On Optimizing the Backoff Interval for Random Access Schemes

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***Abstract*— To improve the channel throughput and the fairness of random access channels, we propose a new backoff algorithm, namely, Sensing Backoff Algorithm (SBA). A novel feature of the SBA scheme is the sensing mechanism, in which every node modifies its backoff interval according to the results of the sensed channel activities. In particular, every active node sensing the successful transmission decreases its backoff interval by an additive factor of the transmission time of a packet. In order to find the optimum parameters for the SBA scheme, we have studied the optimum backoff intervals as a function of different number of active nodes (N) in a single transmission area with pure ALOHA type channels. We have found that the optimum backoff interval should be $4N$ times the transmission time of a packet when the random access channel operates under a pure-ALOHA scheme. Based on this result, we have numerically calculated the optimum values of the parameters for SBA, which are independent of N. The SBA scheme operates close to the optimum backoff interval. Furthermore, its operation does not depend on the knowledge of N. The optimum backoff interval and the SBA scheme are also studied by simulative means. It is shown that the SBA scheme out-performs other backoff schemes, such BEB and MILD. As a point of reference, the SBA scheme offers a channel capacity of 0.19 when N is 10, while the MILD scheme can only offer 0.125. The performance gain is about 50%.**

I. Introduction

In shared-channel ad hoc networks, one single channel is shared by several geographically distributed communication nodes. Without central control, a Multiple Access Control (MAC) protocol is needed to resolve access collisions. The simplest MAC scheme is to allow packets to be sent immediately when they arrive at idle nodes; this scheme is known as ALOHA. More sophisticated MAC schemes employ the ALOHA mechanism to reserve the channel for packet transmissions (e.g., the Packet Reservation Multiple Access (PRMA) [1]).

Packet collisions in multiple access exist due to the spatial distribution of nodes, lack of central access coordinating entity, and the randomness of packet transmissions. Collision resolution algorithms based on "tree" traverse or "splitting" have been proposed and studied [2]. Usually, the schemes operate in a slotted manner and rely on the channel *feedback*, indicating 0, 1, or more than 1 senders (in ternary feedback) have sent packets in the previous time slot. In the case of binary feedback, the presence or absence of packet transmission should be detected.

In radio environment, however, channel feedback such as packet collisions can hardly be detected, even though successful packet transmission can be overheard by all nodes in range. This is different to the assumption of imperfect channel feedback or asymmetric feedback [3], since under the asymmetric feedback assumption it is a probability distribution that some nodes will be able to detect packet collisions. In radio environment, only the colliding senders notice the packet collisions due to the lack of the acknowledgment from their receiver(s).

Another approach is the use of the random backoff technique. In order to avoid repeated collisions between the same nodes upon detection of a collision, the sender is required to wait for a random period of time before it retries. This random period is referred to as *retransmission delay* or simply *backoff*. Backoff algorithms, which usually adaptively change the retransmission delay according to the traffic load, are implemented to address the dynamic network conditions and to improve the performance of such system.

In a backoff algorithm, the duration of the backoff is usually selected randomly in the range of 0 and some maximum time duration, which we refer to as the *backoff interval* (B). The backoff interval is dynamically controlled by the backoff algorithm. Setting the length of the backoff interval is, however, not a trivial task. On one hand, with a fixed number of ready nodes, small backoff intervals do not reduce the correlation among the colliding nodes to low enough level. This results in still too high probability of collisions, lowering the channel throughput. On the other hand, large backoff intervals introduce unnecessary idle time on the channel and increase the average packet delay, also degrading the scheme's performance.

High channel throughput and low delay are the two fundamental characteristics of a good backoff algorithm, but not the only two. Fairness among competing nodes should also be considered. In designing backoff algorithms, one should avoid algorithms with high channel throughput and low delay, but poor fairness.

Many backoff algorithms have been proposed in the technical literature. However, as discussed in the following section, some problems still remain unresolved. For instance, what is the backoff interval maximizing the throughput with fair access from active nodes? Is a backoff scheme operating

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at this optimum backoff interval and supporting maximum throughput, or at least close to it? How much does a scheme degrade in performance when it doesn't operate at the optimum point? In this paper, we study the problem of setting optimum backoff interval as a function of the number of active nodes (N). Our study shows that the optimum backoff interval should be $4N$ times the transmission time of a data packet when the random access channel operates under a pure-ALOHA scheme. We further propose a new backoff algorithm, named Sensing Backoff Algorithm (SBA). In the SBA scheme, each node dynamically changes its backoff interval according to the results of the sensed channel activities.

The paper is organized in the following way: Section II discusses previous related work. The SBA scheme is introduced in Section III. Section IV presents our study of the optimum backoff interval in a fully connected network with a known N. The optimum parameters of the SBA protocol are investigated in Section V, followed by the performance evaluation in Section VI. Section VII concludes the work.

II. RELATED WORK

Many backoff schemes have been proposed and studied in the technical literature. Binary Exponential backoff (BEB) is an algorithm being widely used in the MAC layer protocols [4] [5] [6]. In BEB, each node doubles the backoff interval up to the maximum backoff interval (B_{max}) after a collision occurs and decreases the backoff interval to the minimum value (B_{min}) after a successful transmission. We summarize BEB by the following set of equations:

$$\begin{cases} x \leftarrow min(2x, B_{max}) & \text{upon collision} \\ x \leftarrow B_{min} & \text{upon successful transmission} \end{cases}$$

where x is the backoff interval value. The values of the B_{min} and B_{max} are pre-determined based on the possible range of number of active nodes and the traffic load of a network. For example, B_{min} and B_{max} are usually set to 2 and 1024, respectively, in Ethernet.

The simplicity and good performance of BEB contribute to its popularity. Unfortunately, the fairness of the BEB scheme is relatively poor in some scenarios [7] [8]. A simple example is a network with two active nodes competing with each other, each of which has enough data traffic to saturate the channel. When one node is successful in its transmission, it decreases its backoff interval to the minimum value. Since the other node wasn't successful in its transmission, it has now to compete with the first node with a larger backoff interval. With high probability, the first node will continue to repeatedly gain access to the channel, while the backoff interval of the second node will be repeatedly doubled until it reaches the maximum value. Consequently, the first node effectively monopolizes the channel, while the second node is deprived from accessing the channel altogether.

