Pulse Oximetry

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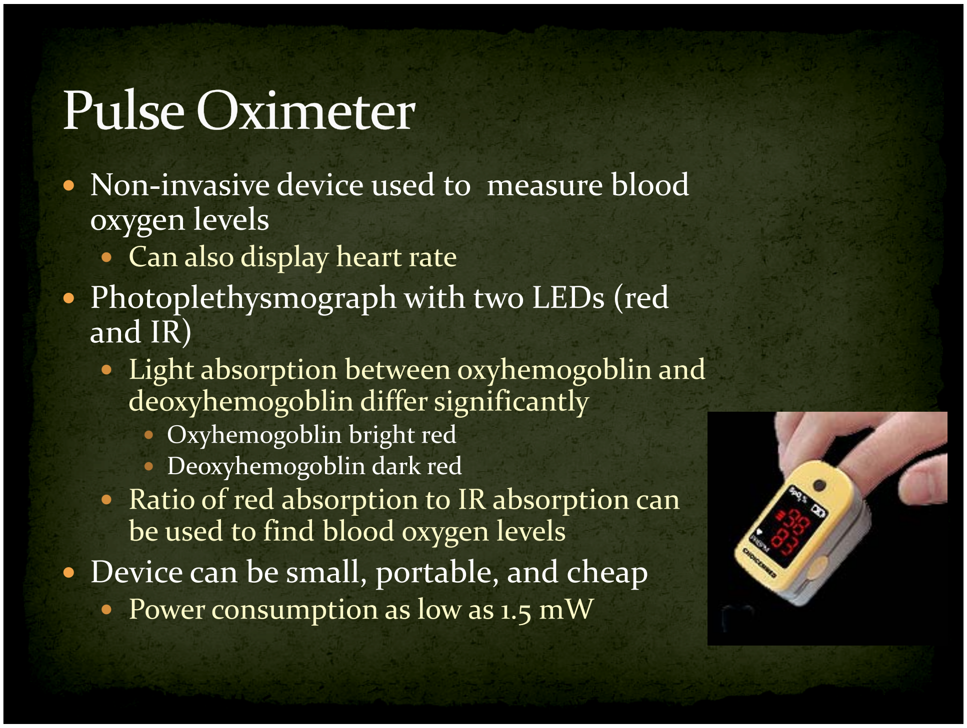
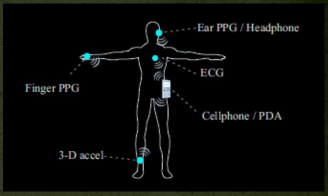


Pulse Oximeter

- Non-invasive device used to measure blood oxygen levels
  - Can also display heart rate
- Photoplethysmograph with two LEDs (red and IR)
  - Light absorption between oxyhemogoblin and deoxyhemogoblin differ significantly
    - Oxyhemogoblin bright red
    - Deoxyhemogoblin dark red
  - Ratio of red absorption to IR absorption can be used to find blood oxygen levels
- Device can be small, portable, and cheap
  - Power consumption as low as 1.5 mW

Applications

- Blood oxygen level one of the critical vital signs
  - Measuring oxygen needs of patient important
- Patient’s oxygenation is unstable
  - Intensive care units
  - Emergency rooms
  - Operating rooms
  - Pilots in unpressurized cabins
- Use in Body Area Networks
  - Monitor many biological signals and record them in a PDA/Cell phone for fitness/wellness monitoring
    - High portability and low cost important
    - Low power



Principle of Operation

- Hemoglobin changes from bright red to dark red
  - Shine red LED through finger and measure absorption of red light
  - Measured intensity related to percentage of oxygenated hemoglobin in blood
    - But also affected by other things i.e. skin, bone, and muscles
  - Arteries’ thickness vary with breathing
    - Take ratio of light intensity at peak and trough of heartbeat cycle
      - Measurement is independent of absolute light intensity
      - Independent of other tissues (depends only on arterial blood)

