ECE 4760: Digital Communications	**Spring 2016**
Lecture 13 — March 22	
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Finish OFDM; Start Synchronization and Training

13.1 OFDM Symbols and Channel Model

Recalling earlier lectures on OFDM, the channel is modeled by the following matrix equation:

$$\underline{Y} = \underline{H}\vec{X} + \vec{Z} \qquad\qquad Z \sim \mathcal{N}(\vec{0}, N\underline{I}) \tag{13.1}$$

The noise, as usual, is characterized by a zero-mean Gaussian distribution with variance N. $\underline{H}$ is the known, causal, Toeplitz matrix that is constructed as follows:

$$\begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_{k_c-1} & h_{k_c-2} & \dots & h_1 & h_0 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & h_{k_c-1} & h_0 \end{bmatrix}$$

Since the input bit stream can be encoded into a vector X with a circulant prefix, the $\underline{H}$ matrix is also circulant.

This causal channel model matrix has a band of non zero values through the lower diagonal (See figure 13.2), of which the first column is the impulse response. The impulse response is indexed from 0 to $k_c - 1$.

The input will be formatted as a series of prefixes, known as guard bands, followed by encoded data. This structure is known as an OFDM symbol. Two symbols are shown below where L is the length of each block of the divided data vector X. Note that no cyclic postfix is required because of the causality of the channel.

$$\underbrace{\underbrace{\vec{X}[L-(k_c-1):L-1]}_{\text{Cyclic Prefix}}\underbrace{\vec{X}[0:L-1]}_{\text{Encoded Data 1}}}_{\text{OFDM Symbol 1}}\underbrace{\underbrace{\vec{X}[2L-(k_c-1):2L-1]}_{\text{Cyclic Prefix}}\underbrace{\vec{X}[L:2L-1]}_{\text{Encoded Data 2}}}_{\text{OFDM Symbol 2}}\dots$$

L is chosen such that the channel remains time invariant over the time span [0,L]. L may be longer or shorter depending on environmental factors such as speed of transmitter or clock skew.

13.2 OFDM Block Diagram

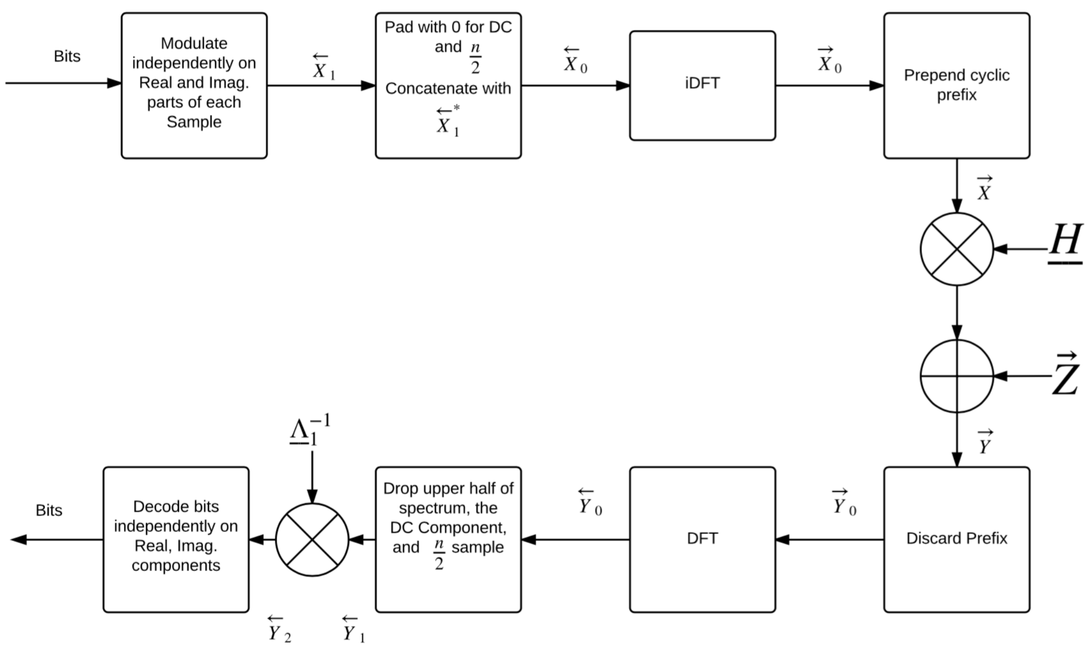

Figure 13.1. OFDM Block Diagram

13.3 OFDM Costs, Benefits and Applications

1. **Costs**
   (a) Dead Time in channel from prefixes
   (b) L cannot be chosen large
   (c) If k_c is large, prefix is large
   (d) Wide band signal ↔ Pulse-like waveform
   (e) Amplifier non-linearity heightened because of pulse-like behavior
2. **Applications**

(a) Sirius XM Radio (terrestrial repeaters)
(b) WiFi
(c) DSL
(d) 5G
(e) European Digital TV Standard

Fun Fact: OFDM isn't used for satellite to earth communication (SiriusXM) because of the absence of multipath (Wireless ISI source) and therefore the channel is nearly diagonal!