To address the problem of unfairness in the BEB scheme, the Multiplicative Increase Linear Decrease (MILD) algorithm was introduced in the MACAW protocol [7]. In the MILD scheme, a collided node increases its backoff interval by multiplying it by 1.5. A successful node decreases its backoff interval by one step, which is defined as the transmission time of the request packet (Request-To-Send, RTS). Since the MACAW protocol assumes that a successful node has a backoff interval that is somehow related to the contention level of the local area, the current backoff interval is included in each transmitted packet. A backoff interval copy mechanism is implemented in each node, to copy the backoff intervals of the overheard successful transmitters. The MILD scheme can be summarized by the following set of equations:

$$\begin{cases} x \leftarrow min(1.5x, B_{max}) & \text{upon collision} \\ x \leftarrow x_{packet} & \text{upon overhearing success} \\ x \leftarrow max(x-1, B_{min}) & \text{upon successful transmission} \end{cases}$$

where x_{packet} is the backoff interval value included in the overheard packet.

The MILD scheme also maintains a backoff interval for each stream instead of each node, in order to improve the fairness. With the copy mechanism, the fairness performance of the MILD scheme is greatly improved. However, the backoff interval stored into the transmitted packets increases the overhead and, thus, the probability of packet collisions. Another adverse effect of the copy mechanism is the migration of the backoff intervals. Suppose there are several areas with different traffic loads in a non-fully connected network, the backoff intervals of these areas will migrate from one area to others through the connecting nodes. The channel throughput in these areas will be degraded, since the backoff intervals do not correctly represent the actual contention levels in these areas.

Aside from the study of the backoff schemes for unslotted random access channels, there are many published works studying the backoff schemes for slotted random access channels. In [9], an Exponential Backoff Scheme has been proposed to control the retransmission probability of each busy node on slotted random access channels. At the beginning of each slot, a busy node "flips" a biased coin according to the retransmission probability, to decide whether or not to transmit in the slot. The operation of the proposed scheme is based on (0, 1, e) channel feedback, in which 0, 1, and e represent idle, successful, and collided channel status, respectively. Each node decreases the retransmission probability by multiplying it with by a factor of q ($0 < q < 1$), when the channel feedback of the previous slot is e (collisions). When the channel feedback is 0 (idle), the retransmission probability is increased by multiplying it with $1/q$. The retransmission probability is unchanged when channel feedback is 1 (success):

$$\begin{cases} x \leftarrow x/q & \text{upon channel idle (0)} \\ x \leftarrow x \cdot q & \text{upon collision (e)} \\ x \leftarrow x & \text{upon success (1)} \end{cases}$$

Simulations were performed to find the optimum value of q for different network scenarios.

In [10], a fair backoff control scheme for IEEE 802.11 based wireless ad hoc network has been proposed. In the scheme, the contention window (backoff interval) is changed according to the received packets and the fair share of channel assigned to each node. In [5], an analytical model to study generalized backoff schemes for the slotted ALOHA scheme is presented.

The difficulty in designing a good backoff algorithm is in how to achieve the optimum operation point with dynamic control of the backoff interval. The BEB scheme operates with high fluctuations of the backoff intervals and it may easily lead to channel domination, as we have discussed. The MILD scheme suffers from the backoff interval migration problem caused by the backoff interval copy mechanism. To address these problems, we propose a new backoff scheme, namely, Sensing Backoff Algorithm (SBA), in the following section.

III. Sensing Backoff Algorithm (SBA)

In general, a backoff algorithm decreases the backoff interval at the successful transmitter and increases that at the collided transmitter. An important design issue is to determine how fast these changes should be and how "other" nodes should respond to the channel activities. The BEB scheme tends to favor the last successful transmitter and "other" nodes do not change their backoff intervals. The MILD scheme varies the backoff interval more gently, while allowing "other" nodes to copy the backoff interval value from the successful packet. The backoff interval copy mechanism improves the fairness performance of the MILD scheme, but it also introduces new problem, namely the backoff interval migration problem.

We propose here a new backoff algorithm, the Sensing Backoff Algorithm (SBA). In the SBA scheme, nodes sensing successful packet transmissions decrease their backoff intervals. Compared with the BEB scheme, this "sensing" mechanism provides much better fairness performance. It also avoids the backoff interval migration problem of the MILD scheme, since the copy mechanism is not used. When its parameters are optimized, the SBA scheme operates at, or close to, the optimum operation point of backoff interval, supporting maximum channel throughput with fair access to active nodes on a shared channel. Furthermore, the operation of the SBA scheme does not require the knowledge of the number of active nodes in a network.

In the SBA scheme, every node that experiences packet collisions multiplies its backoff interval by α ($\alpha > 1$). The transmitter and the receiver of each successful transmission should multiply their backoff intervals by θ ($\theta < 1$). All active nodes overhearing (sensing) a successful transmission are required to decrease their backoff intervals by β steps, where a step is defined as the transmission time of a packet (γ). This *sensing* feature is the novel aspect in the design of our scheme and is responsible for the improvement of the fairness performance. The SBA operation can be summarized by the following set of equations:

$$\begin{cases} x \leftarrow min(\alpha \cdot x, B_{max}) & \text{upon failed transmission at sender} \\ x \leftarrow max(x - \beta \cdot \gamma, B_{min}) & \text{upon sensing successful packet at neighbors} \\ x \leftarrow max(\theta \cdot x, B_{min}) & \text{upon successful transmission at sender and receiver} \end{cases}$$

Before optimizing the parameters of the SBA scheme, we firstly derive the expression for the optimum backoff intervals in a single transmission area, given that the total number of active nodes (N) is known.

IV. Optimum backoff Intervals for Random Access Channels

In order to calculate the optimum backoff interval (B_{opt}) maximizing the channel throughput in a single transmission area with the total number of active nodes (N) known, we use the following assumptions:

- There are N identical nodes in a single local coverage area, in which all nodes are in the range of each other. We assume that the maximum connectivity (number of neighbors of each node) is 100, meaning that $N \leq 100$.
- Any overlap of transmissions at a receiver causes loss of all the colliding packets. We assume that transmission errors occur with much lower probability than packet collisions. Accordingly, packet collisions are the only source of packet error.
- We assume that all nodes are in line-of-sight of each other and the network is operating with radio transmission range less than hundred of meters. Furthermore, the radio signal attenuation on every receiving node is relatively equal and there is no capture effect.
- We assume that a successful transmission can be heard by all nodes, since they are all in the range of each other. However, collisions can only be noticed by the packet transmitter, by means of lack of acknowledgment from its intended receiver. Thus, we assume promiscuous operation mode of all nodes and packet level sensing capability [11].
- Once a packet is successfully received, an acknowledgment packet is sent immediately to the transmitter. We assume that the transmission of the acknowledgment packet uses negligible network resources (e.g., piggybacked on traffic in the reverse direction) and the transmission delay is negligible compared with the random (backoff) waiting time.
- A busy node will not process new packets until it successfully transmits the current packet. No packet preemption is allowed.
- The transmission time of a data packet is γ [time-units][1]. All data packets are of the same size. Due to the assumption of local coverage, the propagation delays are negligible.[2]

We assume that the backoff algorithm operates in the following way:

- When a new packet arrives at a non-idle node (in the backlogged or transmission state), the packet will be put into a queue of infinite size.
- Before the transmission of a packet, a node generates a random backoff waiting time according to the uniform distribution between 0 and B, the length of its backoff interval.[3] All nodes have the same value of B and this value does not change.