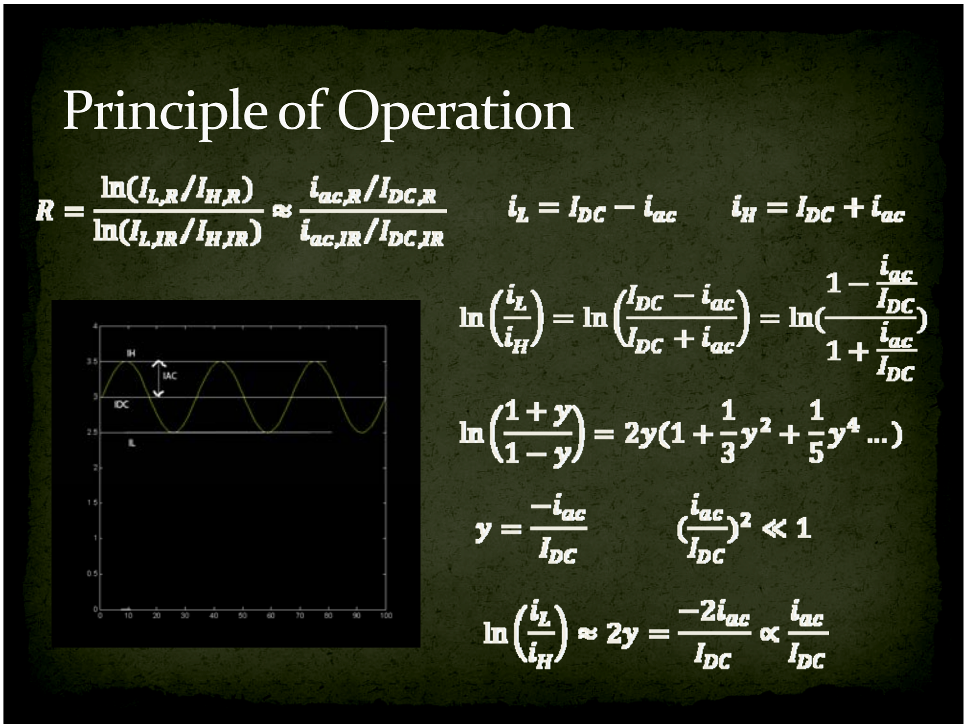
Principle of Operation

- Ratio still exponentially related to:
  - Concentration of hemoglobin in blood
  - Thickness variation of arteries over a heartbeat cycle
  - Absorption coefficient of red light by hemoglobin
- Take natural log to remove exponential dependence
- Find another ratio for infra-red (IR) LED
  - Take natural log
- Take ratio of the two ratios
  - Removes dependence on concentration and thickness variation
    - Unknown dependencies are constant for these two wavelengths
- Absorption coefficients well known
  - Easily adjusted for

Principle of Operation

$$S_pO_2 = \frac{0.81 - 0.18R}{0.63 + 0.11R} x100 \qquad R = \frac{\ln(I_{L,R}/I_{H,R})}{\ln(I_{L,IR}/I_{H,IR})} \approx \frac{i_{ac,R}/I_{DC,R}}{i_{ac,IR}/I_{DC,IR}}$$

- Percentage of hemoglobin that is oxygenated (S_pO_2) is related to the ratio of absorptions (R) [Beer's Law]
  - 0.81,0.18,0.63,0.11 are represent absorption coefficients of oxy/deoxy-hemoglobin for red and IR light
- Ratio of absorptions can be approximated using Taylor expansion



Principle of Operation

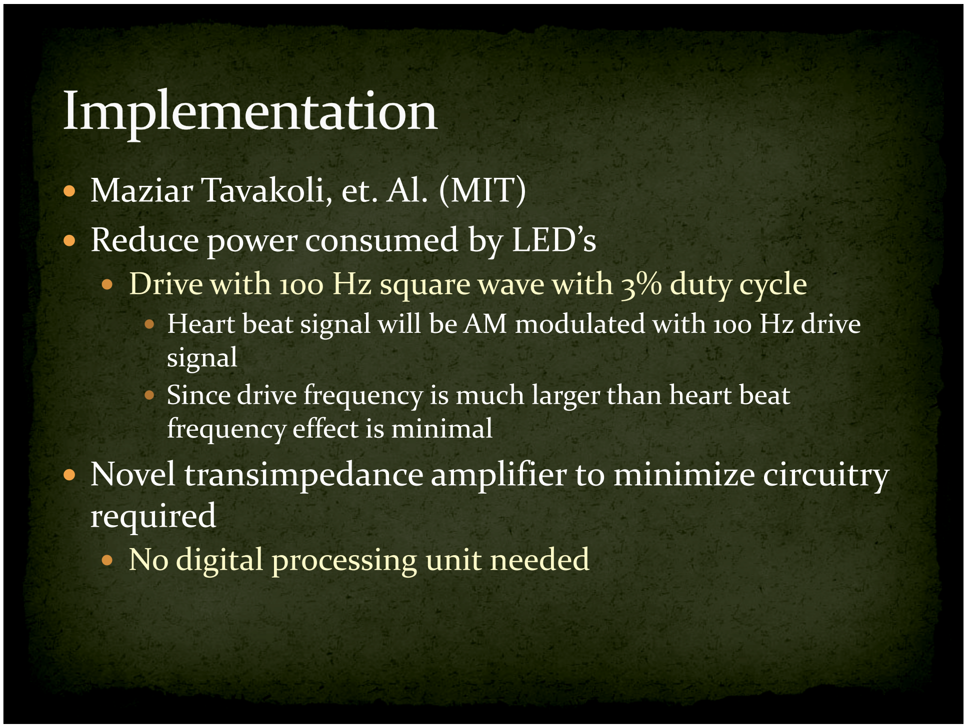
$$R = \frac{\ln(I_{L,R}/I_{H,R})}{\ln(I_{L,IR}/I_{H,IR})} \approx \frac{i_{ac,R}/I_{DC,R}}{i_{ac,IR}/I_{DC,IR}} \qquad i_L = I_{DC} - i_{ac} \qquad i_H = I_{DC} + i_{ac}$$

$$\ln\left(\frac{i_L}{i_H}\right) = \ln\left(\frac{I_{DC} - i_{ac}}{I_{DC} + i_{ac}}\right) = \ln\left(\frac{1 - \frac{i_{ac}}{I_{DC}}}{1 + \frac{i_{ac}}{I_{DC}}}\right)$$

$$\ln\left(\frac{1+y}{1-y}\right) = 2y\left(1 + \frac{1}{3}y^2 + \frac{1}{5}y^4 \ldots\right)$$

