The above results extend readily to Udink ten Cate's [4] generalization of the Kudva–Narendra algorithm and to the model reference adaptive control method of Sebakhy [5]. The method of maintaining linear constraints may be applied to the identification method of Martin–Sanchez [6]; it also extends readily to the discrete single-input–single-output adaptive observer described briefly by Luders and Narendra in [7, section V.A], requiring only the choice $G=I$ for the adaptive gain and the projection of the adaptation excitation vector, which in this case is the auxiliary vector s_k.

References

[1] P. Kudva and K. S. Narendra, "An identification procedure for discrete multivariable systems," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 549–552, Oct. 1974.

[2] K. S. Narendra and L. S. Valavani, "Stable adaptive observers and controllers," *Proc. IEEE*, vol. 64, pp. 1198–1208, Aug. 1976.

[3] G. Kreisselmeier, "Adaptive observers with exponential rate of convergence," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 2–8, Feb. 1977.

[4] A. J. Udink ten Cate, "Gradient identification of multivariable discrete systems," *Elec. Lett.*, vol. 11, no. 5, pp. 98–99, Mar. 1975.

[5] O. A. Sebakhy, "A discrete model reference adaptive system design," *Int. J. Contr.*, vol. 23, pp. 799–804, 1976.

[6] J. M. Martin–Sanchez, "A new solution to adaptive control," *Proc. IEEE*, vol. 64, pp. 1209–1218, Aug. 1976.

[7] G. Luders and K. S. Narendra, "Stable adaptive schemes for state estimation and identification of linear systems," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 841–847, Dec. 1974.

Adaptive Implementation of One-Step-Ahead Optimal Control via Input Matching

C. RICHARD JOHNSON, JR., MEMBER, IEEE, AND EDISON TSE, MEMBER, IEEE

***Abstract*—Directly adapting feedback control parameters to match an optimal input avoids the current limitations of indirect adaptive control and direct adaptive model matching. This original concept, termed input matching, allows combination of stable parameter adjustment algorithms and optimal performance considerations. Furthermore, the emphasis on input matching permits consistent control despite inconsistent parameter identification obviating efforts such as test or probing inputs required to ensure consistent identification.**

The control signal is reduced to a constant weighted sum of the measurable information-state vector components by the use of a one-step-ahead quadratic cost function to govern the behavior of a linear, time-invariant multivariable plant. The control effort from this linear combination proves globally estimable by a vector equation error formulation since the one-step-ahead cost function permits simple *a posteriori* input error calculation. Several simulations demonstrate the behavior of this new multivariable adaptive input matching control method.

I. Introduction

Adaptive control is an outgrowth of automatic control that has attracted significant research effort since the mid-1950's [1]–[3]. These investigations have been motivated by a desire for development of real-time control of incompletely known plants. Limited plant specification is normally assumed to entail unknown, perhaps drifting parameters in a prespecified structural description. Adaptive controllers can be coarsely divided into two large classes of active and passive adaptivity [4], [5]. Actively adaptive controllers, based on Fel'dbaum's dual control theory [6], utilize, in addition to the available real-time information, the knowledge that future observations will be made which will provide further possible performance evaluation and regulate their learning accordingly. Passively adaptive controllers utilize the available real-time measurements but ignore the availability of future observations. This limitation results in accidental learning but generates much simpler control algorithms and therefore will be the technique considered here.

Passive adaptive controllers can be subdivided into two further classifications: indirect and direct, denoting the primary focus of the adaptation mechanism either on plant parameter determination or control parameter determination, respectively. Indirect adaptive control, apparently originally suggested in [7], arbitrarily divides the control task into a plant identification stage providing parameter estimates to a prestructured controller, which utilizes these estimates in generating a control signal as if they were the actual values. Acceptance of this method has led to a tremendous interest in system identification [8]–[11]. Most parameter estimation schemes, however, are inherently open loop and suffer consistency and identifiability constraints in the presence of feedback [12], [13]. Any resultant bias in the parameter estimates could prove disastrous upon insertion in a predetermined control law. This limitation has been artificially circumvented by the injection of a perturbation input [14] or reliance on its inherent presence [15].

The obvious alternative, avoiding the necessity of proper plant identification, is direct adaptive control, which adjusts the available control parameters themselves such that the overall performance of the control system improves. Two broad techniques exist for establishment of convergent control parameter adaptation schemes: search methods and stability analysis. Search techniques, primarily gradient based, arise directly from similar techniques for plant parameter estimation. Normally, however, the gradient of the performance function with respect to the control parameters is not readily determinable and the performance surface is typically multimodal in the control parameters [16]. Alternatively, adaptive control algorithms arising from stability analysis can guarantee global asymptotic stability as a by-product. The widest application of stability theory to adaptive control design has utilized Lyapunov's second method [17], originally [18], [19] via a model reference approach [20], [21]. Model reference adaptive control techniques implement adjustment of reachable parameters in the overall controlled system so that its response to some arbitrary reference signal exactly matches that of a predetermined model due to the same reference. General lack of the ability to satisfactorily alter all of the plant parameters led to the concepts of perfect model following [22], [23] and equicontrollability [24], either of which ensures exact matching with a bounded control effort. Assumption of the capability of exact matching hampers [25]–[27] the current sophisticated schemes of adapting controller parameters solely from plant input and output measurements [28]–[31].

A deterministic direct adaptive scheme is developed in this paper capable of providing both bounded tracking error and bounded control effort by shifting the focus from exact output matching to exact input matching, a concept intimated in [32]–[34]. Input matching, as originated here, melds the globally convergent estimation character of model matching and the robustness of optimally-based input specification. The concept of input matching is motivated by the realization that most control laws for discrete, linear, lumped-parameter time-invariant systems are simply linear combinations of input, output, and driving sequence terms, the "linear-in-the-parameters" [8] structures of which are identifiable by the bulk of parameter estimation schemes. The goal of input formation does not require identifiability, however, only matchability, thereby loosening the restrictions of indirect adaptive control. The idea of observing system behavior and inferring the optimal action that should have been taken completes the scheme by providing a method of error determination for the estimation scheme. The incorporation of control cost in the optimal action inferral expands the technique's applicability beyond the realm of exact output matching. This paper presents the initial application of this new approach to adaptive control.

The next section will discuss the one-step-ahead quadratic cost func-

Manuscript received May 3, 1977; revised September 26, 1977 and April 20, 1978. Paper recommended by R. V. Monopoli, Past Chairman of the Adaptive, Learning Systems, Pattern Recognition Committee. The work of E. Tse was supported by the Office of Naval Research under Contract N00014-75-C-0738.

C. R. Johnson, Jr., was with the Department of Electrical Engineering, Stanford University, Stanford, CA 94305. He is now with the Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

E. Tse is with the Department of Engineering-Economic Systems, Stanford University, Stanford, CA 94305.

0018-9286/78/1000-0865$00.75 ©1978 IEEE

tion to be minimized. A direct adaptive scheme based on input matching utilizing this one-step-ahead definition of optimality is developed in the third section. Simulation studies which furnish insight into the behavior of the proposed adaptive control scheme are presented in Section IV.

II. One-Step-Ahead Optimal Control

The autoregressive-moving-average (ARMA) vector difference equation

$$Y(k+1)=\sum_{i=1}^{v} A_i Y(k-i+1)+\sum_{j=1}^{w} B_j U(k-j+1), \tag{2.1}$$

where Y is an $n\times 1$ output vector, U an $m\times 1$ input vector, and A_i and B_j are, respectively, $n\times n$ and $n\times m$ constant parameter matrices, adequately describes the input–output behavior of a linear, time-invariant lumped-parameter MIMO plant. The plant transfer function matrix, $[I_n-\sum_{i=1}^{v}z^{-i}A_i]^{-1}[\sum_{j=1}^{w}z^{-j}B_j]$, is assumed to be irreducible, which assures [35], [36] that an observable and controllable state-space description exists for the plant described by (2.1). The system underlying (2.1) is therefore assumed to be minimally implemented. If not, the unobservable and/or uncontrollable portions are assumed neglectable due to satisfactory, i.e., stable, behavior.

