# CHANNEL ESTIMATION FOR SPACE-TIME ORTHOGONAL BLOCK CODES

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## ABSTRACT

**Space-time coding is an efficient technique to exploit transmitting and receiving diversities for wireless channels. One of the key components in the design of receivers for space-time codes is channel estimation. For the block codes based on orthogonal design proposed by Alamouti and Tarokh, a new class of pilot assisted channel estimation schemes are presented where pilot symbols are superimposed on the data symbols. Placement strategies are examined for their efficiency in channel estimation and data transmission.**

## 1. INTRODUCTION

A major challenge in space-time communications in a wireless environment is channel estimation. Typically, pilot symbols are necessary to acquire the channel [1]. The insertion of pilot symbols, however, may induce unacceptable overhead and lower the data throughput. Here system designers must consider two contradictory goals. On the one hand, it is desirable to minimize the number of pilot symbols in a data packet so that more information carrying symbols can be transmitted. On the other hand, more pilot symbols result in better channel estimation hence reducing symbol error rate and the need for packet retransmissions.

In this paper, we consider the channel estimation problem in the framework of space-time block codes from orthogonal designs proposed by Tarokh *et al* [2]. Our goal is to examine the effects of the number, power and placement of pilot symbols in data packets. This tradeoff can be examined by the Cramér-Rao Lower Bound (CRLB) and mutual information. Both quantities are functions of the number of pilot symbols inserted in the data packet and the locations of these known symbols. Specifically, we consider the possibility of superimposing pilot symbols on data streams. This enables continuous transmission of information symbols while channel estimation and tracking can be performed simultaneously. It also offers the flexibility to allocate power dynamically for channel estimation and symbol transmission. We show that certain placements of pilots for the orthogonal block codes are equivalent in the sense that they have the identical Fisher information matrices, which implies they have the same CRLBs.

The problem of channel estimation for space-time code was considered by Naguib *et al.* [1]. By using orthogonal pilot symbols, a simple channel estimation algorithm was proposed that used only observations associated with pilot symbols. By ignoring observations from data symbols, such techniques do not lead to efficient estimation. The location of pilot symbols was also fixed in [1]. The idea of using superimposed pilot for channel estimation has been proposed by Makrakis and Feher [6], Hoeher and Tufvesson [4], Manton and Hua [5], Ohno and Giannakis[10]. The analysis presented in these papers, however, do not apply directly to the space-time block codes.

This paper is organized as follows. In Section 2, we present the framework and assumptions. In Subsection 3.1, a general scheme for pilot symbol placement is presented. We also discuss tradeoffs among key factors. In Subsections 3.2 and 3.3 the performance of different placement schemes is examined using Cramér-Rao Lower Bounds and mutual information respectively. Simulations and numerical results are presented in Section 4 and are followed by concluding remarks.

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## 2. MODEL AND ASSUMPTIONS

Consider a multiple antenna system shown in Figure 1 where there are $m$ transmitters and $n$ receivers. In this paper we consider only rate one codes and real symbols. Symbols are transmitted in blocks of size $N$. Each block is transmitted within $N$ symbol periods. For block $t$, $\mathbf{S}(t) \in \mathcal{R}^{m\times N}$ is transmitted from $m$ antennas, and the $k$th column of $\mathbf{S}(t)$ corresponds to the transmitted vector in the $k$th symbol interval. The space-time code proposed by Tarokh *et al.* [2] has the following form

$$\mathbf{S}(t) = \sum_i \mathbf{X}_i s_i(t), \tag{1}$$

where $\{s_1(t), \ldots, s_N(t)\}$ is the block of N transmitted symbols and $\{\mathbf{X}_i \in \mathcal{R}^{m\times N}\}$ is the space-time block code. It is shown in [2],[9] that using $N$ single user detectors in parallel, the best performance is obtained if

$$\mathbf{X}_j \mathbf{X}_i^T = \begin{cases} \mathbf{I} & i = j \\ -\mathbf{X}_i \mathbf{X}_j^T & i \neq j \end{cases} \tag{2}$$

For rate one codes and real symbols it is shown, in the theory of orthogonal designs, that the family $\{\mathbf{X}_i\}_{i=1,\ldots,N}$ exists if and only if $N$=2, 4 or 8.