$$y = \frac{-i_{ac}}{I_{DC}} \qquad \left(\frac{i_{ac}}{I_{DC}}\right)^2 \ll 1$$

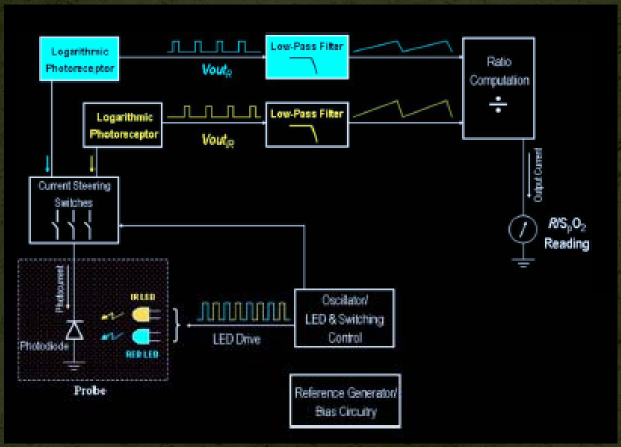
$$\ln\left(\frac{i_L}{i_H}\right) \approx 2y = \frac{-2i_{ac}}{I_{DC}} \propto \frac{i_{ac}}{I_{DC}}$$



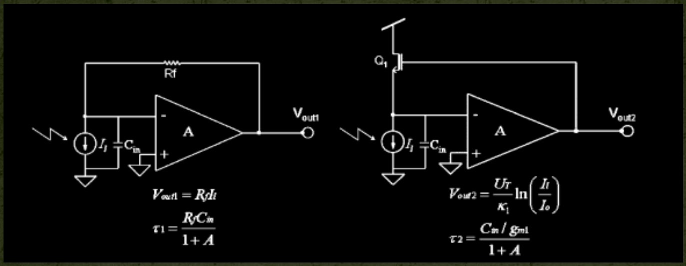
Implementation

- Maziar Tavakoli, et. Al. (MIT)
- Reduce power consumed by LED's
  - Drive with 100 Hz square wave with 3% duty cycle
    - Heart beat signal will be AM modulated with 100 Hz drive signal
    - Since drive frequency is much larger than heart beat frequency effect is minimal
- Novel transimpedance amplifier to minimize circuitry required
  - No digital processing unit needed

Implementation



Transimpedance Amplifier



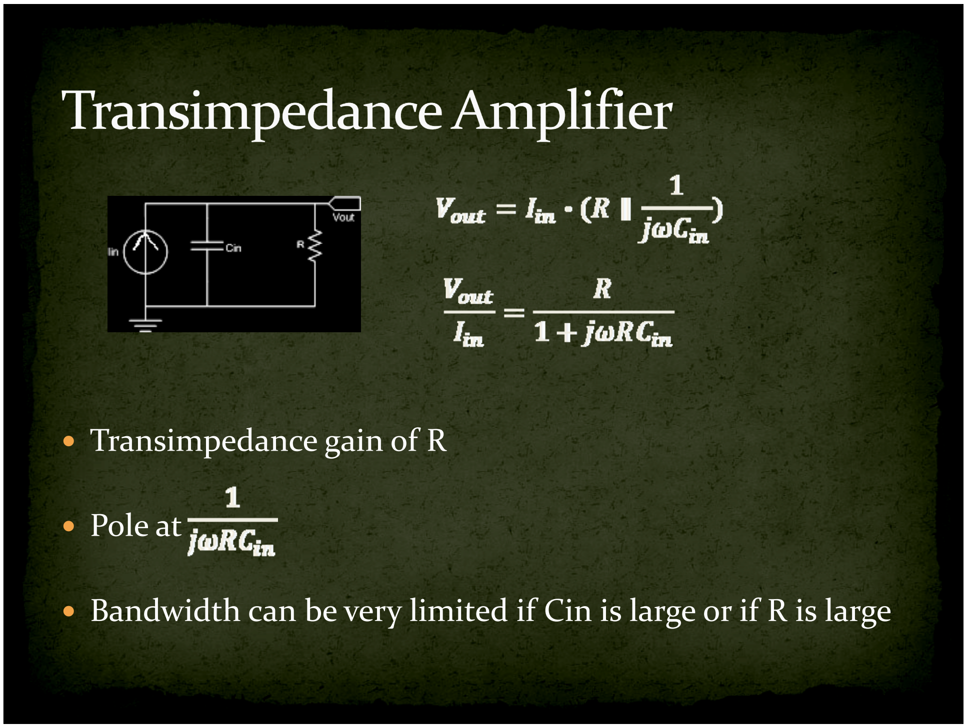
$$V_{out1} = R_f I_l$$

$$\tau_1 = \frac{R_f C_{in}}{1+A}$$

$$V_{out2} = \frac{U_T}{\kappa_1} \ln\left(\frac{I_l}{I_o}\right)$$

$$\tau_2 = \frac{C_{in} / g_{m1}}{1+A}$$

- Takes I as input, outputs V
  - Helps suppress large Cin to increase bandwidth
- Can be made logarithmic
  - Using transistor in subthreshold