The objective in controlling the plant via formation of an appropriate input signal is reasonable tracking of a reference signal while simultaneously maintaining a bounded control effort. The one-step-ahead cost function

$$J(U(k))=\frac{1}{2}\{[R(k+1)-Y(k+1)]^T P[R(k+1)-Y(k+1)] + U^T(k)QU(k)\}, \tag{2.2}$$

where R is the $n\times 1$ reference to be tracked and P and Q are, respectively, $n\times n$ and $m\times m$ symmetric, positive-definite matrices chosen to reflect the relative costs of components of the tracking error and the input vectors, has been previously utilized in hybrid [37] and indirect adaptive [14], [38, ch. 5], [39] control schemes.[1] A heuristic justification for its use in an adaptive context is that insertion of estimated parameters into a control computaton based on a more distant end time would tend to transmit and compound the estimation error.

Minimizing (2.2) with respect to $U(k)$ results in [27], [40]

$$U^*(k)=[Q+B_1^TPB_1]^{-1} \times B_1^TP\left[R(k+1)-\sum_{i=1}^{v}A_iY(k-i+1)-\sum_{j=2}^{w}B_jU(k-j+1)\right]. \tag{2.3}$$

Therefore, the optimal control effort is a linear combination of the elements of the information state vector, i.e., the next desired output (reference), the current and past plant outputs and past plant inputs. Note that only the next value of the reference must be known to apply the present control. Herein, the reference will be considered as specified one, and only one, time unit in advance, Such step-by-step revelation of the reference allows tracking of a broad class of trajectories, including the output of nonlinear or distributed parameter models as well as linear, time-invariant lumped-parameter models, the traditional domain of model-following schemes.

The attractive simplicity of the control input formation is somewhat offset by limitations on the class of plants it can stabilize, due to its "shortsightedness." Stabilization implies that, for any bounded reference R, the input U, the output Y, and every internal state of the system are bounded. If the control input and plant output remain bounded, clearly, the cost function in (2.2) is bounded and, due to the assumed controllability and observability of the plant, all state variables are bounded [41]. Therefore, it is sufficient to investigate the boundedness of U and Y, via transfer function analysis, in assessing the stabilization of the plant in (2.1) by the optimal control of (2.3). The question of the plant's stabilizability degenerates to the possibility of selecting appropriate P and Q such that [40]

$$|z|<1\ \forall z \ni \det C(z)=0 \tag{2.4}$$

where

$$C(z)\equiv\left[z^{\max(v,w)}I_n-\sum_{i=1}^{v}z^{\max(v,w)-i}A_i\right] + \left[\sum_{j=1}^{w}z^{\max(v,w)-j+1}B_j\right]Q^{-1}B_1^TP, \tag{2.5}$$

which is the characteristic equation of the R to Y transfer function. Interpretation of this problem as a multivariable root locus or as constant gain output feedback allows use of the results of current research in resolving this question [40]. The remainder of this work will assume, however, that the choice of the one-step-ahead optimal criterion in (2.2) incorporates a reasonable P and Q that, through (2.3), will stabilize the plant in (2.1). It is demonstrable [40] that this assumption is less restrictive than the assumption of perfect model-following capability.

III. Adaptive Input Matching for One-Step-Ahead Optimal Control

In indirect adaptive control schemes, the parameter estimation problem centers on the plant in (2.1). If the past input sequence and resulting output sequence are available, the $\{A_i, B_j\}$ parameters can be estimated via any standard linear recursive method. The consistency of these estimates hinges on the requirement that the input and output sequences are not linearly related. Artificially imposing separation between parameter estimation and control law determination in following the indirect adaptive control design technique violates this requirement and leads to parameter estimate inconsistency. The alternative approach postulates a linear feedback form for $U(k)$ and adjusts the gain parameters directly. Existing direct adaptive control methods recursively update the control parameters so that the difference between the plant output and a reference signal is minimized. These recursions are controlled by the output error and are therefore termed output matching.

The basic concept of the input-matching method developed in this paper, though still direct, is drastically different from the output-matching methods. Consider (2.3), which can be rewritten as

$$U^*(k)=DR(k+1)+\sum_{i=1}^{v}F_iY(k-i+1)+\sum_{j=2}^{w}G_jU(k-j+1) \tag{3.1}$$

where

$$D=[Q+B_1^TPB_1]^{-1}B_1^TP, \tag{3.2}$$

$$F_i=-DA_i \quad \text{for} \quad i=1,2,\cdots,v, \tag{3.3}$$

and

$$G_j=-DB_j \quad \text{for} \quad j=2,3,\cdots,w. \tag{3.4}$$

Note that if the optimal input sequence (based on possibly suboptimal preceding behavior), the reference history, and the applied input and resultant output records are known, then the control parameters $\{D, F_i, G_j\}$ can be estimated using any standard linear recursive method where the update term is controlled by the "input error" $U^*-\hat{U}$. A particular parameter estimation scheme is given below.

Lemma 1: The adjustment rule

$$\begin{bmatrix} \hat{D}^T(k) \\ \hline \hat{F}_1^T(k) \\ \hline \vdots \\ \hline \hat{F}_v^T(k) \\ \hline \hat{G}_2^T(k) \\ \hline \vdots \\ \hline \hat{G}_w^T(k) \end{bmatrix} = \begin{bmatrix} \hat{D}^T(k-1) \\ \hline \hat{F}_1^T(k-1) \\ \hline \vdots \\ \hline \hat{F}_v^T(k-1) \\ \hline \hat{G}_2^T(k-1) \\ \hline \vdots \\ \hline \hat{G}_w^T(k-1) \end{bmatrix} + h(k)H(k)\odot \begin{bmatrix} R(k) \\ \hline Y(k-1) \\ \hline \vdots \\ \hline Y(k-v) \\ \hline U(k-2) \\ \hline \vdots \\ \hline U(k-w) \end{bmatrix} [U^*(k-1) - \hat{U}(k-1)] \tag{3.5}$$

[1] This cost function could be altered by penalizing the deviation in control effort from a nonzero level as suggested in [40], possibly decreasing the steady-state tracking error.

where $\odot$ denotes element-by-element multiplication, when used to update the parameters of

$$\hat{U}(k)=\hat{D}(k)R(k+1)+\sum_{i=1}^{v}\hat{F}_i(k)Y(k-i+1)+\sum_{j=2}^{w}\hat{G}_j(k)U(k-j+1) \tag{3.6}$$

will cause

$$\hat{U}(k)\rightarrow U^*(k) \quad \text{as} \quad k\rightarrow\infty \tag{3.7}$$

for any initial estimates $\hat{D}(0)$, $\hat{F}_i(0)$, and $\hat{G}_j(0)$, if

$$\exists H_L \quad \text{and} \quad H_U \ni 0<H_L\leqslant H_{ij}(k)\leqslant H_U<\infty$$
$$\forall k;\ \forall i\in[1,\cdots,q];\quad \forall j\in[1,\cdots,m], \tag{3.8}$$

where H_{ij} denotes the ijth element of H and $q=(v+1)n+(w-1)m$,

$$\exists I\in[1,\cdots,q] \quad \text{and} \quad J\in[1,\cdots,m]$$
$$\ni \frac{H_{IJ}(k+1)}{H_{ij}(k+1)}\leqslant\frac{H_{IJ}(k)}{H_{ij}(k)} \qquad \forall i,j,k \tag{3.9}$$

and the scalar $h(k)$ satisfies

$$0<h(k)<\frac{2}{\sum_{i=1}^{q}H_{ij}(k)X_i^2(k)}\ \forall k,\ \forall j\in[1,\cdots,m]. \tag{3.10}$$

where

$$X(k)\triangleq$$
$$\left[R^T(k)\ \vdots\ Y^T(k-1)\ \vdots\ \cdots\ \vdots\ Y^T(k-v)\ \vdots\ U^T(k-2)\ \vdots\ \cdots\ \vdots\ U^T(k-w)\right]. \tag{3.11}$$

Proof: Identifying $W(k)$ with $U^*(k-1)$, $\theta^T(k)$ with $[D(k-1)\ \vdots\ F_1(k-1)\ \vdots\ \cdots\ \vdots\ F_v(k-1)\ \vdots\ G_2(k-1)\ \vdots\ \cdots\ \vdots\ G_w(k-1)]$, and $X^T(k)$ with $[R^T(k)\ \vdots\ Y^T(k-1)\ \vdots\ \cdots\ \vdots\ Y^T(k-v) \quad U^T(k-2)\ \vdots\ \cdots\ \vdots\ U^T(k-w)]$, the adjustment rule (3.5) can be written as (A.3) in the Appendix. The required satisfaction of (3.8), (3.9), and (3.10) allows the application of Theorem A.2, which directly proves (3.7). Q.E.D.