[1]The values of all time variables are in the same time units, which will be omitted for simplicity.

[2]Please note that this does not lead to negligible collision probability, as no carrier sense capability of nodes has been assumed.

[3]We assume Delayed First Transmission (DFT) in our analysis, in which new packet arrivals are subject to the random delay. We have also considered Immediate First Transmission (IFT) in our simulations.

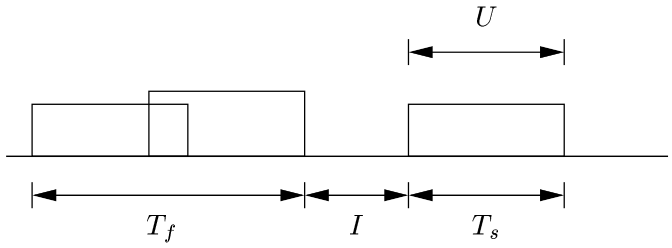

Fig. 1. An Example of Channel Activities

- At the end of the random backoff waiting time, the packet will be sent.
- If the packet transmission is unsuccessful, a new random backoff waiting time will be generated and applied to the packet.

Since unsuccessful packets backoff and retry at a random time later until they are successfully transmitted, the channel throughput is equal to the input traffic load until the arriving packets saturate the channel (at the network capacity). To calculate the channel capacity, we further assume that every node on the single-hop network is always ready to transmit [9] [12].

We now introduce the notion of the "busy period" [13]. A busy period is a period of time with packet transmissions (failed or successful) on the channel (Fig. 1). The period of time between consecutive busy periods is called an idle period (I). The utilization period (U) is the time within a successful period, when the useful data is sent. According to [13], the channel throughput (S) of a shared channel can be expressed as:

$$S \cong \frac{P_s \cdot \overline{U}}{P_s \cdot \overline{T_s} + (1 - P_s) \cdot \overline{T_f} + \overline{I}}, \tag{1}$$

where P_s is the probability of successful packet transmissions, $\overline{U}$, $\overline{T_s}$, $\overline{T_f}$, and $\overline{I}$ are the average duration of the utilization period, the duration of the successful busy period, the duration of the failed busy period, and the duration of the idle period, respectively.[4]

We first study the probability of one node transmitting in a short period of time Δt, where $\Delta t \ll B$. Since a fixed backoff interval, B, is used, with DFT and with uniformly distributed backoff waiting time, the mean inter-arrival time at each node is $B/2$. Hence, the average transmission arrival rate on the shared channel due to one node is $2/B$. So,

$$\text{Prob}\{\text{A node starts transmission in } \Delta t\} = \frac{2}{B} \cdot \Delta t.$$

For the first transmitted packet on the channel after each idle period (I), the probability of success is the probability that all other nodes are silent in the period of time that the packet is being transmitted on the channel (γ) [14][5]:

$$\begin{aligned} P_s &= \text{Prob}\{\text{None of the other nodes transmit in } \gamma\} \\ &= \left(1 - \frac{2\gamma}{B}\right)^{N-1}. \end{aligned} \tag{2}$$

[4]Equation (1) is an approximation, because we have replaced each random variable with its average value.

[5]We assume that the transmissions at different nodes are independent.

We calculate the average idle time ($\overline{I}$) by approximating the arrivals of all nodes by a Poisson arrival process. The total arrival rate is $N \cdot 2/B$, so the average idle time is [13]:

$$\overline{I} = \frac{1}{N \cdot 2/B} = \frac{B}{2N}. \tag{3}$$

The average successful period $\overline{T_s}$ and the average utilization period $\overline{U}$ are both γ. The average failed period can be expressed as (see Appendix I):

$$\begin{aligned} \overline{T_f} &= \left(\frac{B}{B-2\gamma}\right)^{N-1} \cdot \\ &\left[\frac{B}{2} - \frac{B}{2} \cdot \frac{N-1}{N} \cdot \frac{1 - \left(\frac{B-2\gamma}{B}\right)^{N}}{1 - \left(\frac{B-2\gamma}{B}\right)^{N-1}}\right] + \gamma. \end{aligned} \tag{4}$$

Applying (2), (3), and (4) into (1), the channel throughput as a function of N and B can be obtained as:

$$S = \frac{\left(1 - \frac{2\gamma}{B}\right)^{N-1} \cdot \gamma}{\left(1 - \frac{2\gamma}{B}\right)^{N-1} \cdot \gamma + \left(1 - \left(1 - \frac{2\gamma}{B}\right)^{N-1}\right) \cdot \overline{T_f} + \frac{B}{2N}}, \tag{5}$$

where $\overline{T_f}$ is given by (4).

To find the optimum B (B_{opt}), we numerically solve the equation of $\partial S/\partial B = 0$ for different N. After some manipulations on $\partial S/\partial B = 0$, it can be proved that

$$\lim_{N\to\infty} \frac{B_{opt}(N)}{N\gamma} = 4.$$

Thus we approximate the equation

$$\left.\frac{\partial S}{\partial B}\right|_{B=B_{opt}} = 0$$

by

$$B_{opt}/\gamma - 4N = 0$$

and show both results in Fig. 2. As discussed below, we have verified that the approximation is good even for small N, and thus we conclude that

$$B_{opt}(N) = 4N\gamma, \tag{6}$$

where γ is the transmission time of a packet.

An intuitive explanation for the value of B_{opt} given by (6) is discussed below: Pure ALOHA channel achieves its maximum throughput of $1/(2e)$ at $G = 0.5$ under the Poisson arrival assumption [15]. In a network with large N and large backoff interval B, the maximum channel throughput can also be achieved with $G = 0.5$. Since the packet transmissions arrive at each node at a normalized rate of $2\gamma/B$, the total rate of arrival is $2N\gamma/B$. Solving the equation of $2N\gamma/B = 0.5$, we obtain the optimum backoff intervals (B_{opt}) for different N, as per (6).