Note: End of Prelim 1 Material

13.4 Synchronization and Learning

Now that OFDM has solved the problem of ISI, the next step in the channel model is to add periods of silence before and after the transmission. Now $\vec{X}$ in equation 13.1 features a length of zeros before and after the encoded data. The noise is also of unknown variance but remains to be modeled by a Gaussian distribution. H remains a Toeplitz matrix and is unknown in this situation.

$$\underline{Y} = \underline{H}\vec{X} + \vec{Z} \qquad Z \sim \mathcal{N}(\vec{0}, N\underline{I}) \tag{13.2}$$

$$\vec{X} = \begin{bmatrix} zeros(L_1) \\ \vec{X}_0 \\ \vdots \\ \vec{X}_{n-1} \\ zeros(L_2) \end{bmatrix}$$

To measure the variance of the noise, the silence period before the transmission can be utilized as a calibration period. By taking the variance of a set of silent samples, one can estimate the variance of the noise well. This method will only apply in a thermal noise limited system.

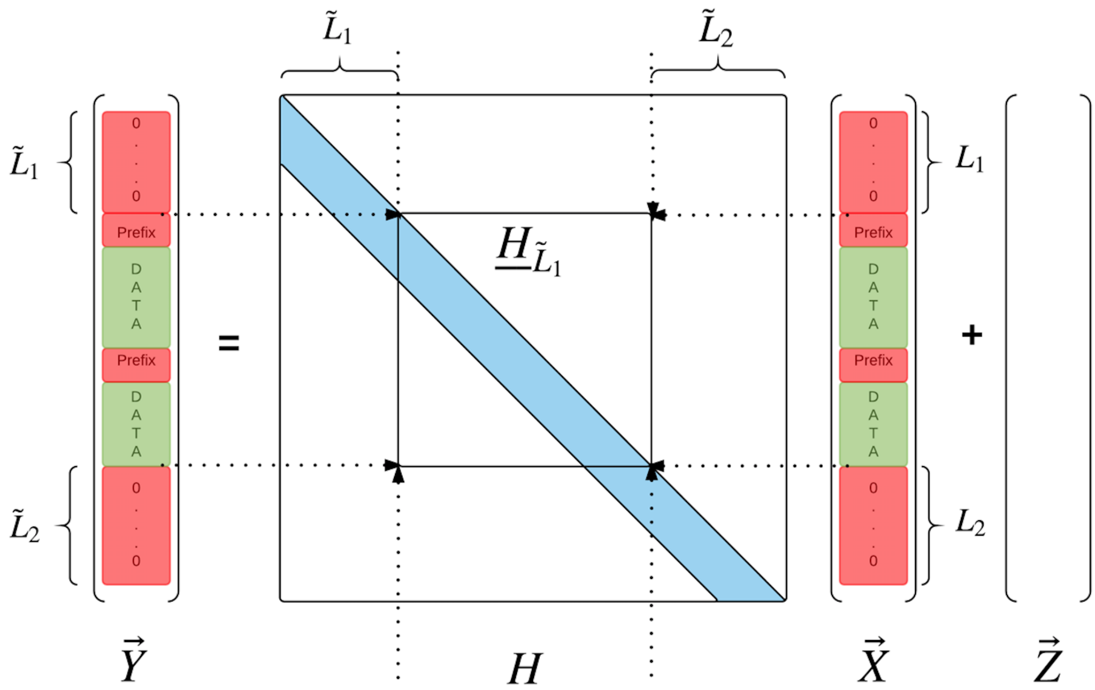

Figure 13.2. OFDM Synchronization

The figure above shows the periods of silence L_1 and L_2, and the corresponding estimates at the output of the system $\tilde{L}_1$ and $\tilde{L}_2$. The green sections denote samples that carry data and are decoded, the red shows samples that are discarded, and the blue indicates non-zero elements of the $\underline{H}$ matrix. $\tilde{L}_1$ and $\tilde{L}_2$ determine which sub-matrix of the overall $\underline{H}$ is chosen as the channel matrix, denoted as $\underline{H}_{\tilde{L}_1}$.

Question: What properties do we want $\tilde{L}_1$ and $\tilde{L}_2$ to satisfy?

If the length of the transmission is hard coded, $\tilde{L}_2$ is irrelevant. If the transmission is not of determined length, choosing a very large $\tilde{L}_2$ will raise computation time as the matrix will be larger. Assuming $\tilde{L}_2$ is hardcoded, choosing $\tilde{L}_1$ is very small, the sub-matrix can end up being upper triangular, which means the impulse response is non-causal. Therefore, $\tilde{L}_1$ must be large enough to be large enough such that the matrix is lower triangular.

If $\tilde{L}_1$ is chosen too large, then the prefix grows proportionately and gives rise to a very inefficient OFDM implementation. Both data rate will suffer as well as power consumption.

13.4.1 Determining $\tilde{L}_1$

Most of these methods are hacks and there is really no closed form solution to this problem. One way to go about finding $\tilde{L}_1$ is to threshold the received power and detecting when the channel begins to show signs of a transmission, and then applying minor adjustments based on how long the channel takes to respond. A more sophisticated approach lies in taking a running average of a batch of samples and comparing the next sample to the average. This provides added robustness towards noise. as a particularly large spike could set off a false alarm in the threshold method.