Unfortunately, the sequential mechanization of (3.5) and (3.6) requires exact knowledge of the information state vector in addition to the value of the past optimal control effort. Reliance on reception of the optimal control signal only one sample instant too late clearly appears impractical. However, this optimal control effort can be calculated after the fact.

Lemma 2: The error between the optimal control input of (3.1) and the estimated input of (3.6) is calculable *a posteriori* from

$$U^*(k-1)-\hat{U}(k-1)=[Q+B_1^TPB_1]^{-1}[B_1^TP(R(k)-Y(k))-Q\hat{U}(k-1)]. \tag{3.12}$$

Proof: If the control estimate in (3.6) is applied to the plant in (2.1) for some choice of $\hat{D}(k)$, $\hat{F}_i(k)$ and $\hat{G}_j(k)$ and the next output $Y(k+1)$ is measured, then rearranging (2.1)

$$\sum_{i=1}^{v}A_iY(k-i+1)+\sum_{j=2}^{w}B_jU(k-j+1)=Y(k+1)-B_1\hat{U}(k). \tag{3.13}$$

So, from (3.1) the optimal control effort $U^*(k)$, given the same past input and output record, would have been

$$U^*(k)=[Q+B_1^TPB_1]^{-1}B_1^TP[R(k+1)-Y(k+1)+B_1\hat{U}(k)]. \tag{3.14}$$

Subtracting $\hat{U}(k)$ from both sides of (3.14) and reindexing yields (3.12). Q.E.D.

These two lemmas provide the necessary tools for the statement of the full algorithm adaptively implementing one-step-ahead optimal control of (2.1) via input matching.

Theorem 1: Given the structure, i.e., m, n, v, and w, of an irreducible ARMA description (2.1) of a linear, time-invariant lumped-parameter MIMO plant, foreknowledge of the single matrix B_1 is sufficient to allow globally consistent adaptive control via (3.5), (3.6), and (3.12) eventually minimizing (2.2) provided (2.1) is stabilized by (2.3) for the chosen P and Q.

Proof: The combination of Lemmas 1 and 2 satisfies (3.7). The only internal information necessary about the plant in order to adapt (3.6) via (3.5) and (3.12) is the value of B_1. The convergence of the performance to that of the optimal control system, which employs the optimal feedback gains of (3.2), (3.3), and (3.4) in (3.1) from the initial instant onward can be substantiated by a stable open-loop estimation interpretation, since upon adequate satisfaction of (3.7) the output error becomes an unforced system governed by the coefficients of $C(z)$ in (2.5) which is assumed stable [27]. Q.E.D.

The remaining problem is the estimation of B_1. A very crude estimate of B_1 is obtainable by attributing all of the next output after a large input vector to its transmission through B_1. If B_1 is roughly estimated by some appropriate procedure, the use of an inaccurate estimate $\hat{B}_1$ in (3.12) will certainly affect the convergence of $\hat{D}$, $\hat{F}_i$, and $\hat{G}_j$ in (3.6) via (3.5). The question is how severely an inaccurate *a priori* estimate for B_1 will alter the convergent values of $\hat{D}$, $\hat{F}_i$, and $\hat{G}_j$ from their desired optimal values in (3.2), (3.3), and (3.4). This is answered by the following lemma.

Lemma 3: If a bounded estimate $\hat{B}_1$ replaces B_1 in (3.12), and subsequently (3.5), then implementing the adaptive controller of Theorem 1 will result in the convergence of $\hat{U}(k)$ to $U^*(k)$ in (3.1) with (3.2) replaced by

$$D=[Q+\hat{B}_1^TPB_1]^{-1}\hat{B}_1^TP \tag{3.15}$$

if

1) a pair $h_1(k)$ and $H_1(k)$ exists, satisfying (3.8), (3.9), and (3.10), as well as

$$h(k)H(k)\odot\left\{Z(k)[Q+\hat{B}_1^TP\hat{B}_1]^{-T}\right\}$$
$$=h_1(k)H_1(k)\odot\left\{Z(k)[Q+\hat{B}_1^TPB_1]^{-T}\right\}, \qquad \forall k, \tag{3.16}$$

where

$$Z(k)\triangleq X^T(k)[\hat{B}_1^TP(R(k)-Y(k))-Q\hat{U}(k-1)]^T \tag{3.17}$$

and

2) use of (3.15), (3.3), and (3.4) in the control law (3.1) does not destroy the stability of the controlled system.

Proof: The replacement of a bounded $\hat{B}_1$ for B_1 in (3.12) requires that

$$\hat{U}(k-1)=Q^{-1}\hat{B}_1^TP(R(k)-Y(k)) \quad \text{as} \quad k\rightarrow\infty \tag{3.18}$$

for the input matching error to converge to zero. Paraphrasing the development of the measured error in the proof of Lemma 2 by assuming that (3.18) provides the optimal input based on $\hat{B}_1$, $\hat{U}^*$, yields

$$\hat{U}^*(k-1)-\hat{U}(k-1)=[Q+\hat{B}_1^TPB_1]^{-1}[\hat{B}_1^TP(R(k)-Y(k))-Q\hat{U}(k-1)], \tag{3.19}$$

which disagrees with the error $\hat{E}$ generated by substitution of $\hat{B}_1$ for B_1 in (3.12)

$$\hat{E}(k-1)=[Q+\hat{B}_1^TP\hat{B}_1]^{-1}[\hat{B}_1^TP(R(k)-Y(k))-Q\hat{U}(k-1)]. \tag{3.20}$$

If $\hat{E}(k-1)$ rather than $\hat{U}^*(k-1)-\hat{U}(k-1)$ is utilized in (3.5), satisfaction of (3.16) is sufficient to allow use of Lemma 1 to prove that $\hat{U}(k-1)$ will converge as indicated in (3.18). The reasonableness of convergence to this approximate control effort certainly requires the stability of the suboptimal overall scheme, as assured by condition (2). Q.E.D.

Note that if $Q\equiv\mathbf{0}$ and $\hat{B}_1$ and B_1 are invertible, then the D in (3.15) matches the desired optimal value of B_1^{-1} from (3.2), i.e., if (3.16) is satisfied, then for exact input matching despite an inaccurate estimate of B_1 the overall scheme is asymptotically optimal.

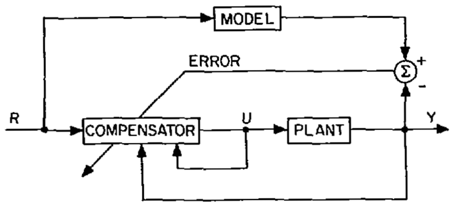

Fig. 1. Exact output/model matching.

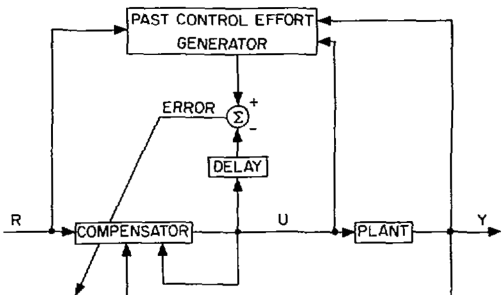

Fig. 2. Input matching.

For the special class of SI systems, satisfaction of (3.16) is ascertained simply since $Q+B_1^TPB_1$ and therefore both $Q+\hat{B}_1^TPB_1$ and $Q+\hat{B}_1^TP\hat{B}_1$ are scalars.

Corollary 1: For SI systems, if

$$\operatorname{sgn}[Q+\hat{B}_1^TPB_1]=\operatorname{sgn}[Q+\hat{B}_1^TP\hat{B}_1], \tag{3.21}$$

where $\operatorname{sgn}(x)=x/(|x|)$, and $h(k)$ is chosen satisfying

$$0<h(k)<\frac{2}{\sum_{i=1}^{q} H_iX_i^2(k)}[Q+\hat{B}_1^TPB_1]^{-1}[Q+\hat{B}_1^TP\hat{B}_1] \tag{3.22}$$

then (3.7) will be satisfied with (3.15) replacing (3.2) in (3.1).

Proof: Equations (3.21) and (3.22) clearly satisfy Lemma 3. Q.E.D.

Remarks:

1) Exact output matching represented by Fig. 1 requires development of sophisticated adaptive algorithms to properly utilize the output error in adjusting the compensator parameters. The input matching error in Fig. 2 is more easily utilized in established parameter adaptation schemes via a direct equation error formulation (such as in the Appendix) because it is not separated from the adapted element by the unknown plant as in adaptive output matching. Generation, from information state vector measurements, of control efforts that would have been optimal in the past and necessary for input error determination, therefore becomes the central pursuit.