In Fig. 3, we show the throughput comparison of using the approximate optimum backoff intervals from (6) and using the optimum values from numerical results in Fig. 2. It can be seen that the throughput degradation due to the approximation is always less than 2%, except for $N = 2$, where the degradation is about 10%. Hence, we approximate the optimum backoff interval for a network with N active nodes to be $4N$ times the

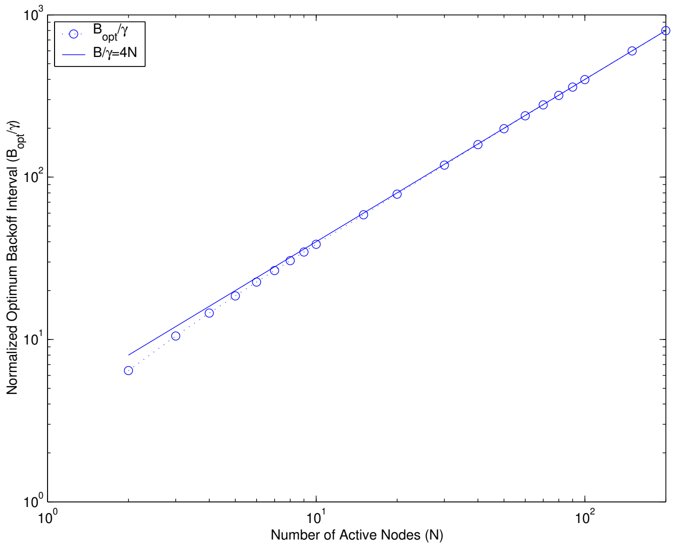

Fig. 2. Normalized Backoff Intervals

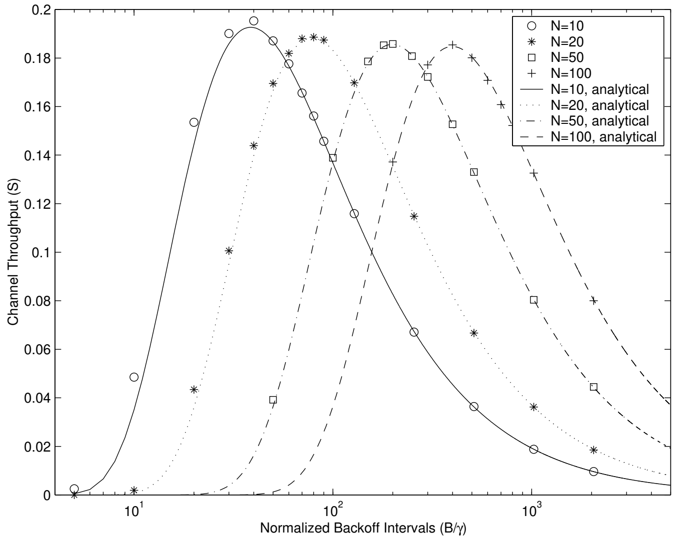

Fig. 4. Channel Throughput as a Function of B for Different N

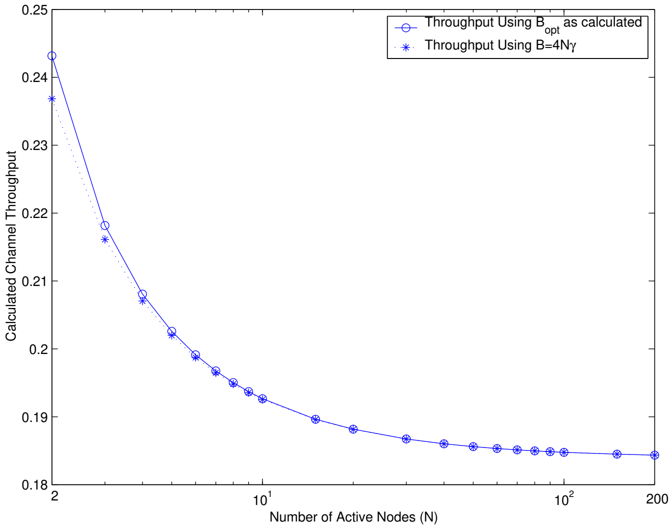

Fig. 3. Throughput Degradation by Using $B_{opt} = 4N\gamma$

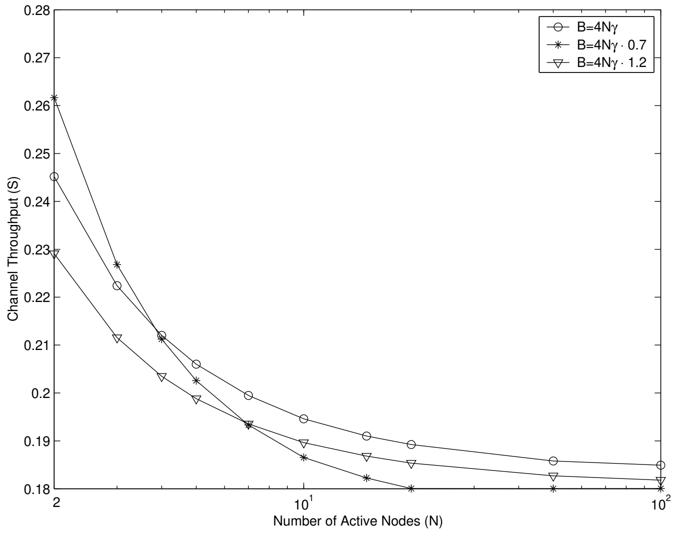

Fig. 5. Throughput Performance of $B = 4N\gamma \cdot \phi$

transmission time of a packet. When more precision is desired, the optimum backoff interval for a network with $N = 2$ should be $B_{opt}(2) = 6\gamma$.

From Fig. 3, it can also be observed that, as N increases, the throughput performance of an optimal backoff scheme, as shown in (5), approaches the value of 0.184 (i.e. $\frac{1}{2e}$), which is the maximum throughput of pure ALOHA scheme. This performance is achieved with the use of (6). Please note that the backoff scheme operates in the unstable region of pure ALOHA scheme.

In Fig. 4, we verify, analytically and by means of simulation, the value of the optimum backoff interval in (6). We show the channel throughput of a fully connected network as a function of fixed backoff interval (B) for different number of active nodes (N). Simulation results are presented as discrete points[6], while analytical results in (5) are shown as curves. Close match is achieved between the simulative results and the analytical results, although some noticeable discrepancy can be observed when N and B are small. We have verified that the optimum value of the backoff intervals is about $4N\gamma$ for the results shown. On one hand, smaller B leads to lower channel throughput, because of the larger probability of repeated collisions. On the other hand, larger B drives nodes into a defer state too often with the channel being idle in larger fraction of time, lowering the channel throughput as well, as shown in the graph. (The latter phenomenon is the result of the assumption that a busy node doesn't process new packets until it successfully transmits the current one.)

In Fig. 5, we show the throughput performance of the optimum backoff algorithm with imperfect knowledge of N. From the figure, one can find that even if the uncertainty of N is in the range of 0.7 or 1.2 times its actual value, the throughput performance is still quite good; i.e., the performance degradation is less than 5%. The figure also demonstrates that

[6] In our simulations, we have assumed that the channel data rate is 1 Mbps and that the data packet length is 2000 bits.

the performance of $B = 4N\gamma$ is generally better than the other two values of the backoff interval. The only exception is for small value of N (i.e., N=2 or 3), under which condition our approximation becomes less accurate.