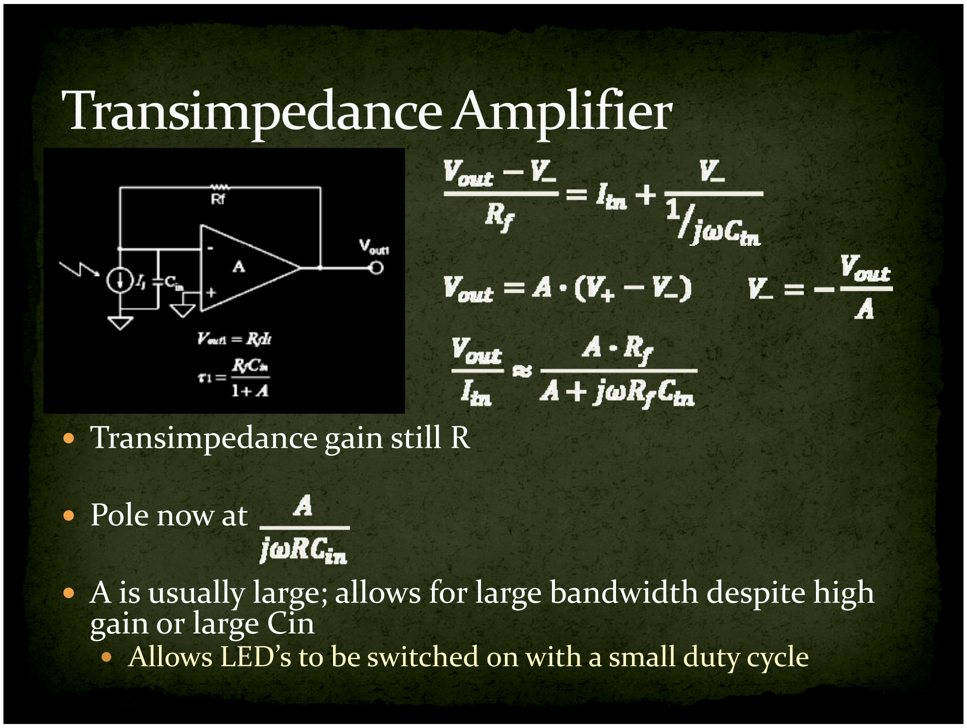


Transimpedance Amplifier

$$V_{out} = I_{in} \cdot \left(R \parallel \frac{1}{j\omega C_{in}}\right)$$

$$\frac{V_{out}}{I_{in}} = \frac{R}{1 + j\omega R C_{in}}$$

- Transimpedance gain of R
- Pole at $\frac{1}{j\omega R C_{in}}$
- Bandwidth can be very limited if Cin is large or if R is large