2) The rate of convergence of the actual output to the optimal output, once adaptation is complete, is at the mercy of the chosen design. This shift in specification of a component of the overall convergence rate from the adjustable multipliers in the adaptive algorithm to the designer's choice of P and Q reflects the shift from the parallel model reference adaptive system (MRAS) form [21] of recent attempts utilizing output error in updating either control [31] or plant [42] parameter estimates to a series-parallel MRAS form focusing on input matching. A possible sacrifice in convergence speed is more than offset by the broader applicability of the control scheme.

3) A problem with the parameter identification of MIMO ARMA models, such as that of the controller in (3.1) or the plant in (2.1), is the multiplicity of specifications of the entries of A_i and B_j yielding equivalent input–output transfer function descriptions. This has prompted the consideration of unique characterizations of such equations [43], [44]. However, if the delay-line lengths v and w in (2.1) are accurately specified along with B_1, then due to the plant's asserted irreducibility, the A_i and remaining B_j in (2.1) are uniquely specified [27]. If the v and w in (3.6) are different from the correct values in (3.1), however, (3.6) need not provide a unique parameter estimate of (3.1) by obeying (A.7) at some point in the estimation scheme. Clearly, the estimates for v and w used in (3.6) must be greater than or equal to the respective values in (3.1) in order for $\hat{U}(k)$ to equal $U(k)$, again due to the plant's irreducibility.

4) Note that in the SISO case, (3.21) proves true if, for the scalar B_1

$$\operatorname{sgn}(B_1)=\operatorname{sgn}(\hat{B}_1), \tag{3.23}$$

a condition required in several earlier adaptive control schemes based on output matching [20], [45]–[48]. Furthermore, in the SISO case, if Theorem A.1 is satisfied so that the parameter estimates display consistent convergence to the optimal control parameters of (3.15), (3.3), and (3.4), then B_1 can be exactly calculated from the convergent value of D in (3.15) by

$$B_1=D^{-1}-P^{-1}\hat{B}_1^{-T}Q. \tag{3.24}$$

(Also determinable, excluding the numerical difficulties of near singularity, in the MIMO case if both D^{-1} and $\hat{B}_1^{-T}$ exist.) Then, either the $\hat{F}_i$ and $\hat{G}_j$ could be recalculated to reflect the use of (3.2) or, if the B_1 in (3.24) were still assumed inexact, due perhaps to incomplete parameter convergence, the adaptation procedure could be restarted. This tiering of decision-making mechanisms is a rudimentary example of the layered approach to learning systems recently formulated in [49]. The suggestion of successive regeneration of the adaptive scheme with an "improved" estimate of B_1 also raises the possibility of incorporating such a redetermination of B_1 at each iteration, possibly providing an overall consistent scheme.

5) If $B_1=\cdots=B_{N-1}=0$, but $B_N\neq 0$, i.e., the plant in (2.1) includes a transport lag, basically an N-step delay occurs between action and observation of the reaction necessary for correcting the feedback gains.

IV. Simulations

The direct adaptive implementation of one-step-ahead optimal control via input matching espoused in Theorem 1 will be examined in this section via simulations. Examples display several salient features of adaptive input matching for one-step-ahead optimal control: consistent control without consistent identification, insensitivity to parameter ambiguity due to order over-estimation; robustness despite output measurement noise, ability to track jump parameter changes; and feasibility of two-stage adaptation relaxing the necessary *a priori* knowledge of B_1. The examples are limited to SISO plants in order to more clearly display these valuable characteristics, though MIMO examples have also been successfully simulated [27], [50].

A. Consistent Control Without Consistent Identification

Consider the unstable SISO plant

$$H_1(z)=\frac{z+1.1}{z^2-1.6z+1.28}=\frac{z+1.1}{(z-1.13e^{j\pi/4})(z-1.13e^{-j\pi/4})}, \tag{4.1}$$

the inverse of which is also unstable. This plant is stabilizable by one-step-ahead optimal control for positive values of ρ between 0.3 and 35 in $P=\rho\bar{P}$, with $Q^{-1}B_1^T\bar{P}=1$, as is easily verified by a simple root-locus plot. For $\rho=10$ the steady-state response to a unit-step input is

$$Y_{ss}|_{(R(z)=1/(1-z^{-1}))}=\frac{10(1)(1+1.1)}{(1-1.6+1.28)+10(1)(1+1.1)}=0.97 \tag{4.2}$$

which is very close to the unit reference.

The indirect one-step-ahead optimal control of $H_1(z)$ achieved in the virtual steady-state environment of the step response, neglecting the perturbation input found necessary in [14], can prove satisfactory as shown in Fig. 3. (Note that the clipping on this and other plots is a characteristic of the plotting routine, not the actual curves.) The "adaptive step response," an established evaluator-comparator for adaptive control schemes [17], [18], [20], of the simultaneous equation error identification [51] and one-step-ahead optimal control of $H_1(z)$, given an accurate initial estimate of the unity numerator gain and initially zero

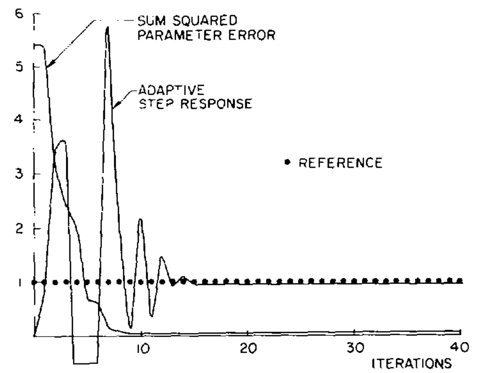

Fig. 3. Simultaneous identification and one-step-ahead optimal control: unit step response (plant: $H_1(z)$; initial estimates: $\hat{B}_1(0)=1$, $\hat{A}_1(0)=\hat{A}_2(0)=\hat{B}_2(0)=0$).

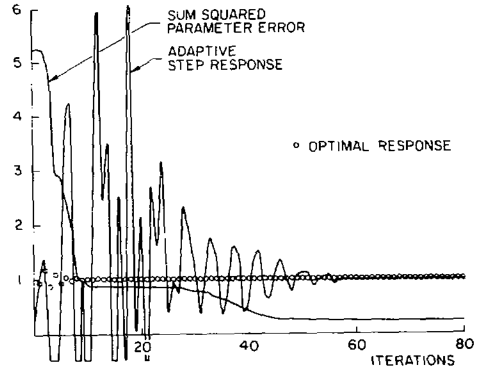

Fig. 5. One-step-ahead optimal control via adaptive input matching: unit step response (plant: $H_1(z)$; initial estimates: $\hat{D}(0)=0.9$, $\hat{F}_1(0)=\hat{F}_2(0)=\hat{G}_2(0)=-1.1$).

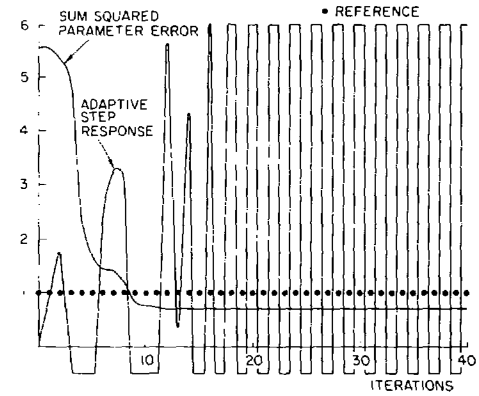

Fig. 4. Simultaneous identification and one-step-ahead optimal control: unit step response (plant: $H_1(z)$; initial estimates: $\hat{B}_1(0)=\hat{B}_2(0)=\hat{A}_1(0)=\hat{A}_2(0)=1$).

