

Study on Power Azimuth Spectrum of Wireless Channel in Microcell Environments

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Abstract—This paper discussed the spatial or angular spreading properties of wireless propagation in the urban microcell environment, which is of great interests of future mobile technology, especially in case of using smart antennas. Based on the elliptical scattering model, the power azimuth spectrum in microcell environments is derived and it is found that the result approximately followed a truncated Laplacian distribution. The parameter of the Laplacian function is determined by regression. The results can provide insight into the spatial spreading properties of the wireless channel and can be used easily for further research.

Keywords—power azimuth spectrum; microcell environment

I. INTRODUCTION

As increased utilization of wireless channel for growing wireless services, there are increasing needs to explore the characteristic of the radio channel to support various technical system and to accommodate more services to more users. The power distribution upon the angle of arrival (AOA) is now of great importance for using smart antenna or other techniques with spatial treatment of the incoming signals. Conventional approach to describe the channel properties, such as level-crossing rate, average fade duration, auto-covariance and coherence distance of fading are all dependent on the power azimuth spectrum [1]. However, given works have not intensively studied the power azimuth spectrum of wireless channels, while more investigations and discussions were focused on the AOA probability density function (pdf). In this paper, the power azimuth spectrum for microcell environment is studied based on the elliptical scattering model, and then the spectrum is approximately expressed by the truncated Laplacian. Furthermore, the parameters of the Laplacian function are also achieved on conditions of different path loss exponents and different shapes of the scattering region.

II. MODEL AND ASSUMPTIONS

As shown in Fig.1, an elliptical region of uniformly distributed scattering objects is used to model the spreading [2]. The foci are at the base station and the mobile respectively. The major axis of the ellipse is given by $c\tau_m$, where τ_m is the

maximum delay associated with scattering objects within the ellipse, and c is the speed of light. It is obvious, using the elliptical model means that only those multipath components with delay less than or equal to τ_m are considered [2].

Provided that τ_m is chosen large enough, nearly all of the power of the multipath signals could be accounted in by the model.

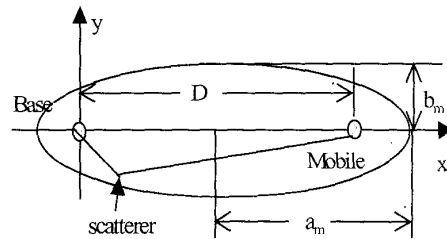


Figure 1. Geometry of Scattering objects

The parameters a_m and b_m shown in Fig.1 are the semi-major axis and semi-minor axis values which are given by

$$a_m = \frac{c\tau_m}{2} \quad (1)$$

$$b_m = \frac{1}{2} \sqrt{c^2\tau_m^2 - D^2} \quad (2)$$

The eccentricity of the ellipse can be expressed by

$$e = \frac{D}{2a_m} \quad (3)$$

It is assumed that both transmitting and receiving antennas are omni-directional. And the following well known additional assumptions [2] for the elliptical model are also used:

- 1) The signals are carried by plane waves that propagate in the horizontal plane;

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2) Each scattering object is treated as an omni-directional re-radiating element whereby the plane wave, on arrival, is reflected directly to the mobile or the base station without influence from other scattering objects;

3) The loss caused by the reflection of scattering objects are same to each other and so will not affect our power azimuth spectrum results.

4) The propagation path loss follows the $1/r^n$ law.

III. DERIVATION OF POWER AZIMUTH SPECTRUM

Due to the symmetry of the ellipse with respect to the base station and the mobile, the result at the base is also valid for the mobile. Therefore, we could only consider the power azimuth spectrum at the base station.

Firstly, we derive the power azimuth spectrum in the case of that scattering objects are only uniformly distributed on the border of the ellipse with foci at the base station and mobile, which means there is no objects located in the ellipse region. The semi-major axis value of the ellipse is a , and satisfies $\frac{D}{2} \leq a \leq a_m$.

This ellipse can be described by the polar equation with respects to the polar coordinates (r, α) as:

$$r = \frac{a(1-k^2)}{1-k \cos \alpha} \quad (4)$$

where k is the eccentricity of the ellipse and determined by

$$k = \frac{D}{2a} \quad (5)$$

In this case, the power azimuth spectrum is simply a mapping of the power distributed in differential element of the arc (ds) to power distributed in differential element of the AOA ($d\alpha$). The relationship is

$$ds = \sqrt{r^2 + \left[\frac{dr}{d\alpha}\right]^2} d\alpha \quad (6)$$

Because $P(ds) \propto ds$, so

$$P(\alpha) \propto \sqrt{r^2 + \left[\frac{dr}{d\alpha}\right]^2} = \frac{D}{2} p_k(\alpha) \quad (7)$$

where

$$p_k(\alpha) = \frac{(1-k^2)}{k(1-k \cos \alpha)^2} \sqrt{1+k^2-2k \cos \alpha} \quad (8)$$