Under the *quasi-static flat fading* model, the received signal matrix for block $t$ is given by

$$\begin{aligned} \mathbf{Y}(t) &= \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) \\ &= \mathbf{A}\sum_{i=1}^{N} \mathbf{X}_i s_i(t) + \mathbf{N}(t), \end{aligned} \tag{3}$$

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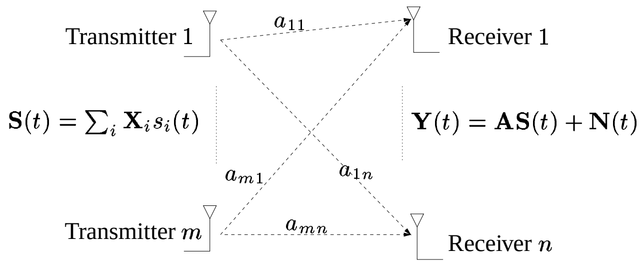


**Fig. 1**: An $m$-transmitter $n$-receiver space-time system.

where $\mathbf{A} \in \mathcal{C}^{n\times m}$ is the channel matrix and $\mathbf{N}(t)$ is the additive Gaussian noise.

In the sequel we need the received signal and the parameters represented as column vectors. Define

$$\mathbf{y}(t) \triangleq \text{vec}\left(\mathbf{Y}(t)\right), \mathbf{n}(t) \triangleq \text{vec}\left(\mathbf{N}(t)\right), \quad (4)$$

$$\mathbf{a}(t) \triangleq \text{vec}\left(\mathbf{A}^T\right). \quad (5)$$

We then have

$$\mathbf{y}(t) = \left(\mathbf{I}_n \otimes \sum_{i=1}^{N} \mathbf{X}_i^T s_i(t)\right) \mathbf{a} + \mathbf{n}(t). \quad (6)$$

In this paper, we assume (i) the noise $\mathbf{n}(t) \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ sequence is i.i.d.; (ii) the symbols $s_i(t)$ are real and $E\{s_i^2(t)\} = 1$.

## 3. CHANNEL ESTIMATION WITH SUPERIMPOSED PILOT SYMBOLS

### 3.1. Placement of Superimposed Pilot Symbols

Pilot symbols are usually embedded in the data stream for channel estimations. Typically, the pattern of pilot placement is periodic. In this paper, we consider the placement of pilot symbols in cycles of $N$ blocks, and we only need to specify the placement for one cycle.

We generalize the placement of pilot symbols by allowing superimposed training. In particular, each transmitted symbol $s_i(t)$ consists of the pilot part $v_i(t)$ and information carrying part $u_i(t)$

$$s_i(t) = \sqrt{1-\gamma_i(t)}v_i(t) + \sqrt{\gamma_i(t)}u_i(t), \quad (7)$$

where power is allocated via $\gamma_i(t) \in [0, 1]$. Here we assume $u_i(t)$ is zero mean, unit variance, and $v_i(t) \in \{\pm 1\}$. The placement of pilot symbols is equivalent to the specification of the $N \times N$ power allocation matrix $\mathbf{\Gamma} = [\mathbf{\Gamma}_{ij}]$, where $\mathbf{\Gamma}_{ij} = \gamma_i(j), \forall\, i, j = 1, \ldots, N$.

From this general family, we'll consider three special cases shown in Fig.2.

*Horizontal Placement- $P_H$:* The horizontal placement is described by

$$\begin{array}{l} \mathbf{\Gamma}_{1j} = \gamma,\ \forall j = 1, \ldots, N \\ \mathbf{\Gamma}_{kj} = 1,\ \forall k = 2, \ldots, N, j = 1, \ldots, N \end{array} . \quad (8)$$

The horizontal placement has the advantage that all the blocks within a cycle have the same structure, that simplifies estimation at reception and allows more efficient tracking for time-varying channels. Other advantages of this scheme will be showed later.