V. Calculating the Parameters for SBA

Based on the above calculation of the optimum backoff intervals, we can find the optimum values of α, β, and θ for the SBA scheme proposed in Section III. We study the sum of the backoff intervals of all nodes on the network (B_N) by calculating the net change of B_N (ΔB_N) over a period of time (t). The net change should approach zero asymptotically, when the system is in equilibrium. Hence we can obtain the relation among α, β, and θ.

The net change of B_N can be calculated as:

$$\Delta B_N(t) = \Delta B_N^s(t) + \Delta B_N^c(t),$$

where $\Delta B_N^s(t)$ and $\Delta B_N^c(t)$ are the net change of B_N due to the successful transmissions and the collided transmissions, respectively, in the period of time (t). In the calculation of $\Delta B_N^s(t)$ and $\Delta B_N^c(t)$, we assumed that these successful transmissions and collided transmissions are sent by nodes with a backoff interval of $\overline{B}$, the average of backoff interval over the period of time (t). Our objective is to find optimum values of α, β, and θ to maintain $\overline{B}$ as close as possible to $B_{opt} = 4N\gamma$, to maximize the network throughput.

After each successful transmission, the transmitter and the receiver change their backoff interval from $\overline{B}$ to $\theta\overline{B}$, with a net change in B_N of $2(\theta-1)\overline{B}$. All other nodes decrease their backoff intervals by β steps, with a net change in B_N of $-\beta\gamma(N-2)$. So, $\Delta B_N^s(t)$ can be expressed as:

$$\Delta B_N^s(t) = \rho^s(t) \cdot \left[2(\theta-1)\overline{B} - \beta\gamma(N-2)\right],$$

where $\rho^s(t)$ is the total number of successful transmissions in the period of time t.

After each collided transmission, the packet transmitter multiplies its backoff interval by α, with a net change in B_N of $(\alpha-1)\overline{B}$. So, $\Delta B_N^c(t)$ can be expressed as:

$$\Delta B_N^c(t) = \rho^c(t) \cdot (\alpha-1)\overline{B},$$

where $\rho^c(t)$ is the total number of collided packets in the period of time t.

As the net change of B_N should approach zero asymptotically, $\lim_{t\to\infty} \frac{\Delta B_N(t)}{t} \approx 0$, i.e.,

$$\lim_{t\to\infty} \frac{\rho^s(t)\cdot\left[2(\theta-1)\overline{B} - \beta\gamma(N-2)\right] + \rho^c(t)\cdot(\alpha-1)\overline{B}}{t}$$

or

$$\lim_{t\to\infty} \frac{\rho^s(t)\overline{B}}{t}\left\{\left[2(\theta-1) - \frac{\beta\gamma(N-2)}{\overline{B}}\right] + \frac{\rho^c(t)}{\rho^s(t)}\cdot(\alpha-1)\right\}$$

should equal to zero.

So, the relation among α, β, and θ is:

$$(\alpha-1)\cdot\lim_{t\to\infty}\frac{\rho^c(t)}{\rho^s(t)} = 2(1-\theta) + \frac{(N-2)\beta\gamma}{\overline{B}}.$$

We give the derivation of $\lim_{t\to\infty}\frac{\rho^c(t)}{\rho^s(t)}$ in Appendix II and present the result here:

$$\lim_{t\to\infty}\frac{\rho^c(t)}{\rho^s(t)} = \left(\frac{B}{B-2\gamma}\right)^{2N-2} - 1, \quad (7)$$

where, according to our assumption, B is the average value of the backoff intervals ($\overline{B}$).

So the relation between α, β, and θ becomes:

$$\left[\left(\frac{2N}{2N-1}\right)^{2N-2} - 1\right](\alpha-1) = 2(1-\theta) + \frac{(N-2)\beta}{4N}. \quad (8)$$

The value of α controls the promptness of the SBA scheme in responding to traffic load change. As an example, we use $\alpha = 1.2$ in the following calculation and defer the discussion on the choice of α to the section of "Performance Evaluation". By allowing N to take values of either 10 or infinity in (8), we obtain the following equations that allow to calculate the values of β and θ:

$$\begin{cases} \alpha = 1.2 \\ 1.72(\alpha-1) = 2(1-\theta) + \frac{\beta}{4} \\ 1.52(\alpha-1) = 2(1-\theta) + \frac{8\beta}{40}. \end{cases} \quad (9)$$

The solution to the above equation set is (α,β,θ) = (1.2, 0.8, 0.93)[7].

VI. Performance Evaluation

We have run simulations to evaluate the performance of the SBA scheme. The set of optimum parameters (1.2, 0.8, 0.93) that we chose in Section V for (α,β,θ) is simulated and compared with some other choices of values. The channel throughput of SBA using the optimum set of parameters is compared with the throughput of the MILD scheme, the BEB scheme, and a *genie algorithm*, which assumes the perfect knowledge of the total number of active nodes on the fully connected network. We also compared the performance of the SBA scheme and the MILD scheme in regards to fairness and delay. In our performance evaluation, we have assumed that the channel data rate is 1 Mbps and that the data packet length is 2000 bits. The minimum and the maximum value of backoff intervals (B_{min} and B_{max}) are 2 and 1024, respectively. Initially every node has a backoff interval of $B = B_{min} = 2$ and all nodes are always ready to send.

Figure 6 shows the channel throughput of the SBA scheme with different sets of values of (α, β, and θ) when α is fixed at 1.2. The graph confirms that (1.2, 0.8, 0.93) is the optimum value set for (α, β, and θ) in the SBA scheme when α is 1.2. Operating with the parameter set of (1.2, 0.8, 0.93), the SBA scheme offers a channel capacity from 0.186 to 0.245, when N is in the range of (2, 100).

Figure 7 presents the channel throughput of the SBA scheme with different values of α. We modified the first equation in the

[7] We believe that selecting a matching point of infinity nodes is necessary to asymptotically guarantee the best throughput. Different selections of the second matching point may slightly change the protocol parameters β and θ. However, the differences are not significant. For instance, when we select the second matching point in the range of $[3, 100]$, β is changed from 0.78 to 0.88 and θ is changed from 0.925 to 0.938. The performance of our SBA scheme is still very good according to Fig. 6 in Section VI. Furthermore, the performance of our SBA scheme is guaranteed by the robustness of backoff schemes regarding to some deviation of B from B_{opt}, as shown in Fig. 5.