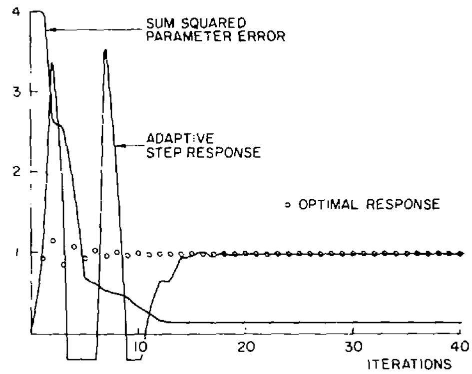

Fig. 6. One-step-ahead optimal control via adaptive input matching: unit step response (plant: $H_1(z)$; initial estimates: $\hat{D}(0)=0.9$, $\hat{F}_1(0)=\hat{F}_2=(0)=\hat{G}_2(0)=0$).

guesses for the remaining transfer-function coefficients, proves stable and approximately optimal, in the sense of (4.2), despite the slight residual sum of the squared parameter errors. Convergence is rapid in this noiseless case. However, for the different initial estimate of all the transfer-function coefficients as unity, with the identical step-size weight formulation [51, section 4.6], the identifier is "satisfied" by a significantly more erroneous parameter set that, when incorporated in (3.1), generates an unstable response, as indicated by Fig. 4.

The direct approach of exact output matching, epitomized by the "adaptive inverse control" in [34], though avoiding the pitfalls of an ill-posed identification task, would fare no better due to its attempted cancellation of the unstable numerator in (4.1) [52]. In [53] the limiting of the input-signal magnitude is suggested, which leads to the possibly undesirable limit cycle oscillation of bang-bang control. A second corrective procedure suggested in [53] is the more careful choice of a reference signal. Rather than attempting to adaptively track, e.g., a step, a suitable approximation to the step should be followed. The only truly suitable facsimile is a step response retaining the nonminimum phase zero, thereby not requiring its cancellation. But this requires exact knowledge of a plant parameter which is assumed unknown *a priori*.

Input matching avoids these restrictions. For the initial estimates of $\hat{D}(0)=0.9$ and $\hat{F}_i(0)=\hat{G}_j(0)=-1.1$ corresponding to the unity plant parameter guesses of the divergent simultaneous identification and control example, input matching has significantly bounded the adaptive step response within forty iterations and optimally stabilized the output in significantly less than eighty iterations, as illustrated in Fig. 5, using the same step-size weighting formula. Beginning with the zero estimates relating to the initial guesses of the convergent indirect example, the adaptive step response in Fig. 6 maintains a lower overshoot. The convergence times are roughly equivalent. Note, more importantly however, that adaptive input matching achieves consistent optimal control despite inexact control parameter identification, as expected from Theorem 1. A constant residual sum-squared control parameter error remains in both Figs. 5 and 6. Therefore the probing signals [54] or perturbation inputs [14] required to assure consistent plant identification in indirect schemes are unnecessary in input matching since the objective is control signal formation rather than control-law determination.

B. Insensitivity to Order Overestimation

The almost-inverse control choice of $Q^{-1}B_1^TP=10$ used in determining the optimal control of

$$H_2(z)=\frac{2.5z-1}{z^2-0.8z+0.4}=\frac{2.5(z-0.4)}{(z-0.632e^{j0.28\pi})(z-0.632e^{-j0.28\pi})} \tag{4.3}$$

improves the underdamped plant step response to a practically deadbeat behavior. Fig. 7 demonstrates the postulated convergence of the adaptive step response achieved via input matching adaptive control of (4.3) despite the overestimation of the plant order, in this case by one to third order. Despite the considerable residual sum of the squared error between the convergent control parameters and their respective minimal-order optimal control law counterparts, the response proves optimal within forty iterations. The wild initial gyrations are due to the short lag in overcoming the large initial parameter estimate errors.

C. Robust Behavior Despite Noise

Despite the exclusion of noise from the theoretical development, the adaptive step response in Fig. 8 for input matching control of $H_2(z)$ seems centered about the noise-free optimal response despite the uniform $[-0.25, 0.25]$ sequentially uncorrelated output measurement noise. Input matching appears to maintain robustness even in the presence of zero-mean noise impinging on the output values in (3.5), (3.6), and (3.12). Analytical examination of the scheme's behavior in a noisy environment could rely on a proper extension of its deterministic Lyapunov stability foundation through related stochastic invariant set theorems [55].

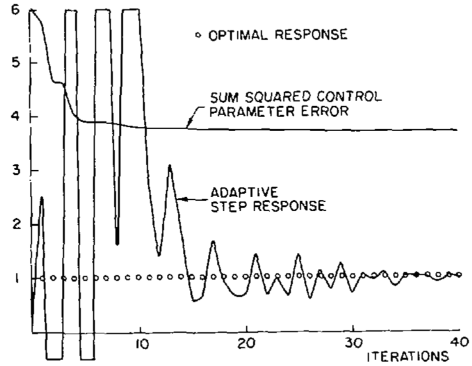

Fig. 7. Convergent behavior despite plant order over-estimation and severe parameter estimate bias: unit step response (plant: $H_2(z)$; initial estimates: $\hat{D}(0)=\hat{F}_3(0)=\hat{G}_3(0)=1$, $\hat{F}_1(0)=\hat{F}_2(0)=\hat{G}_2(0)=-1$).

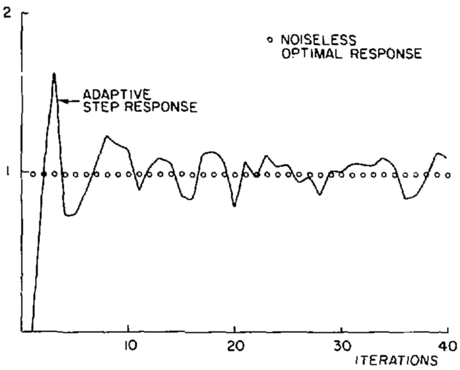

Fig. 8. Adaptive input matching with output measurement noise: unit step response (plant: $H_2(z)$; initial estimates: $\hat{D}(0)=\hat{F}_1(0)=\hat{F}_2(0)=\hat{G}_2(0)=0$; output measurement noise: uniform $[-0.25, 0.25]$).

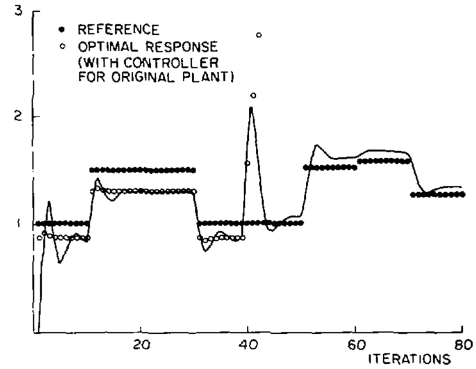

Fig. 9. Adaptive adjustment to abrupt plant parameter change: pulse train response (plant: $H_2(z)$ becomes $H_3(z)$ on fortieth iteration; initial estimates: $\hat{D}(0)=\hat{F}_1(0)=\hat{F}_2(0)=\hat{G}_2(0)=0$).

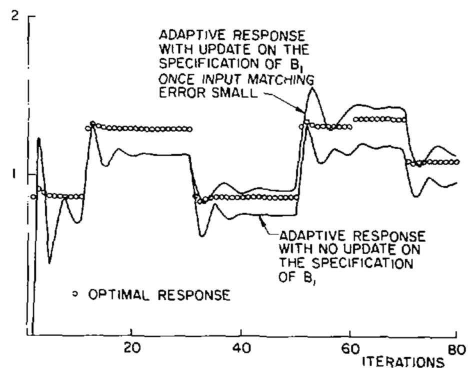

Fig. 10. Two-stage adjustment for inaccurate estimate of B_1: pulse train response (plant: $H_2(z)$; initial estimates: $\hat{D}(0)=\hat{F}_1(0)=\hat{F}_2(0)=\hat{G}_2(0)=0$, $\hat{B}_1(0)=1.2$).

D. Recovery from Plant Parameter Change

In Fig. 9, the plant $H_2(z)$, on the fortieth iteration, abruptly becomes

$$H_3(z)=\frac{2.5z-1}{z^2-1.6z+0.4}=\frac{2.5(z-0.4)}{(z-1.29)(z-0.31)} \tag{4.4}$$

by the doubling of the denominator plant parameter weighting $Y(k)$ in the difference equation description of $Y(k+1)$. If the optimal controller for $H_2(z)$, with $\rho=1$, remains in use after the change to the unstable $H_3(z)$, an unstable response results. The adaptive input matching controller, however, rapidly adjusts to this change in parameters and continues to reasonably track the reference pulse train. This abrupt parameter variation is characteristic of any plant suffering instantaneous structural or environmental alteration such as component failure [56]. Similarly, this control scheme can be expected to properly adjust to slowly time-varying plant parameters, a widely touted strength of adaptive mechanisms [57].