*Vertical Placement - $P_V$:* The vertical placement is described by

$$\begin{array}{l} \mathbf{\Gamma}_{k1} = \gamma,\ \forall k = 1, \ldots, N \\ \mathbf{\Gamma}_{kj} = 1,\ \forall k = 1, \ldots, N, j = 2, \ldots, N \end{array} \quad (9)$$

This scheme is similar to the classical training schemes where a block contains only pilot symbols. The disadvantage of this scheme is that it is vulnerable to noise bursts during the transmission of the training block.

*Uniform placement - $P_U$:* The uniform placement scheme is described by

$$\mathbf{\Gamma}_{kj} = \gamma, \forall\, k, j = 1, \ldots, N \quad (10)$$

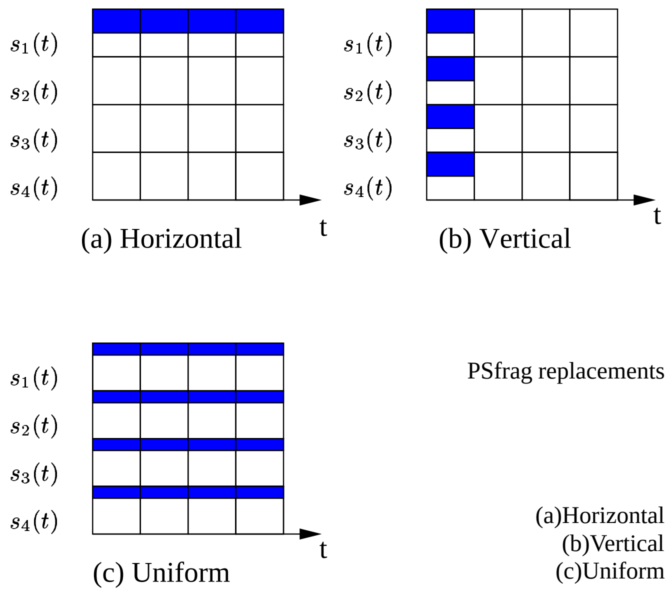


**Fig. 2**: Special placement schemes; the black regions represent the training symbols

The choice of $\mathbf{\Gamma}$ affects both CRLB of the channel estimates and the amount of information that can be transmitted through the channel.

### 3.2. Cramér-Rao Lower Bound (CRLB)

The CRLB provides the lower bound on the mean square error (MSE) for all unbiased estimators. The ratio between the MSE of an estimator and the CRLB measures the efficiency of the algorithm in utilizing the information available in the observation. In general, CRLB is a function of of the channel matrix and the symbol placement scheme. Therefore, different designs of the power allocation matrix lead to different CRLB.

We consider estimating the channel vector $\mathbf{a}$ using the entire data record from one cycle $\{\mathbf{y}(1), \cdots, \mathbf{y}(N)\}$. Define

$$\mathbf{y} \triangleq [\mathbf{y}^T(1), \cdots, \mathbf{y}^T(N)]^T. \quad (11)$$

The likelihood function for the channel parameter $\mathbf{a}$ is then given by

$$p(\mathbf{y}; \mathbf{a}) = \mathcal{N}(\mathbf{y}; \boldsymbol{\mu}(\mathbf{a}), \mathbf{C}(\mathbf{a})), \quad (12)$$

where $\mathcal{N}(\mathbf{y};\boldsymbol{\mu}(\mathbf{a}),\mathbf{C}(\mathbf{a}))$ is the pdf of the normal distribution with mean $\boldsymbol{\mu}(\mathbf{a})$ and covariance $\mathbf{C}(\mathbf{a})$ given by

$$\boldsymbol{\mu}(\mathbf{a}) = \begin{bmatrix} \mathbf{I}_n \otimes \sum_{i=1}^N \mathbf{X}_i^T \sqrt{1-\boldsymbol{\Gamma}_{i1}} v_i(1) \\ \vdots \\ \mathbf{I}_n \otimes \sum_{i=1}^N \mathbf{X}_i^T \sqrt{1-\boldsymbol{\Gamma}_{iN}} v_i(N) \end{bmatrix} \mathbf{a} \qquad (13)$$

$$\mathbf{C}(\mathbf{a}) = \mathrm{diag}\left[\mathbf{C}_{11}(\mathbf{a}), \ldots, \mathbf{C}_{N,N}(\mathbf{a})\right]. \qquad (14)$$