When k grows from e to 1, the ellipse given by (4) will "sweep" the whole scattering region. Thus, in order to get the whole power azimuth spectrum with the scattering objects are not only located on the elliptical border but also uniformly

distributed in the ellipse, we only need to integrate $p_k(\alpha)$ given in (8) with respect to k over the range from e to 1. The path loss and power difference should also be considered as weighting factors in the integration. Since it is assumed that the scattering objects are uniformly distributed, the power distributed on the differential area $|dS|$ is just proportional to $|dS|$. And $|dS|$ can be calculated using

$$|dS| = \frac{\pi D^2}{4} \frac{2-k^2}{k^3 \sqrt{1-k^2}} |dk| = \frac{\pi D^2}{4} w(k) |dk| \quad (9)$$

where

$$w(k) = \frac{2-k^2}{k^3 \sqrt{1-k^2}} \quad (10)$$

Now, the power azimuth spectrum for the elliptical model can be expressed

$$S(\alpha) = c_1 \int p_k(\alpha) w(k) \frac{1}{(2a)^n} dk \quad (11)$$

where the constant c_1 is set to make their integral to be equal to the given total received power, $1/(2a)^n$ denotes the path loss, $w(k)$ is the weighting factor denoting the power distributed on differential area $|dS|$ and is given by (10). $p_k(\alpha)$, which is given by (8), represents the power azimuth spectrum of the arriving power reflected by scattering objects in $|dS|$.

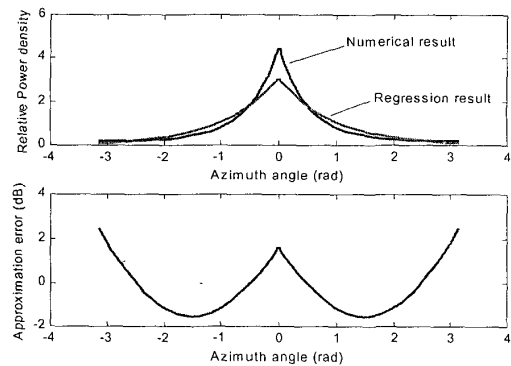


Figure 2. Power azimuth spectrum & Laplacian approximation

Since (11) seems too complicated to solve, numerical method is used. When $n=4$, $e=0.6$ and $c_1=1$, power azimuth spectrum is shown in the upper section of Fig.2 as the Numerical result.

It is apparent that the power azimuth spectrum is not a Gaussian function, but can be approximated better by an Laplacian function. Using Laplacian function to model the power azimuth spectrum in outdoor environment has already been proposed in [3]. The power azimuth spectrum function can be expressed as

$$S(\alpha) = \frac{c_2}{\sqrt{2}\sigma} \exp\left[-\frac{\sqrt{2}|\alpha|}{\sigma}\right] \quad (12)$$

where $\alpha \in [-\pi, +\pi]$.

The parameter σ controls the spread of the function, while the constant c_2 is set such that its integral is equal to the received power.

Assuming (12) is valid, then

$$\ln(S(\alpha)) = -\frac{\sqrt{2}}{\sigma}|\alpha| + \ln\left(\frac{c_2}{\sqrt{2}\sigma}\right) \quad (13)$$

The linear regression to the numerical data was carried out so that σ can be obtained from the gradient. When $n=4$ and $e=0.6$, we got $\sigma=1.30$ and $c_2=5.73$. Substituting them into (12), we can plot the Laplacian function to approximate the numerical result. It is also shown in the upper section of Fig.2 as the regression result.

If the approximation error is defined as

$$error = 10 \lg(N_{numerical} - R_{e regression}) \quad (14)$$

We plot it in the lower section of Fig.2. It is apparent that the error is not that large and would be tolerable to engineering purpose. Only for $\alpha \approx \pi$, the difference between the simulation and regression exceeds 2dB.

For further use, the spread parameter σ is obtained using different path loss exponent n and shape of the ellipse e . The results are shown in Table.I. The table may be used as a quick

reference to evaluate σ when given n and e (using linear interpolation).

TABLE I. σ (RAD) ON DIFFERENT CONDITIONS

n \ e	0.4	0.5	0.6	0.7	0.8
2	2.15	1.76	1.47	1.23	1.02
4	1.61	1.46	1.30	1.14	0.98
6	1.30	1.24	1.16	1.06	0.94

It is obvious that when n increases, σ will decrease. However, when n is unchanged but e increases, σ will also decrease when the ellipse becomes "narrower". All these agree with our common sense very well.

IV. CONCLUSION REMARKS

In this paper, the power azimuth spectrum in the microcell propagation environment is derived analytically, and it is found it could be approximated simply by a truncated Laplacian function with tolerable errors, the spread parameters in the Laplacian function are also determined on different conditions. This approximation can easily be used in the further analysis and simulation of different microcell wireless channels.

REFERENCES

- [1] W.C.Jakes, Ed., "Microwave Mobile Communications", New York: IEEE Press, 1974
- [2] Richard B.Ertel and Jeffrey H.Reed, "Angle and Time of Arrival Statistics for Circular and Elliptical Scattering Models", IEEE Journal on selected areas in communications, Vol.17, No.11, Nov 1999, pp.1829-1839
- [3] K.I.Pedersen, P.E.Mogensen and B.H.Fleury, "Power azimuth spectrum in outdoor Environments", in Electronics Letters, Vol.33, No.18, Aug.1997, pp.1583-1584