E. Two-Stage Adaptation

Finally, the suggestion, arising from (3.24), to recover from inaccurate estimates of the needed plant parameter B_1 by its redetermination upon the convergence guaranteed by Corollary 1 is followed. Fig. 10 shows the biased output resulting in attempting to control $H_2(z)$ to follow the optimal response of $\rho=1$ with a choice of $\hat{B}_1(0)$ slightly less than half the correct value. Incorporation of a second adaptive stage testing the consistent smallness of the error in (3.20) and updating $\hat{B}_1$ via (3.24) once this error is sufficiently reduced narrows the gap between the optimal and adaptive responses to the reference pulse train. The first correction noticeably occurs around the thirtieth iteration. Since the reference has assumed basically only two different levels to that point, dissatisfaction of (A.7) and therefore exact parameter convergence cannot be expected. In fact, the improvement in B_1 is observably incomplete at this early stage. The feasibility of continuing this strategy is limited, however, only by the recurrent satisfaction of Corollary 1 and therefore appears plausible.

V. Concluding Remarks

The primary contribution of this paper is the exposition of the concept of implementing adaptive control via input matching. This concept shifts the focus of the adaptive task and bypasses several of the restrictions of the current techniques of simultaneous identification and control and adaptive exact-output matching. The concept of adaptive input formation reduces the problem to one of parameter estimation once the structure of the control law is established. While output matching requires the development of ingenious algorithms utilizing the prestructured error between a desired response and the plant output, the emphasis is reversed for input matching to a necessity for the clever production of an input matching error to be utilized in easily established globally convergent adaptation algorithms.

The theoretical advancements of adaptive one-step-ahead control via input matching expand the range of feasible applications of adaptive control. The control of power systems subject to component failure would utilize the adaptability of input matching to jump parameters [56] in a predominantly steady-state environment. Application to economic systems with goals practically established in terms of optimal loss functions [58], [59] would require the multivariable optimal control basis of the adaptive input matching scheme developed in this paper. The direct structure of adaptive optimal-input matching could also be used to model the adaptive human control element in man-machine systems [60], [61].

Appendix

This appendix will establish a multivariable parameter estimation technique capable of providing consistent estimates, in most cases, of the entries in the $q\times m$ parameter matrix θ in the multivariable linear combination

$$W(k)=\theta^T X(k) \tag{A.1}$$

from perfect measurements of the $q\times 1$ input vector X and the $m\times 1$ output vector W despite the error in the initial estimate of θ. The parameter estimation algorithm, termed adaptive due to its capability to respond to slow time variations in θ, extends the scalar equation error formulation in [51, pp. 193–208] based on Lyapunov's stability theory for discrete-time systems, first presented in English in [62]. The parameter estimates $\hat{\theta}$ based on observations of X and W can be used to predict the vector output via

$$\hat{W}(k)=\hat{\theta}^T(k)X(k). \tag{A.2}$$

Theorem A.1: If the estimated parameter matrix $\hat{\theta}$ in (A.2) is updated by

$$\hat{\theta}(k+1)=\hat{\theta}(k)+h(k)H(k)\odot\left(X(k)[W(k)-\hat{W}(k)]^T\right), \tag{A.3}$$

where $\odot$ denotes element by element matrix multiplication,

$$\exists H_L \text{ and } H_U \ni 0<H_L\leqslant H_{ij}(k)\leqslant H_U<\infty$$
$$\forall k;\ \forall i\in[1,\cdots,q];\quad \forall j\in[1,\cdots,m] \tag{A.4}$$

where H_{ij} denotes the ijth element of H,

$$\exists I\in[1,\cdots,q]\quad\text{and}\quad J\in[1,\cdots,m]\ni\frac{H_{IJ}(k+1)}{H_{ij}(k+1)}\leqslant\frac{H_{IJ}(k)}{H_{ij}(k)}\qquad\forall i,j,k, \tag{A.5}$$

and the scalar $h(k)$ satisfies

$$0<h(k)<\frac{2}{\sum_{i=1}^{q}H_{ij}(k)X_i^2(k)}\ \forall k,\qquad \forall j\in[1,\cdots,m]. \tag{A.6}$$

Then, unless

$$\left(X_i(k)=0\quad\text{or}\quad W_j(k)-\hat{W}_j(k)=0\right)\quad\text{and}\quad\theta-\hat{\theta}(k)\neq\mathbf{0}\quad\forall k>k_1 \tag{A.7}$$

where $X_i(k)$ (or $W_j(k)$) is the ith (or jth) element in the column vector $X(k)$ (or $W(k)$), for any initial estimate $\theta(0)$

$$\hat{\theta}(k)\to\theta\quad\text{as}\quad k\to\infty \tag{A.8}$$

where θ is the parameter matrix in (A.1).

Proof: From (A.1) and (A.2), (A.3) can be rewritten as

$$\hat{\theta}(k+1)=\hat{\theta}(k)+h(k)H(k)\odot\left(X(k)X^T(k)\left[\theta-\hat{\theta}(k)\right]\right). \tag{A.9}$$

Defining the parameter error as

$$\tilde{\theta}(k)\triangleq\theta-\hat{\theta}(k) \tag{A.10}$$

and accordingly rewriting (A.9) after subtracting both sides from θ yields

$$\tilde{\theta}(k+1)=\tilde{\theta}(k)-h(k)H(k)\odot\left[X(k)X^T(k)\tilde{\theta}(k)\right] \tag{A.11}$$

which is a linear, time-varying, free, discrete-time system.

Define the scalar-valued Lyapunov function candidate

$$V(\tilde{\theta},k)\triangleq H_{IJ}(k)\operatorname{tr}\left[\tilde{\theta}^T(k)\left(\tilde{\theta}(k)\ominus H(k)\right)\right] \tag{A.12}$$

where $\ominus$ denotes element-by-element matrix division. Obviously, $V(\mathbf{0},k)=0$. Choosing $\alpha(\cdot)$ and $\beta(\cdot)$ as

$$\alpha(\|\tilde{\theta}\|)=\left[H_L\operatorname{tr}(\tilde{\theta}^T\tilde{\theta})\right]/H_U \tag{A.13}$$

and

$$\beta(\|\tilde{\theta}\|)=\left[H_U\operatorname{tr}(\tilde{\theta}^T\tilde{\theta})\right]/H_L \tag{A.14}$$

and defining the norm as

$$\|S\|\triangleq\operatorname{tr}(S^TS) \tag{A.15}$$

satisfactorily bounds V.

Since

$$\begin{aligned}\Delta V(\tilde{\theta},k)&\triangleq V(\tilde{\theta},k+1)-V(\tilde{\theta},k)\\&=H_{IJ}(k+1)\operatorname{tr}\left[\tilde{\theta}^T(k+1)\left(\tilde{\theta}(k+1)\ominus H(k+1)\right)\right]\\&\quad-H_{IJ}(k)\operatorname{tr}\left[\tilde{\theta}^T(k)\left(\tilde{\theta}(k)\ominus H(k)\right)\right]\end{aligned} \tag{A.16}$$

then

$$\begin{aligned}\Delta V(\tilde{\theta},k)=&\sum_{j=1}^{m}\sum_{i=1}^{q}\tilde{\theta}_{ij}^2(k+1)\left[\frac{H_{IJ}(k+1)}{H_{ij}(k+1)}-\frac{H_{IJ}(k)}{H_{ij}(k)}\right]\\&+H_{IJ}(k)\sum_{j=1}^{m}\sum_{i=1}^{q}\left[\frac{\tilde{\theta}_{ij}^2(k+1)-\tilde{\theta}_{ij}^2(k)}{H_{ij}(k)}\right].\end{aligned} \tag{A.17}$$

Equations (A.4) and (A.5) ensure that the first double sum in (A.17) is always nonpositive; therefore, (A.17) can be replaced by

$$\Delta V(\tilde{\theta},k)\leqslant H_{IJ}(k)\sum_{j=1}^{m}\sum_{i=1}^{q}\frac{\tilde{\theta}_{ij}^2(k+1)-\tilde{\theta}_{ij}^2(k)}{H_{ij}(k)}. \tag{A.18}$$

Utilizing (A.10) to expand (A.18) yields

$$\begin{aligned}\Delta V(\tilde{\theta},k)\leqslant H_{IJ}(k)&\sum_{j=1}^{m}\sum_{i=1}^{q}\frac{\hat{\theta}_{ij}(k+1)-\hat{\theta}_{ij}(k)}{H_{ij}(k)}\\&\times\left[\hat{\theta}_{ij}(k+1)+\hat{\theta}_{ij}(k)-2\theta_{ij}\right].\end{aligned} \tag{A.19}$$