Transimpedance Amplifier

$$V_{out1} = R_f I_l$$

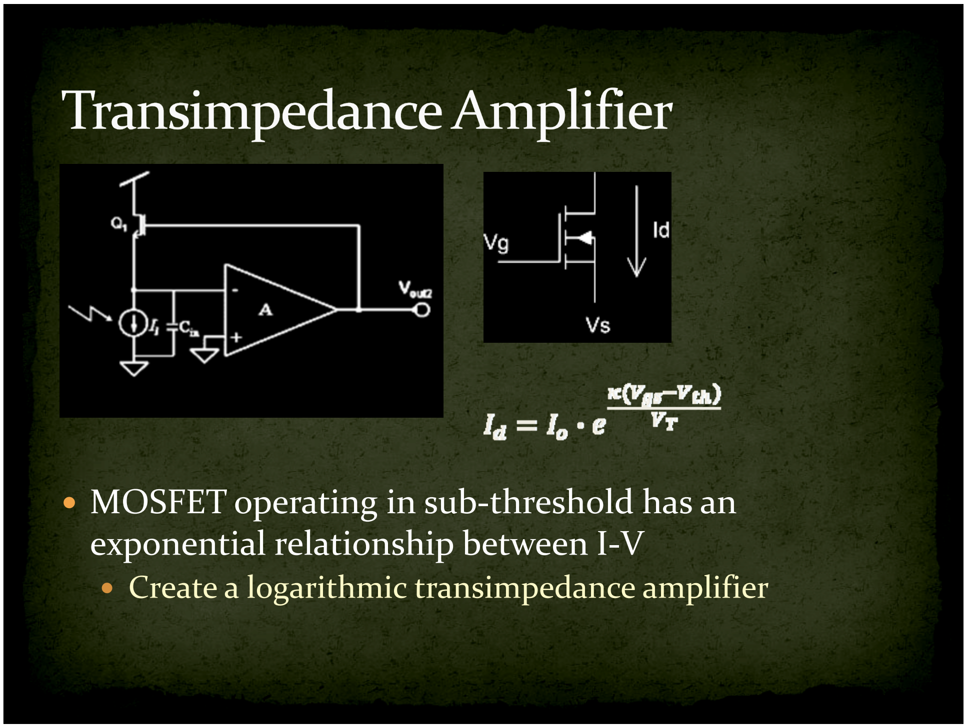
$$\tau 1 = \frac{R_f C_{in}}{1 + A}$$

$$\frac{V_{out} - V_-}{R_f} = I_{in} + \frac{V_-}{{}^{1}\!/_{j\omega C_{in}}}$$

$$V_{out} = A \cdot (V_+ - V_-) \qquad V_- = -\frac{V_{out}}{A}$$

$$\frac{V_{out}}{I_{in}} \approx \frac{A \cdot R_f}{A + j\omega R_f C_{in}}$$

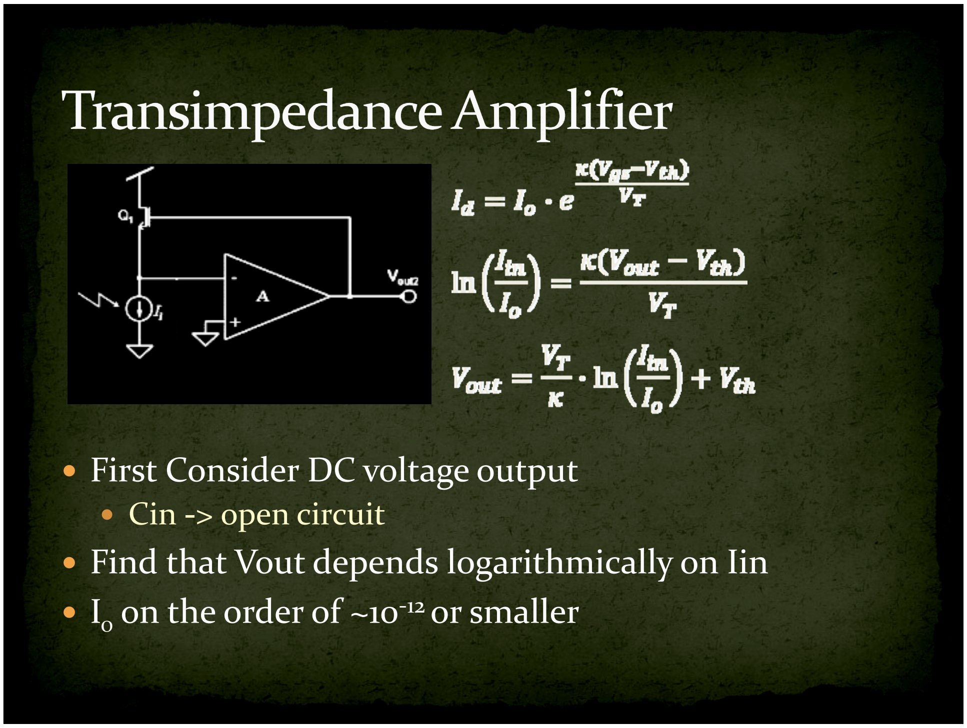
- Transimpedance gain still R
- Pole now at $\frac{A}{j\omega R C_{in}}$
- A is usually large; allows for large bandwidth despite high gain or large Cin
  - Allows LED's to be switched on with a small duty cycle



Transimpedance Amplifier

$$I_d = I_o \cdot e^{\frac{\kappa(V_{gs}-V_{th})}{V_T}}$$

- MOSFET operating in sub-threshold has an exponential relationship between I-V
  - Create a logarithmic transimpedance amplifier



Transimpedance Amplifier

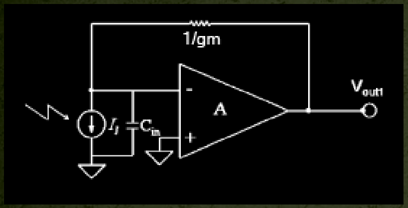
$$I_d = I_o \cdot e^{\frac{\kappa(V_{gs}-V_{th})}{V_T}}$$

$$\ln\left(\frac{I_{in}}{I_o}\right) = \frac{\kappa(V_{out} - V_{th})}{V_T}$$

$$V_{out} = \frac{V_T}{\kappa} \cdot \ln\left(\frac{I_{in}}{I_o}\right) + V_{th}$$

- First Consider DC voltage output
  - Cin -> open circuit
- Find that Vout depends logarithmically on Iin
- I_o on the order of ~10^{-12} or smaller

Transimpedance Amplifier



$$V_{out} = I_{in}\left(\frac{1}{g_m A} \| \frac{1}{j\omega C_{in}}\right)$$

$$\frac{V_{out}}{I_{in}} = \frac{1}{g_m A + j\omega C_{in}}$$

- Pole at $\frac{A}{C_{in} \cdot {}^{1}/_{g_m}}$
- $\frac{1}{g_m}$ is usually small, especially in sub-threshold
  - Good bandwidth

Transimpedance Amplifier

$$V_{out} = \frac{V_T}{\kappa} \cdot \ln\left(\frac{I_{in}}{I_o}\right) + V_{th}$$

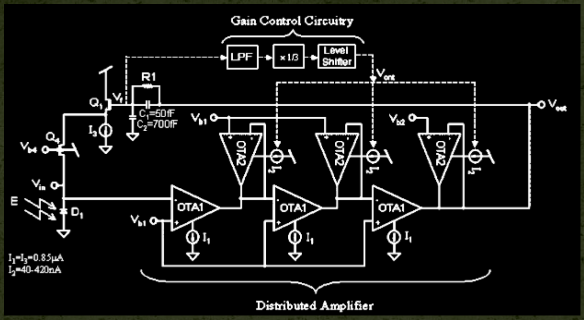
$$v_{out,ac} = \frac{dV_{out}}{dI_{in}} \cdot i_{ac} = \frac{V_T \cdot 1/I_o}{\kappa \cdot I_{in}/I_o} \cdot i_{ac}$$

$$v_{out,ac} = \frac{V_T}{\kappa} \cdot \frac{i_{ac}}{I_{DC}}$$

$$R = \frac{\ln(I_{L,R}/I_{H,R})}{\ln(I_{L,IR}/I_{H,IR})} \approx \frac{i_{ac,R}/I_{DC,R}}{i_{ac,IR}/I_{DC,IR}}$$

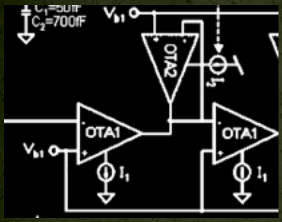
- Taylor expand the DC expression for $V_{out}(I_{in})$
- v_{out}, AC proportional to ratio of i_{ac} to I_{DC}
- Exactly what we need to find R
  - Do not need to find i_{ac} and I_{DC} separately