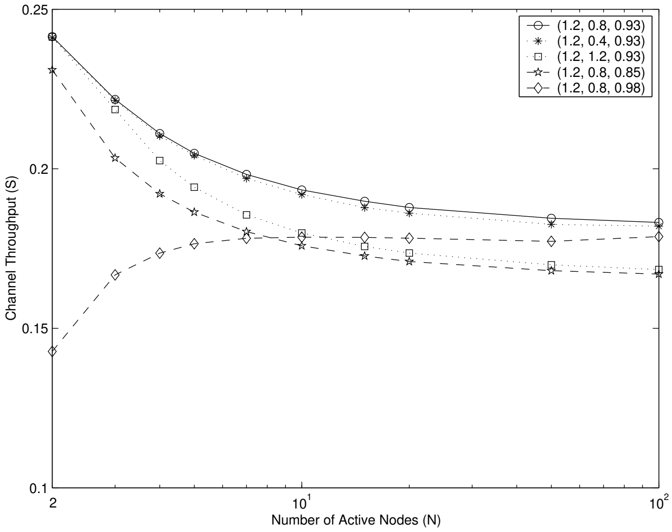

Fig. 6. Performance of SBA with Different Parameters ($\alpha = 1.2$)

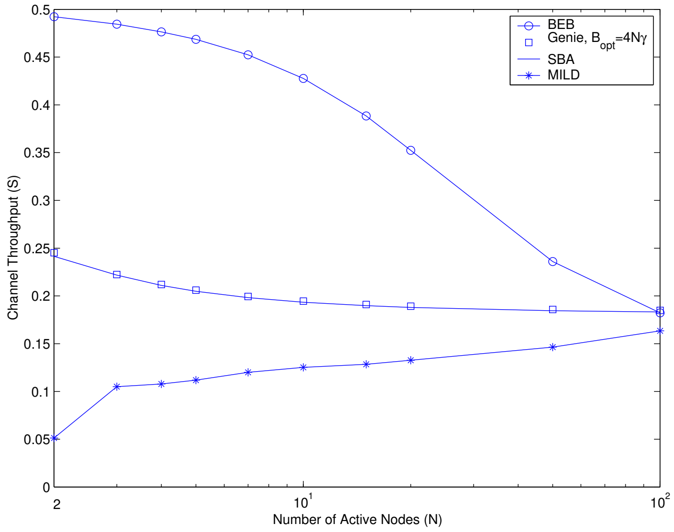

Fig. 8. Performance of BEB, SBA, MILD, Genie ($B_{opt} = 4N\gamma$)

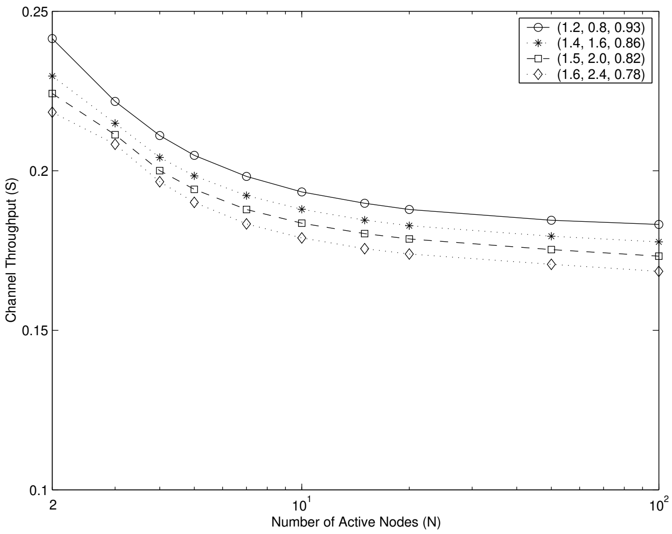

Fig. 7. Performance of SBA with Different α

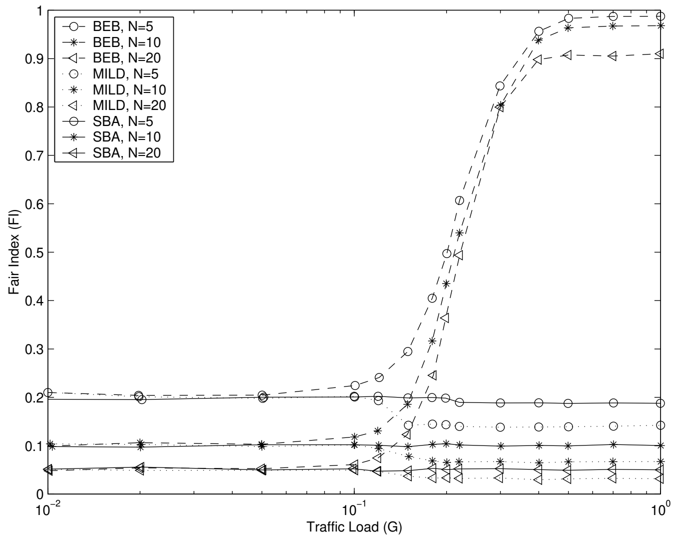

Fig. 9. Fairness Performance of BEB, MILD, and SBA

equation set (9) and solved for β and for θ. We found that as α increases (better responsiveness to the changes in the traffic load), the throughput performance degrades. However, as α increases from 1.2 to 1.4 and further to 1.6, the throughput degradation is only about 5% and 10%, respectively, which is the performance penalty due to the higher responsiveness to the changes in the traffic load.

The channel throughput performance of the SBA scheme is compared with the performance of the other algorithms in Fig. 8. The figure depicts the channel throughput of the SBA scheme, the MILD scheme, the BEB scheme, and the genie algorithm ($B = 4N\gamma$). The genie algorithm with $B = 4N\gamma$ serves as the "upper bound" in the comparison, since it assumes the perfect knowledge of the total number of active nodes in the network (N), which, practically, is unknown to the backoff algorithm. We want to point out that the high throughput of the BEB scheme is achieved by allowing one node to dominate the channel and penalizing the other nodes, resulting in unfair channel sharing. We defer the discussion of fairness to Fig. 9.

Figure 8 demonstrates that the SBA scheme operates very closely to the genie algorithm. The performance of the SBA scheme approaches the "upper bound", with a channel throughput of about 0.18 - 0.24, depending on the value of N. Note that one of the salient features of the SBA scheme is that it does not require the knowledge of N. Yet it can achieve performance close to that of the genie algorithm. The performance gain of SBA over MILD is about 50%, with larger gain for smaller N.

The throughput curve of the MILD scheme shows that it operates away from the optimum backoff interval. In fact, the MILD scheme lowers the backoff interval too slowly (only by one step). Hence, the backoff interval tends to be large. As N increases, the backoff interval (bounded by B_{max}) is closer to the optimum values, leading to higher throughput.

Figure 9 compares the performance of fairness of the BEB scheme, the MILD scheme, and the SBA scheme. In this figure, we show the Fairness Index (FI)[8] of these schemes as a function of traffic load (G) for different nodal densities. The Fairness Index (FI) is calculated as the probability that the previous successful node becomes the next successful transmitter. The FI, thus, indicates the instantaneous domination in the channel sharing. In [10] and [16], FI is calculated as the ratio of maximum and minimum throughput shared by all nodes, which might hide channel domination by calculating average throughput.