Defining

$$\tilde{W}(k)\triangleq W(k)-\hat{W}(k)=\left[\theta-\hat{\theta}(k)\right]^TX(k) \tag{A.20}$$

results in

$$\begin{aligned}\Delta V(\tilde{\theta},k)\leqslant H_{IJ}(k)&\sum_{j=1}^{m}\sum_{i=1}^{q}h(k)\tilde{W}_j(k)\\&\times\left[h(k)H_{ij}(k)X_i^2(k)\tilde{W}_j(k)-2\tilde{\theta}_{ij}(k)X_i(k)\right]\end{aligned} \tag{A.21}$$

or

$$\begin{aligned}\Delta V(\tilde{\theta},k)\leqslant H_{IJ}(k)\Bigg\{&h^2(k)\left[\sum_{j=1}^{m}\sum_{i=1}^{q}\tilde{W}_j^2(k)H_{ij}(k)X_i^2(k)\right]\\&-2h(k)\left[\sum_{j=1}^{m}\tilde{W}_j^2(k)\right]\Bigg\}.\end{aligned} \tag{A.22}$$

Therefore, (A.6) would convert (A.22) to

$$\Delta V(\tilde{\theta},k)\leqslant 0. \tag{A.23}$$

In order to utilize Lyapunov's main stability theorem, $\Delta V(\tilde{\theta},k)$ must be negative definite rather than negative semi-definite as shown by (A.24). The inequality in (A.24) is possibly an equality for any $\tilde{\theta}\neq 0$ only if the summation element in (A.21) is zero for some $\tilde{\theta}\neq 0$. This is averted by disallowing (A.7) for all time after some $k=k_1$. Reconsidering $\hat{\theta}(k_1)$ as the initial estimate, despite previous behavior, completes the verification of (A.12) as a Lyapunov function. Therefore, $\tilde{\theta}(k)$ is asymptotically stable in the large which proves (A.8) regardless of the initial $\tilde{\theta}$. Q.E.D.

The injunction against (A.7) and the failure of the algorithm to update the parameter estimates despite their error prohibits two major difficulties of parameter identifiability [38, ch. 4], [63], [64]: input sufficiency and parameter resolvability. The resolution of parameter ambiguity, however, is actually of secondary importance in a control situation. The consistent construction of the output of the estimator for use as the control input more realistically depicts the estimator's practical use. Therefore, global unbiased convergence of the output estimate irrespective of the occurrence of (A.7) is detailed in the following theorem.

Theorem A.2: If the estimated parameter matrix $\hat{\theta}$ in (A.2) is updated by (A.3) while satisfying (A.4), (A.5), and (A.6), then

$$\hat{W}(k)\to W(k)\quad\text{as}\quad k\to\infty. \tag{A.24}$$

Proof: If (A.7) is not satisfied, then (A.8) is satisfied for any initial estimate of $\hat{\theta}$ and from (A.20) it is obvious that (A.24) is true. Alternatively, if (A.7) proves correct, despite the cause, the offending terms do not offer any nonzero terms to the calculation of $\tilde{W}$. Therefore, due to the consistent convergence of the remainder of $\hat{\theta}$ irrespective of $\hat{\theta}(0)$, implicit in (A.7), $\tilde{W}$ must be zero which supports (A.24). Q.E.D.

A more complete presentation of this estimation scheme appears in [65].

References

[1] D. D. Donalson and F. H. Kishi, "Review of adaptive control system theories and techniques," in *Modern Control Systems Theory*, C. T. Leondes, Ed. New York: McGraw-Hill, 1965, pp. 228–284.

[2] R. B. Asher, D. Andrisani II, and P. Dorato, "Bibliography on adaptive control systems," *Proc. IEEE*, vol. 64, pp. 1226–1240, Aug. 1976.

[3] E. Tse, "State of art and needs in adaptive stochastic control," *Chemical Process Control: AIChE Symposium Series*, vol. 72, no. 159, pp. 195–205, 1976.

[4] E. Tse and Y. Bar-Shalom, "An actively adaptive control for linear systems with random parameters via the dual control approach," *IEEE Trans. Automat. Contr.*, vol. AC-18, pp. 109–117, Apr. 1973.

[5] ——, Actively adaptive control for nonlinear stochastic systems," *Proc. IEEE*, vol. 64, pp. 1172–1181, Aug. 1976.

[6] A. A. Fel'dbaum, "Dual control theory I-IV," *Automation and Remote Control*, vol. 21, no. 9, pp. 874–880, Apr. 1961, vol. 21, no. 11, pp. 1033–1039, June 1961, vol. 22, no. 1, pp. 1–12, Aug. 1961, vol. 22, no. 2, pp. 109–121, Sept. 1961, also collected in *Optimal and Self-Optimizing Control*, R. Oldenburger, Ed. Cambridge, MA: MIT Press, 1966, pp. 458–496.

[7] R. E. Kalman, "Design of a self-optimizing control system," *Trans. ASME*, vol. 80, no. 2, pp. 468–478, Feb. 1958, also in *Optimal and Self-Optimizing Control*, R. Oldenburger, Ed. Cambridge, MA: MIT Press, 1966, pp. 440–449.

[8] K. J. Åström and P. Eykhoff, "System identification—a survey," *Automatica*, vol. 7, no. 2, pp. 123–162, Mar. 1971.

[9] R. E. Nieman, D. G. Fisher, and D. E. Seborg, "A review of process identification and parameter estimation techniques," *Int. J. Contr.*, vol. 13, no. 2, pp. 209–264, Feb. 1971.

[10] P. Eykhoff, Ed., *Identification and System Parameter Estimation, Parts 1 and 2*. Amsterdam: North-Holland, 1973.

[11] T. Kailath, Ed., *IEEE Trans. Automat. Contr.* (*Special Issue on System Identification and Time-Series Analysis*), vol. AC-19, Dec. 1974.

[12] P. E. Wellstead and J. M. Edmunds, "Least-squares identification of closed-loop systems," *Int. J. Contr.*, vol. 21, no. 4, pp. 689–699, Apr. 1975.

[13] T. Söderström, L. Ljung, and I. Gustavsson, "Identifiability conditions for linear multivariable systems operating under feedback," *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 837–840, Dec. 1976.

[14] G. N. Saridis and R. N. Lobbia, "Parameter identification and control of linear discrete-time systems," *IEEE Trans. Automat. Contr.*, vol. AC-17, pp. 52–60, Feb. 1972, and "Comments on 'Parameter identification and control of linear discrete-time systems'," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 442–443, June 1975.

[15] D. P. Turtle and P. H. Phillipson, "Simultaneous identification and control," *Automatica*, vol. 7, no. 4, pp. 445–453, July 1971.

[16] L. Hasdorff, *Gradient Optimization and Nonlinear Control*. New York: Wiley, 1976.

[17] D. P. Lindorff and R. L. Carroll, "Survey of adaptive control using Liapunov design," *Int. J. Contr.*, vol. 18, no. 5, pp. 897–914, Nov. 1973.

[18] B. Shackcloth and R. L. Butchart, "Synthesis of model reference adaptive systems by Liapunov's second method," in *Theory of Self-Adaptive Control Systems*, P. H. Hammond, Ed. New York: Plenum, 1966, pp. 145–152.

[19] P. C. Parks, "Liapunov redesign of model reference adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-11, pp. 362–367, July 1966.

[20] C. C. Hang and P. C. Parks, "Comparative studies of model reference adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-18, pp. 419–428, Oct. 1973.

[21] I. D. Landau, "A survey of model reference adaptive techniques—theory and applications," *Automatica*, vol. 10, no. 4, pp. 353–379, July 1974.

[22] H. Erzberger, "Analysis and design of model following control systems by state space techniques," in *Proc. 1968 JACC*, Ann Arbor, MI, June 1968, pp. 572–581.

[23] S. G. Tzafestas and P. N. Paraskevopoulos, "On the exact model matching controller design," *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 242–246, Apr. 1976.

[24] R. T. Curran, "Equicontrollability and the model-following problem," Ph.D. dissertation, Stanford Elec. Laboratories, Stanford University, Stanford, CA, Tech. Rep. 6303-2, July 1971.

[25] P. Kudva and V. Gourishankar, "On the stability problem of multivariable model-following systems," *Int. J. Contr.*, vol. 24, no. 6, pp. 801–805, Dec. 1976.