Denote $\mathbf{w}_i = \left(\mathbf{I}_n \otimes \mathbf{X}_i^T\right)\mathbf{a}$, and we have

$$\mathbf{C}_{tt} = \sum_{i=1}^N \mathbf{w}_i \mathbf{w}_i^T \boldsymbol{\Gamma}_{it} + \sigma^2 \mathbf{I}_{mN}. \qquad (15)$$

Consider that the channel parameters are real numbers. The elements of the Fisher information matrix (FIM) are given by:

$$\begin{aligned}[\mathbf{I}(\mathbf{a})]_{ij} = & \left[\frac{\partial \boldsymbol{\mu}(\mathbf{a})}{\partial \mathbf{a}_i}\right]^T \mathbf{C}^{-1}(\mathbf{a}) \left[\frac{\partial \boldsymbol{\mu}(\mathbf{a})}{\partial \mathbf{a}_j}\right] + \\ & \frac{1}{2}\mathrm{tr}\left[\mathbf{C}^{-1}(\mathbf{a})\frac{\partial \mathbf{C}(\mathbf{a})}{\partial \mathbf{a}_i}\mathbf{C}^{-1}(\mathbf{a})\frac{\partial \mathbf{C}(\mathbf{a})}{\partial \mathbf{a}_j}\right].\end{aligned} \qquad (16)$$

Because of the independence of the unknown symbols, matrix $\mathbf{C}$ has a block diagonal structure, which makes it possible to calculate the FIM as a sum of the FIMs for each block.

**Lemma 1** *The inverse of the matrix $\mathbf{C}_{tt}$ is given by*

$$\mathbf{C}_{tt}^{-1} = \sum_{i=1}^N \mathbf{w}_i \mathbf{w}_i^T \Delta_{it} + \rho \mathbf{I}$$

*where $\rho \triangleq \frac{1}{\sigma^2}$ and $\Delta_{it} \triangleq -\frac{\boldsymbol{\Gamma}_{it}}{\sigma^2(||\mathbf{a}||^2 \boldsymbol{\Gamma}_{it} + \sigma^2)}$.*

Using properties of the orthogonal codes $\{\mathbf{X}_i\}$, we have the following theorems.

**Theorem 1** *If $N = 2$ or $N = 4$ the FIM of the channel parameters is the same for the horizontal and vertical placement. It follows that the CRLBs of the parameters estimated from the two placement schemes are the same.*

Note that the theorem above does not hold for N=8. In section 4.2 some numerical examples are given.

**Theorem 2** *Consider only one block, and $N = 2$ or $N = 4$. Then the FIM of the placement scheme $\boldsymbol{\Gamma} = [\gamma_1 \mathbf{e}_{i1}, \ldots, \gamma_N \mathbf{e}_{i_N}]$ does not depend on the choice of the unit vectors $\mathbf{e}_{i_n}$ ( i.e., on the indices $i_n, n = 1, \ldots, N$) or on the BPSK training sequence used.*

This theorem says that we can change the position of the training symbol in a block of the horizontal placement without affecting the performance of the scheme. Again this theorem does not hold for $N = 8$.

### 3.3. Mutual Information

Mutual information between the input and the output measures the efficiency and reliability of data transmission. In [8, 7], Adireddy and Tong examined the effect of pilot symbols on the capacity of the system and showed that placing known symbols judiciously can improve the transmission rate significantly. For space-time code systems, the placement of known symbols again play an important role.

