] W.Luo and A.Ephremides, "Stability of N interacting queues in random-access Systems," *IEEE Tran. Info. Theory*, vol. 45, pp. 1579–1587, 1999.

[40] W.Szpankowski, "Stability conditions for some mulitqueue distributed systems: Buffered random access systems," *Adv. Appl. Probab.*, vol. 26, pp. 498–515, 1994.

[41] A. Pakes, "Some conditions for ergodicity and recurrence of markov chains," *Operations Research*, vol. 17, pp. 1058–1061, 1969.

[42] P.Billingsley, *Probability and Measure*, vol. 3. New York, NY: Wiley Inter-Science, 1995.

[43] I.S.Gradshteyn and I.M.Ryzhik, *Table of Integrals, Series and Products*, vol. 4. New York, NY: Academic Press, 1980.

[44] R.S.Strichartz, *The Way of Analysis*, vol. 2. Sudbury, MA: Jones and Bartlett, 2000.