[26] U. Shaked, "Design of general model-following control systems," *Int. J. Contr.*, vol. 25, no. 1, pp. 57–79, Jan. 1977.

[27] C. R. Johnson, Jr., "Adaptive implementation of one-step-ahead optimal control via input matching," Ph.D. dissertation, Stanford University, Stanford, CA., Apr. 1977.

[28] A. E. Pearson and K.-S.V.R. Vanguri, "A synthesis procedure for parameter adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-16, pp. 440–449, Oct. 1971.

[29] I. D. Landau and B. Courtiol, "Design of multivariable adaptive model following control systems," *Automatica*, vol. 10, no. 5, pp. 483–494, Sept. 1974.

[30] R. V. Monopoli, "Model reference adaptive control with an augmented error signal," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 474–484, Oct. 1974.

[31] L. Mabius and H. Kaufman, "An adaptive flight controller for the F-8 without explicit parameter identification," in *Proc. 1976 IEEE Conf. on Decision and Control*, Clearwater, FL, Dec. 1976, pp. 9–14.

[32] B. H. Swanick, "A high-speed deterministic adaptive controller," *Int. J. Contr.*, vol. 15, no. 5, pp. 833–838, May 1972.

[33] K. J. Åström and B. Wittenmark, "On self-tuning regulators," *Automatica*, vol. 9, no. 2, pp. 185–199, Mar. 1973.

[34] J. M. Martin-Sanchez, "A new solution to adaptive control," *Proc. IEEE*, vol. 64, pp. 1209–1218, Aug. 1976.

[35] B. W. Dickinson, T. Kailath, and M. Morf, "Canonical matrix fraction and state-space descriptions for deterministic and stochastic linear systems," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 656–667, Dec. 1974.

[36] J. Aracil and C. G. Montes, "External description of multivariable systems," *Int. J. Contr.*, vol. 23, no. 3, pp. 409–420, Mar. 1976.

[37] D. J. Sandoz, "Optimal control of linear multivariable systems based on discrete output feedback," *Proc. IEE*, vol. 120, pp. 1439–1444, Nov. 1973.

[38] R. C. K. Lee, *Optimal Estimation, Identification, and Control*. Cambridge, MA: MIT Press, 1964.

[39] V. Maletinsky and W. Schaufelberger, "Suboptimum adaptive control," in *Fourth Int'l. Conf. on Digital Comp. Appl. to Process Control, part I*, M. Mansour and W. Schaufelberger, Eds. Berlin: Springer-Verlag, 1974, pp. 129–143.

[40] C. R. Johnson, Jr., "On single-stage optimal control," in *Proc. 1978 IEEE Southeastcon*, Atlanta, GA, Apr. 1978, pp. 511–514.

[41] C.-T. Chen, "Stability of linear multivariable feedback systems," *Proc. IEEE*, vol. 56, pp. 821–828, May 1968.

[42] K. S. Narendra and L. S. Valavani, "Stable adaptive observers and controllers," *Proc. IEEE*, vol. 64, pp. 1198–1208, Aug. 1976.

[43] J. Valis, "On-line identification of multivariable linear systems of unknown structure from input output data," in *Proc. Second Prague IFAC Symp. on Identification and Process Parameter Estimation*, Prague, Czechoslovakia, June 1970, paper 1.5.

[44] D. S. Spain, Jr., "Identification and modelling of discrete, stochastic linear systems," Ph.D. dissertation, Stanford University, Stanford, CA, Aug. 1971.

[45] R. M. Dressler, "An approach to model-referenced adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-12, pp. 75–80, Feb. 1967.

[46] H. H. Choe and P. N. Nikiforuk, "Inherently stable feedback control of a class of unknown plants," *Automatica*, vol. 7, no. 5, pp. 607–625, Sept. 1971.

[47] V. A. Yacubovich, "On a method of adaptive control under conditions of great uncertainty," in *Proc. Fifth IFAC World Congress*, Paris, France, June 1972, paper 37.3.

[48] O. A. Sebakhy, "A discrete model reference adaptive system design," *Int. J. Contr.*, vol. 23, no. 6, pp. 799–804, June 1976.

[49] R. G. Smith, T. M. Mitchell, R. A. Chestek, and B. G. Buchanan, "A model for learning systems," in *Proc. Int. Joint Conf. on A.I.*, Cambridge, MA, Aug. 1977, pp. 338–343.

[50] C. R. Johnson, Jr., "Adaptive control of structurally varying plants via input matching," in *Proc. 1978 IEEE Southeastcon*, Atlanta, GA, Apr. 1978, pp. 347–350.

[51] J. M. Mendel, *Discrete Techniques of Parameter Estimation: The Equation Error Formulation*. New York: Marcel Dekker, 1973.

[52] C. R. Johnson, Jr. and M. G. Larimore, "Comments on 'A new solution to adaptive control'," *Proc. IEEE*, vol. 65, pp. 587–588, Apr. 1977, and "Corrections to 'Comments on "A new solution to adaptive control"'," *Proc. IEEE*, vol. 65, p. 1607, Nov. 1977.

[53] J. M. Martin-Sanchez, "Author's reply" (to [52]), *Proc. IEEE*, vol. 65, p. 587–588, Apr. 1977.

[54] J. S-C. Yuan and W. M. Wonhan, "Probing signals for model reference identification," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 530–538, Aug. 1977.

[55] H. J. Kushner, *Introduction to Stochastic Control*. New York: Holt, Rinehart, and Winston, 1971, ch. 8.

[56] D. D. Sworder, "Control of systems subject to sudden change in character," *Proc. IEEE*, vol. 64, pp. 1219–1225, Aug. 1976.

[57] B. Widrow, J. M. McCool, M. G. Larimore, and C. R. Johnson, Jr., "Stationary and nonstationary learning characteristics of the LMS adaptive filter," *Proc. IEEE*, vol. 64, pp. 1151–1162, Aug. 1976, also in *Apsects of Signal Processing*, Giorgo Tacconi, Ed. Dordecht: D. Reidel, 1977, pp. 355–393.

[58] R. S. Pindyck, "An application of the linear quadratic tracking problem to economic stabilization policy," *IEEE Trans. Automat. Contr.*, vol. AC-17, pp. 287–300, June 1972.

[59] G. C. Chow, *Analysis and Control of Dynamic Economic Systems*. New York: Wiley, 1975.

[60] D. L. Kleinman and S. Baron, and W. H. Levison, "An optimal control model of human response, part I: theory and validation, part II: prediction of human performance in a complex task," *Automatica*, vol. 6, no. 3, pp. 357–383, May 1970.

[61] G. O. Burnham, J. Seo, and G. A. Bekey, "Identification of human driver models in car following," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 911–915, Dec. 1974.

[62] R. E. Kalman and J. E. Bertram, "Control system analysis and design via the 'second method' of Lyapunov: II. Discrete-time systems, *Trans. ASME: J. Basic Eng.*, vol. 82, ser, D, pp. 394–399, June 1960.

[63] R. M. Staley and P. C. Yue, "On system parameter identifiability," *Inform. Sci.*, vol. 2, no. 2, pp. 127–138, Apr. 1970.

[64] E. Tse and J. J. Anton, "On the identifiability of parameters," *IEEE Trans. Automat. Contr.*, vol. AC-17, pp. 637–646, Oct. 1972 and E. Tse and H. Weinert, "Correction and extension of 'On the identifiability of parameters'," *IEEE Trans. Automat. Contr.*, vol. AC-18, pp. 684–688, Dec. 1973.

[65] C. R. Johnson, Jr., "Adaptive parameter matrix and output vector estimation via an equation error formulation," in *Proc. 9th Pittsburgh Conference on Modeling and Simulation*, Pittsburgh, PA, Apr. 1978.

Controllability and Observability of Linear Time-Invariant Compartmental Models

RAYMOND M. ZAZWORSKY, MEMBER, IEEE, AND HAROLD K. KNUDSEN, MEMBER, IEEE

***Abstract*—The problems of complete controllability and complete observability of linear time-invariant compartmental models with general**

Manuscript received August 2, 1977; revised June 1, 1978. Paper recommended by M. Sain, Chairman of the Linear Systems Committee.

R. M. Zazworsky is with the Department of Mathematical Sciences, United States Air Force Academy, Colorado Springs, CO 80840.

H. K. Knudsen is with the Department of Electrical Engineering and Computer Science, University of New Mexico, Alburquerque, NM 87131.

0018-9286/78/1000-0872$00.75 ©1978 IEEE