If the channel is known, for Gaussian input symbols we have

$$I(\mathbf{y};\mathbf{u}) = \log|\det(\mathrm{cov}(\mathbf{y}))| - \log|\det(\sigma^2 \mathbf{I})|$$

Using the properties of the space-time block codes we have :

$$det(\mathbf{C}(\mathbf{a})) = \sigma^{2N(N^2-N)} \prod_{t=1}^N \prod_{k=1}^N (q\boldsymbol{\Gamma}_{kt} + \sigma^2). \qquad (17)$$

**Theorem 3** *The mutual information between the input and the output does not change under any permutation of the coefficients $\boldsymbol{\Gamma}$. If we constrain the amount of power that can be used to transmit pilot signals, i.e.,*

$$\sum_{t,k=1,\ldots,N} \boldsymbol{\Gamma}_{kt} = P = const,$$

*then the uniform scheme with*

$$\boldsymbol{\Gamma}_{kt} = \frac{P}{N^2} \;\; \forall i = 1 \ldots N, t = 1 \ldots N \qquad (18)$$

*maximizes the mutual information between the input and the output.*

## 4. SIMULATIONS AND NUMERICAL RESULTS

### 4.1. Simulations

The channel estimation algorithm that was used was a semiblind Maximum Likelihood Algorithm using the scoring method. The numerical results that are presented were obtained using the following setup. The training symbols were binary, $+1$, $-1$, the parameters values were $N = 4$, $m = 3$, $n = 5$, number of blocks considered $n_b = 32$. The channel coefficients were random numbers $\mathbf{a}_i \in (0, 1] \, \forall \, i$[1]. 300 Monte Carlo simulations were performed. We evaluated the sum of the CRLBs for all parameters.

### 4.2. Numerical Results

Fig.3 shows the CRLB and estimation variance of the ML parameters vs. the power of noise, $\sigma^2$. In our simulation, the ML estimator was initialized randomly, and it usually converged in less than 10 iterations. We observed that for reasonable allocations of power $\gamma < 0.7$, the MSE of the ML estimator is close to CRLB. In Fig.4 shows the CRLB vs. $\gamma$ for different powers of noise. It can be observed that the training schemes with less power allocated to training are more sensitive to noise.

[1] the following channel was used

$$\begin{aligned}\mathbf{a} \;\; = \;\; & [0.56, 0.74, 0.78, 0.77, 0.63, 0.69, 0.88, 0.54, \\ & 0.68, 0.55, 0.70, 0.83, 0.56, 0.58.0.68]^T\end{aligned}$$

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In Fig.5 the horizontal placement scheme is compared with the uniform one using the CRLB. It can be observed that the uniform placement scheme has higher CRLB than the horizontal scheme, and the difference is significant at high SNR. However, at low SNR the two schemes perform similarly.

In Fig.6 compares the horizontal and vertical placement for a code with $N = m = 8$. At the current resolution the two placement schemes appear identical, but their FIMs are different and their is also a small difference between the CRLBs.

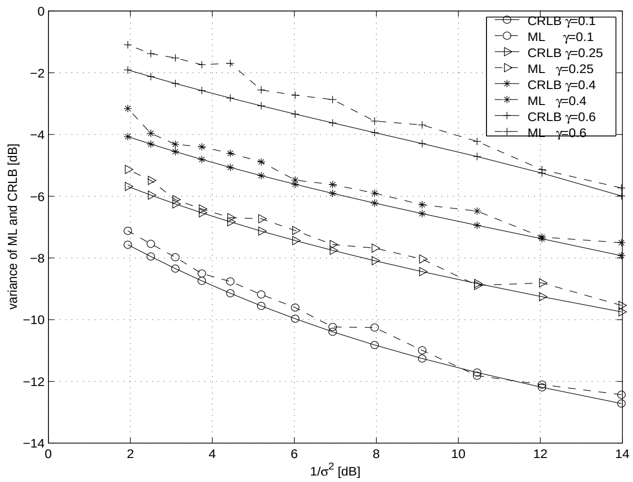


**Fig. 3**: CRLB (continous line) and the variance of the ML estimator (dashed line) for $P_H$. The values of $\gamma$ are shown in the legend.