Distributed Amplifier



- By distributing gain over multiple stages, can improve gain-bandwidth product
- While gain's multiple, time constants add
  - Accumulate gain faster then bandwidth lost

Distributed Amplifier



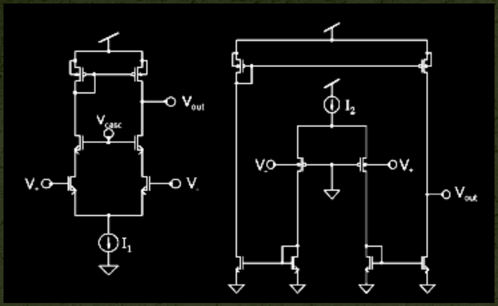
$$i_{out} = g_m(V_+ - V_-)$$

- OTA2 acts as a resistor

$$\frac{V_-}{i_{out}} = R = -\frac{1}{g_{m2}}$$

- OTA1 and OTA2 combine for a gain of $-\frac{g_{m1}}{g_{m2}}$
- Total gain of $-(\frac{g_{m1}}{g_{m2}})^3$

Distributed Amplifier

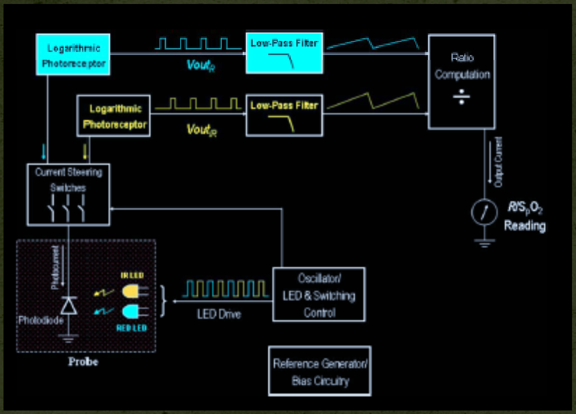


- OTA1 is a standard cascode differential amplifier
- OTA2 uses body terminal to achieve a small g_{m2}
  - Results in higher overall gain

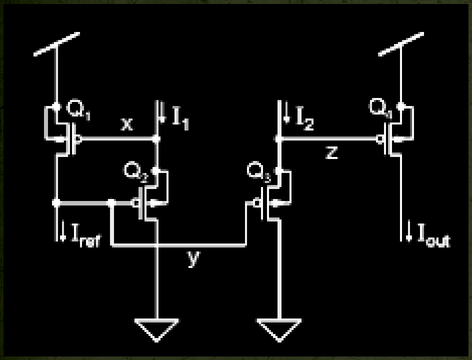
Distributed Amplifier

- Novel amplifier design achieves logarithmic transimpedance amplification
  - Logarithmic output matches desired output for oximetry
- High gain bandwidth product allows for LED to be switched quickly
  - Small duty cycle allows for significant power savings