From Fig. 9, we can observe that the FI of the three compared scheme are about the same when traffic load is lower than 0.1. The FI level is about $1/N$, which represents the randomness of traffic generation. However, as the traffic load increases, the FI value of the BEB scheme increases sharply. Under high traffic load, the FI value of the BEB scheme is about 0.9 to 0.99, depending on the number of active nodes in the network. These FI reveals the significant channel domination characteristic of the BEB scheme.

The FI value of the SBA scheme stays at almost the same level of $1/N$ as the traffic load changes from 0.01 to 1. This shows the good fairness performance of the SBA scheme over wide range of traffic loads: when there are N active nodes on the network, the successful transmitter has a probability of $1/N$ to be the next transmitter. The fairness performance of the MILD scheme shows an interesting pattern. The FI value is lower when the traffic load is higher than the channel capacity, meaning that successful nodes are too "generous" after their successful transmissions. The explanation of this result is that a node has to schedule its new transmission after a successful transmission, while the timer of the other nodes have already been running, although their waiting time was generated based on the same backoff interval. Hence the other nodes have higher probability of winning the next round of the competition.

Note that the MILD scheme offers good fairness performance, because of the use of the backoff interval copy mechanism. However, this increases the overhead of the transmitted packets and, thus, increases the probability of packet collisions, as discussed before. Furthermore, in a non-fully connected network, the adverse effect of the copy mechanism is the migration of backoff intervals into areas with different contention levels. The SBA scheme provides reasonable fair access to all active nodes in the network, without the need to resort to the backoff interval copy mechanism, thus avoiding this problem altogether.

Figure 10 presents the delay performance of the SBA scheme and the MILD scheme. In the graph, we show the delay performance of networks with N equal to 5, 10, and 20. We can see from the graph that with reasonable average packet delay, the SBA scheme offers 50-80% higher channel capacity than the MILD scheme does.

In the same figure, we have also shown the performance of the Immediate First Transmission (IFT) mode of the SBA scheme and compared its performance with that of the DFT

[8]More precisely, the index should be called Unfairness Index. But we followed [10] and [16] and used the term, Fairness Index, as it has been defined there.

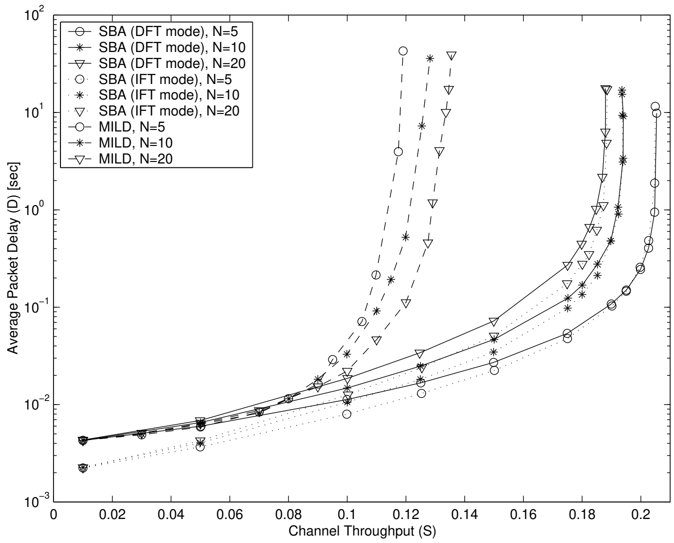

Fig. 10. Delay Performance of SBA and MILD

TABLE I
PERFORMANCE OF MILD AND SBA ON A MULTI-HOP NETWORK

Number of Nodes	25	50	100
Throughput of MILD	0.79	0.78	0.70
Throughput of SBA	1.02	1.00	0.96

mode SBA scheme. Operating in the IFT mode, a packet that arrived at an idle node will be transmitted immediately. In contrast, in the DFT mode of operation, this packet would be subject to the random delay. The average packet delay of the IFT mode is somewhat lower than that of the DFT mode. This is more noticeable in the light traffic load condition, under which the first transmissions have higher probability of success. Under heavy traffic load condition, however, the probability of first transmission being successful is lower. Hence, the effect of IFT mode is less noticeable. However, both the IFT and the DFT modes offer approximately the same channel capacity.

Finally, Table I compares the throughput performance of the MILD scheme and the SBA scheme in a multi-hop network. The network size is 400 meters by 400 meters, while the radio transmission range is 100 meters. We used different number of nodes (25, 50, and 100) in the network and placed them randomly within the network area. The throughput results show that the SBA scheme out-performs the MILD scheme by about 30%, with all the nodal densities in the multi-hop network that we have simulated.

VII. SUMMARY AND CONCLUSIONS

In shared channel ad hoc networks, a single channel is shared by a number of nodes. Packet collisions may take place as a result of the random transmissions from active nodes. After collisions, nodes need to backoff and retry at a later time. The process of backoff is managed by the backoff algorithm, implemented in the MAC layer protocol. Channel throughput, packet delay, and fairness are the three main concerns in designing backoff algorithms. Good backoff algorithms should be able to achieve high channel throughput and low packet delay, while maintaining fairness among active nodes.

A new backoff algorithm, which we have termed the Sensing Backoff Algorithm (SBA), has been proposed in this paper and its performance evaluated. In the SBA scheme, each node dynamically changes its backoff interval according to the results of the sensed channel status. We have derived and verified the optimum setting of the backoff interval value (B) with the knowledge of the number of active nodes (N) in a fully connected network, when the MAC operates in an unslotted ALOHA access scheme. We found that, when the random access channel operates with a pure-ALOHA scheme, this optimum value should be $4N\gamma$, where γ is the transmission time of a packet. Based on this result, we calculated the optimum parameters for the SBA scheme.

Our study has shown that the SBA scheme operates close to the optimum, maximizing the network throughput with fair access from active nodes, without the precise knowledge of the number of active nodes. Comparing with the MILD scheme, SBA does not require additional control fields to be added to the packets, reducing the overhead and vulnerable time of each transmitted packet. Furthermore, the SBA algorithm does not use the backoff copy mechanism, avoiding the problem of the backoff interval migration among areas with different contention levels. The SBA scheme provides fairness performance comparable to that of MILD, both of which are much better than that of BEB. It is shown that the SBA scheme out-performs the MILD scheme in throughput performance. As a point of reference, the SBA scheme offers a channel capacity of 0.19 for $N = 10$, while the MILD scheme provides capacity of 0.125 in this case. The performance gain is about 50%.