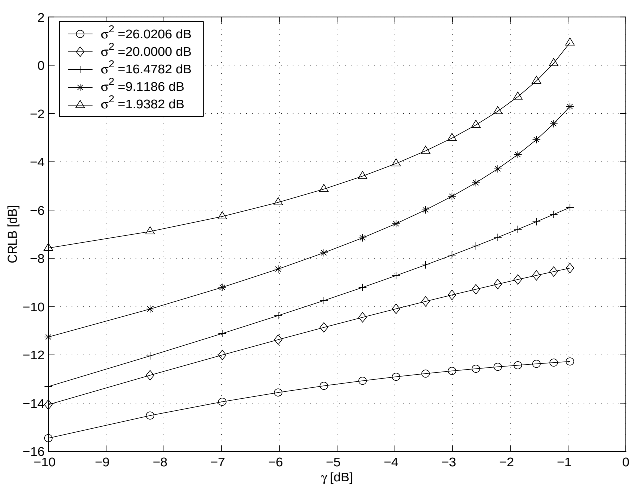


**Fig. 4**: Horizontal placement, CRLB vs. $\gamma$, $\sigma^2$ is a parameter.

## 5. CONCLUSIONS

In this paper we have presented a channel estimation approach for space-time block codes. The training is done by superimposed pilot symbols and the estimation is ML semiblind. We have introduced the horizontal placement of the training symbols that is more convenient than the "classical" vertical training scheme and provides the same performance. We have also compared different placement schemes using the CRLB for channel estimation and the mutual information between the input and the output.

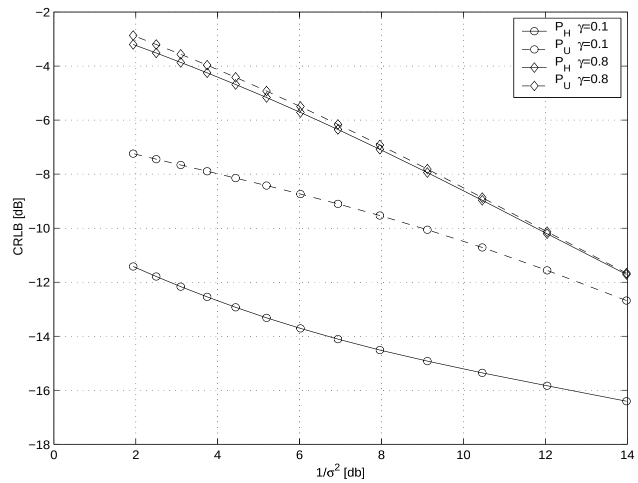


**Fig. 5**: CRLB for different values of $\gamma$ for the horizontal scheme (continuous line) and uniform scheme (dashed line)

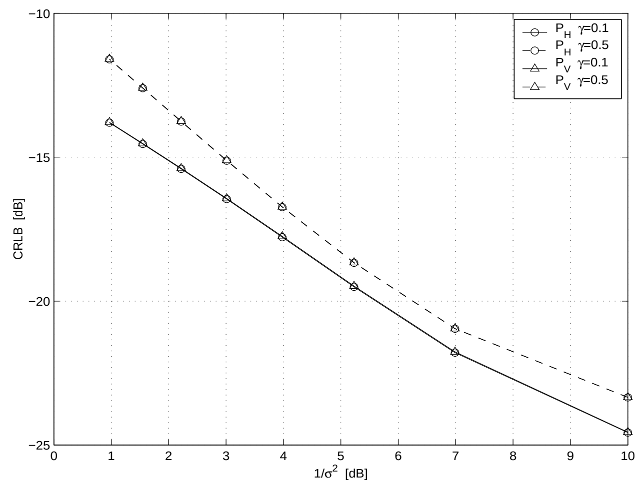


**Fig. 6**: CRLB for $P_U$ and $P_V$ for $N = m = 8$. The plots appear identical at the current resolution. $\gamma$ is a parameter.

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