Ratio Computation



Current Divider



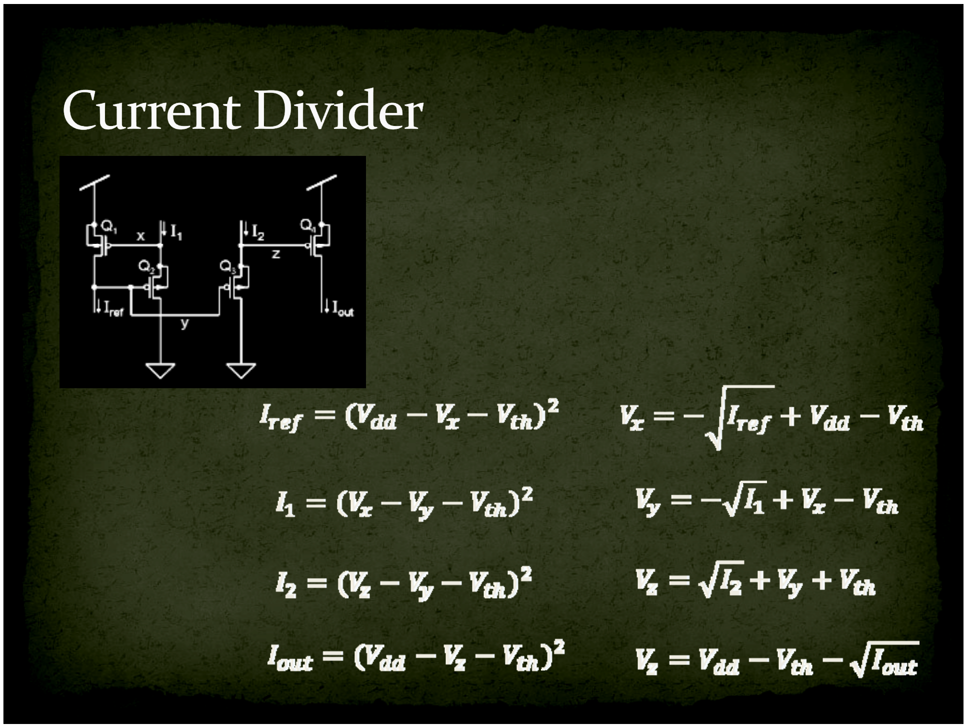
$$I_{DS} \propto (V_{gs} - V_{th})^2$$

$$I_{ref} = (V_{dd} - V_x - V_{th})^2$$

$$I_1 = (V_x - V_y - V_{th})^2$$

$$I_2 = (V_z - V_y - V_{th})^2$$

$$I_{out} = (V_{dd} - V_z - V_{th})^2$$



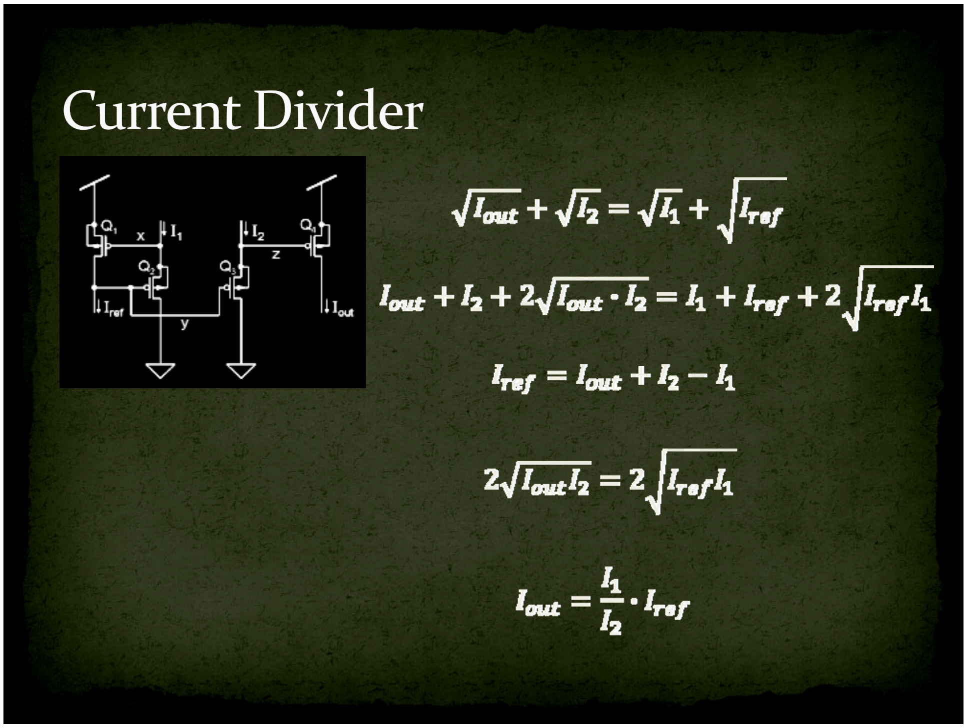
Current Divider

$$I_{ref} = (V_{dd} - V_x - V_{th})^2 \qquad V_x = -\sqrt{I_{ref}} + V_{dd} - V_{th}$$

$$I_1 = (V_x - V_y - V_{th})^2 \qquad V_y = -\sqrt{I_1} + V_x - V_{th}$$

$$I_2 = (V_z - V_y - V_{th})^2 \qquad V_z = \sqrt{I_2} + V_y + V_{th}$$

$$I_{out} = (V_{dd} - V_z - V_{th})^2 \qquad V_z = V_{dd} - V_{th} - \sqrt{I_{out}}$$



Current Divider

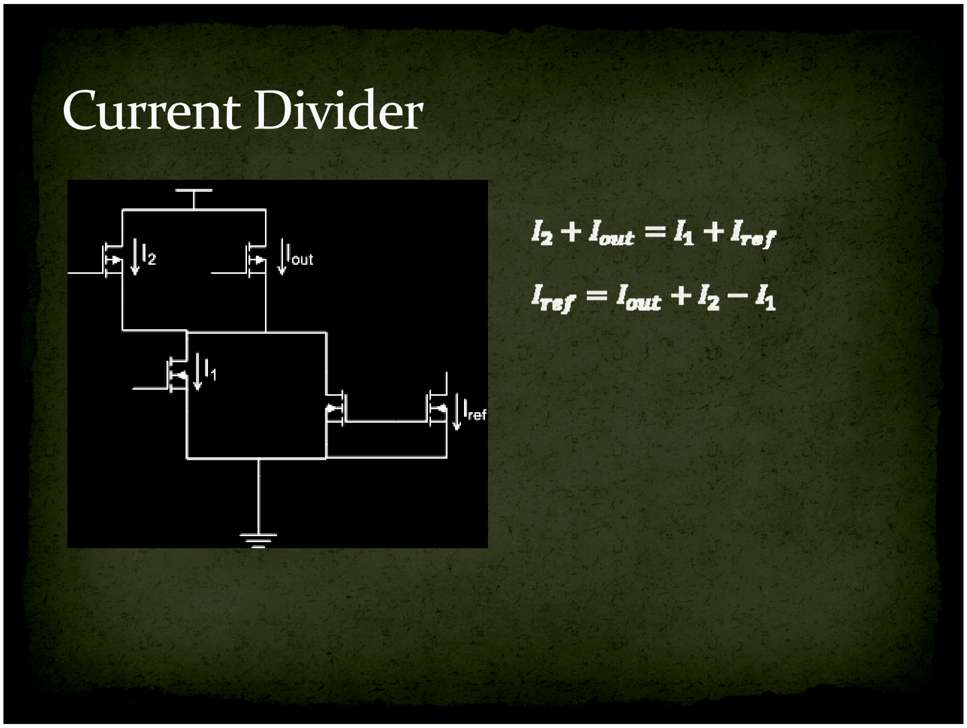
$$\sqrt{I_{out}} + \sqrt{I_2} = \sqrt{I_1} + \sqrt{I_{ref}}$$