In our performance evaluations, all nodes have the same initial settings and they are always ready to send. A question is how well the SBA scheme behaves under un-aligned settings, i.e., nodes starting with different backoff intervals and turning on and off from time to time. Whether the SBA scheme is able to guarantee the re-alignment of the backoff interval of all nodes is an important performance characteristic of the proposed scheme. Since the SBA scheme guarantees the long-term average of the backoff intervals of all nodes to be the optimum backoff value and the backoff intervals fluctuate over time, we envision that it is able to re-align such heterogeneous network settings. We defer such detailed discussions to our future work due to space limit.

Our result of the optimum backoff interval with the knowledge of N ($B_{opt} = 4N\gamma$) is derived based on the assumption of unslotted random access channel, but should be applicable in other schemes as well. Another contribution of this paper is the analytical model of backoff controlled random access channels. Additionally, our analytical framework can also be extended to other types of MAC schemes such as FAMA [11], IEEE 802.11 DCF [12], and DBTMA [17]. Finally, the optimum parameters of the SBA scheme can be derived for other MAC schemes with the approach used in this paper.

APPENDIX I
AVERAGE FAILED PERIODS ($\overline{T_f}$)

The method we use to calculate the average failed periods ($\overline{T_f}$) is similar to what Takagi and Kleinrock used in [14]. The duration of a failed busy period F consists of a number (L) of packet inter-arrival times whose duration are less than γ (denoted by $t_1,\ t_2,\ \cdots,\ t_L$) terminated by a full length of γ (Fig. 11):

$$F = t_1 + t_2 + \cdots + t_L + \gamma.$$

All t_n's are independent and identical distributed. The cumulative distribution function can be calculated as:

$$\begin{aligned}\text{Prob}[t_i \le t] &= \frac{\text{Prob[at least one transmission in t]}}{\text{Prob[at least one transmission in }\gamma]} \\ &= \frac{1-(1-\frac{2t}{B})^M}{1-(1-\frac{2\gamma}{B})^M} \\ &= \frac{B^M-(B-2t)^M}{B^M-(B-2\gamma)^M} \quad 0 \le t \le \gamma,\ 1 \le i \le L,\end{aligned}$$

where M is the number of nodes that may send their packets in the period of time.

The probability density function is:

$$\begin{aligned}f(t) &= \frac{\partial}{\partial t}\text{Prob}[t_i \le t] \\ &= \frac{2M(B-2t)^{M-1}}{B^M-(B-2\gamma)^M} \quad 0 \le t \le \gamma,\ 1 \le i \le L.\end{aligned}$$

The expected value of t_i is:

$$\begin{aligned}\overline{t_i} &= \int_0^\gamma t f(t) dt \\ &= \int_0^\gamma \frac{2Mt(B-2t)^{M-1}}{B^M-(B-2\gamma)^M} dt \\ &= \frac{B}{2} - \frac{B}{2}\cdot\frac{M}{M+1}\cdot\frac{1-\left(\frac{B-2\gamma}{B}\right)^{M+1}}{1-\left(\frac{B-2\gamma}{B}\right)^{M}} \quad 1 \le i \le L.\end{aligned}$$

The number of such arrivals is independent of t_i and is geometrically distributed as:

$$\text{Prob}[L = n] = (1-P_a)^{n-1}P_a \qquad n = 1, 2, \cdots,$$

where P_a is the probability that no new transmission will start in the duration of γ seconds:

$$P_a = \left(\frac{B-2\gamma}{B}\right)^M.$$

So the expected value of number of arrivals is:

$$\overline{L} = \frac{1}{P_a} = (\frac{B}{B-2\gamma})^M.$$

The average failed period can be calculated as $\overline{T_f} = \overline{L}\cdot\overline{t_i} + \gamma$, which is

$$(\frac{B}{B-2\gamma})^M\left(\frac{B}{2} - \frac{B}{2}\cdot\frac{M}{M+1}\cdot\frac{1-\left(\frac{B-2\gamma}{B}\right)^{M+1}}{1-\left(\frac{B-2\gamma}{B}\right)^{M}}\right) + \gamma.$$

We approximate M as $N-1$, since there are at most $N-1$ nodes in the network that might start new transmissions in the period of γ. Hence, we have derived (4).

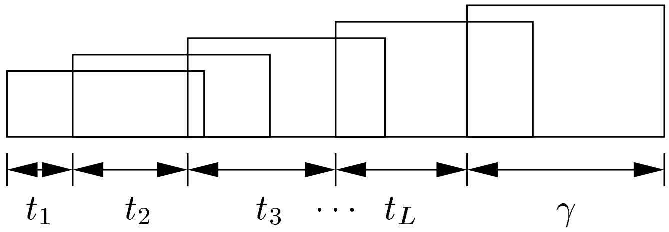

Fig. 11. An Example of Failed Transmission Periods

APPENDIX II
DERIVATION OF (7)

Let $n(t)$ denote the total number of busy periods in the period of time t. The total number of successful packets can be expressed as:

$$\rho^s(t) = n(t) \cdot P_s,$$

where P_s is given by (2).

The total number of collided packets is:

$$\rho^c(t) = n(t) \cdot (1 - P_s) \cdot (\overline{L} + 1),$$

where we have assumed there are $(\overline{L} + 1)$ packets in each failed busy period. $\overline{L}$ is given by

$$\overline{L} = \frac{1}{P_a}$$

in Appendix I.

So the ratio of collided packets and successful packets is:

$$\begin{aligned} \frac{\rho^c(t)}{\rho^s(t)} &= \frac{(1 - P_s) \cdot (\overline{L} + 1)}{P_s} \\ &= \left(\frac{B}{B - 2\gamma}\right)^{2N-2} - 1. \end{aligned}$$

Hence, we have derived (7).

APPENDIX III
SLOTTED VERSION SBA (SSBA)

There are many wireless communication networks operating in slotted fashion. For completeness, we provide a Slotted version of SBA (SSBA) in this appendix.

In SSBA, every node should maintain a backoff interval B and selects a backoff waiting time, in the unit of slots, uniformly from (0, B/γ'],

$$W = \lceil U(0, B/\gamma') \rceil$$

where γ' is the slot duration, which usually should be set to the sum of the packet transmission time (γ) plus guarding time and acknowledgment time. The ceiling function is used to select an integer number of waiting slots.

At the end of each slot, each node updates its backoff interval according to the following algorithm:

$$\begin{cases} x \leftarrow min(\alpha \cdot x, B_{max}) & \text{upon failed transmission} \\ & \text{at sender} \\ x \leftarrow max(x - \beta \cdot \gamma', B_{min}) & \text{upon sensing successful} \\ & \text{packet at neighbors} \\ x \leftarrow max(\theta \cdot x, B_{min}) & \text{upon successful transmission} \\ & \text{at sender and receiver} \end{cases}$$

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