$$I_{out} + I_2 + 2\sqrt{I_{out} \cdot I_2} = I_1 + I_{ref} + 2\sqrt{I_{ref} I_1}$$

$$I_{ref} = I_{out} + I_2 - I_1$$

$$2\sqrt{I_{out} I_2} = 2\sqrt{I_{ref} I_1}$$

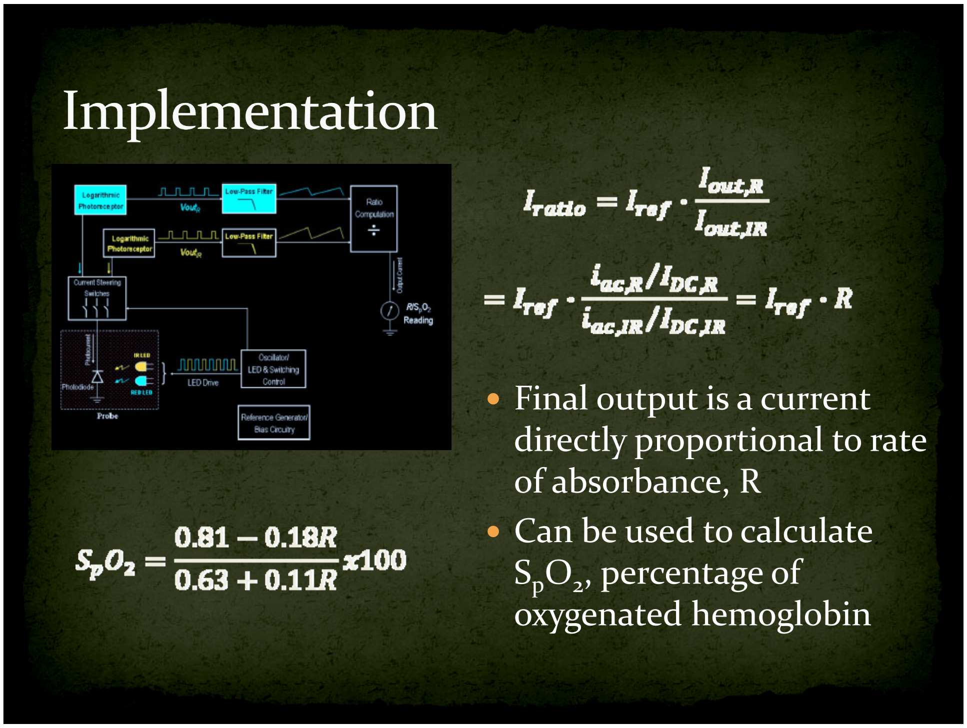
$$I_{out} = \frac{I_1}{I_2} \cdot I_{ref}$$



Current Divider

$$I_2 + I_{out} = I_1 + I_{ref}$$

$$I_{ref} = I_{out} + I_2 - I_1$$



Implementation

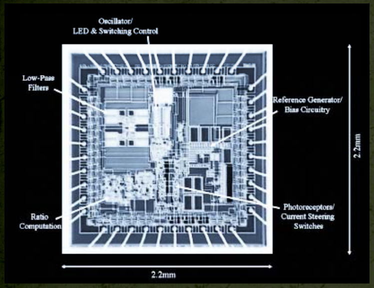
$$I_{ratio} = I_{ref} \cdot \frac{I_{out,R}}{I_{out,IR}}$$

$$= I_{ref} \cdot \frac{i_{ac,R}/I_{DC,R}}{i_{ac,IR}/I_{DC,IR}} = I_{ref} \cdot R$$

- Final output is a current directly proportional to rate of absorbance, R
- Can be used to calculate S_pO_2, percentage of oxygenated hemoglobin

$$S_pO_2 = \frac{0.81 - 0.18R}{0.63 + 0.11R} x100$$

Pulse Oximeter



Pulse Oximeter Version	Total Power Consumption (LED power and processing power, excluding display)	Days in Operation on 4 "AAA" Batteries
Our Chip	4.8mW	60
WristOx® 3100	~55mW	5.2
Xpod	60mW	4.8
Ipod	60mW	4.8
Avant® 4000	71mW	4
PalmSAT® 2500	~80mW	3.6
Onyx® 9500	~120mW	2.4
8500 Series	~130mW	2.2

Citations

- Tavakoli, Maziar, Lorenzo Turicchia, and Rahul Sarpeshkar. "An Ultra-Low-Power Pulse Oximeter Implemented With an Energy-Efficient Transimpedance Amplifier." *IEEE Transactions on Biomedical Circuits and Systems* (2009).
- Baheti, Pawan K., and Harinath Garudadri. "An Ultra Low Power Pulse Oximeter Sensor Based on Compressed Sensing." *Wearable and Implantable Body Sensor Networks* (2009).
- Basu, Arindam, Ryan Robucci, and Paul Hasler. "A Low-Power, Compact, Adaptive Logarithmic Transimpedance Amplifier Operating over Seven Decades of Current." *IEEE Transactions on Circuits and Systems* 54.10 (2007).
- Lopez-Martin, Antonio J., and Alfonso Carlosena. "Current-Mode Multiplier/Divider Circuits Based on the MOS Translinear Principle." *Analog Integrated Circuits and Signal Processing* (2001).