Pricing Multi-period Dispatch Under Uncertainty

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***Abstract*—Price formation in the context of multi-period dispatch of electricity under operational uncertainty is considered. General and partial equilibrium conditions for the locational marginal price (LMP) is examined. It is shown that, when the market participants are provided with limited forward prices, no uniform price exists for the partial equilibrium model in general. As a consequence, market participants have incentives to deviate from the optimal economic dispatch. Taking explicit accounts for ramping constraints, an extension of LMP, referred to as temporal locational marginal pricing (TLMP), is proposed that reflects both generation and opportunity costs of generators. Although discriminative, TLMP and the optimal economic dispatch satisfy both the general and partial equilibrium conditions for which, given the TLMP, dispatch schedules generated individually by profit-maximizing participants match the solution of the centralized social welfare maximization. TLMP is then extended for ex post pricing. The resulting incremental TLMP (iTLMP) is shown to provide revenue adequacy for the system operator and price supports for participants who offer sufficient ramping capability. Numerical simulations are used to demonstrate the performance of LMP, flexible ramping product (FRP), TLMP, and iTLMP.**

***Index Terms*—Multi-period economic dispatch. Locational marginal pricing. General and partial equilibrium.**

I. Introduction

Real-time operations in deregulated electricity markets are based on a *single-period dispatch and pricing model* in which the system operator clears and settles generations on an interval-by-interval basis. A standard implementation is a rolling-window dispatch that provides a look-ahead schedule with only the immediate dispatch realized [1]. The realized dispatch is priced one interval at a time although advisory forward prices for the next few intervals may be provided. The single-period prices are based on either an ex ante or ex post pricing model; the former sets the locational marginal price based on the look-ahead schedule; the latter is based on the realized dispatch using, for example, an incremental dispatch model. In absence of load uncertainty, such an approach results in prices that are consistent with the bids when the ramping capabilities of the generators match well with ramping events in the net-load.

The remarkable growth in renewable generations presents difficult challenges for the system operator to balance the highly volatile and stochastic net-loads. As highlighted in [2], the net-load curves in the operating region of CAISO have been trending toward a duck-shaped profile that contains steep up and down ramps. In 2015, for example, “the 3-hour ramp exceeded 5000 megawatts over 58% of the year, up from only 6 percent of the year in 2011,” as reported in [3].

Volatility in net-load brings difficult challenges in scheduling and pricing of generations. Without accurate forecasts, heuristic techniques such as the rolling-window dispatch are suboptimal and the standard single-period pricing a suspect. Most significant is the lack of guarantee to provide adequate price supports for the generators. Specifically, there is the so-called missing money problem in which a truthful bidder who follows operator’s dispatch signals may wind up loosing money over the scheduling horizon. A rational market participant therefore has incentives to deviate from dispatch instructions.

One remedy for the missing money problem is supplementing real-time pricing with out-of-market uplift payments [4]. Such an approach may have undesirable consequences from an economics perspective; it makes market less transparent, and the settlements used are often hard to justify.

An alternative is to improve upon current pricing schemes with new mechanisms that take explicit consideration of ramping issues. The hope is to find a pricing scheme that better reflects the generation as well as the opportunity costs associated with ramping while maintaining market transparency. An example is the *flexible ramp product* (FRP) [5] that treats ramping capability as a commodity with its prices derived from the dispatch optimization. Although FRP is well motivated, it is not clear whether it can achieve market efficiency and remove incentives of generators to deviate from generation instructions.

A. Summary of results

This paper aims to shed lights on the underlying issues of efficiency and incentive compatibility associated with the single-period dispatch and pricing models. To this end, we apply an equilibrium analysis that helps to address the question on whether a particular pricing scheme gives incentives for market participants to deviate from the optimized dispatch. We show that the standard locational marginal pricing (LMP) does not satisfy the partial equilibrium condition. This means that, if a generator is provided with pricing signals one period at a time, there are incentives to deviate. Indeed, no uniform price can achieve overall market efficiency and satisfy the equilibrium condition in general

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when some of the ramping constraints are binding. This analysis highlights the tension among the needs of achieving market efficiency, providing incentive compatible pricing, and maintaining transparency through uniform pricing.

Next we consider a generalization of LMP, referred to as *temporal locational marginal pricing (TLMP)*, that takes explicit accounts for generators' ramping capabilities and their opportunity costs. As LMP, TLMP is also based on the incremental costs of serving additional unit of demand or providing additional unit of generation at a particular location and time. Unlike LMP, TLMP removes the constraint that all generations at the same time and location are priced equally. Under TLMP, those generators that do not have binding ramping constraints are priced uniformly by LMP whereas generators having binding ramping constraints are priced differently based on their offered ramping capabilities.

We show that TLMP is an optimal equilibrium price in the sense that, *for the given TLMP, if each generator maximizes its profit individually and myopically in each time interval, the resulting generation schedule matches that from the centralized social welfare maximizing dispatch.*

The fact that centralized social-welfare maximizing dispatch can be achieved in a decentralized manner via TLMP one interval at a time is significant. This means that, if the operator has perfect foresight of the future but the generators have no foresight, price signals given to market participants one interval at a time will not only lead to the optimal dispatch for individual participants but also for the overall social welfare optimization.

Next we consider the problem of ex post pricing for the given realized sequence of dispatches. This is motivated by the fact that neither the operator nor the market participants have perfect foresights, and practical dispatches are in general not optimal. Ex post pricing has been implemented by system operators in the U.S. [1], and it has well documented advantages and shortcomings [6]. In general, ex post pricing motivates participants to behave consistently with their bids. The main disadvantage, on the other hand, is that it lacks economic justifications and often relies on parameters that can only be set in an ad hoc fashion.

We apply an incremental dispatch technique to TLMP and refer the resulting pricing as iTLMP. The innovation of iTLMP is the specific way of setting the deviation parameter that ties the individual ramping constraints. This leads to several properties important in practical applications. Specifically, iTLMP guarantees revenue adequacy for the operator. It solves partially the "missing money problem" for the generator in that, a generator with sufficiently high ramping capability is guaranteed to be compensated at levels equal to or above its marginal cost of generation.

B. Related Work

This work is motivated by some of the recent discussions among system operators [5], [7]–[10] on the need of ramping products in response to the emerging net-load profiles as results of renewable integrations, especially on the problem of price support. See also a review of some of the ramping products in [11]. The analytical tools used here are standard LMP theory [12] and equilibrium theory [13], [14]. The derivation of TLMP is an application of the well known envelop theorem applied to each energy resource separately, and the derivation of iTLMP—the ex post version of TLMP—is based on the incremental dispatch model [1]. Some of the pricing issues related to the incremental dispatch model have been discussed in [6], [15].

II. Multiperiod Dispatch, LMP, and Equilibrium

A. Multi-period economic dispatch

We begin with a simplified "one-shot" multi-period economic dispatch model*, :

$$\begin{array}{ll} \text{minimize} & \sum_{t=1}^{T}\sum_{i=1}^{N} f_{it}(P_{it}) \\ \text{subject to} & \text{for all } i \text{ and } t \\ & \sum_{i=1}^{N} P_{it} = P_{0t}, \qquad (\lambda_t) \\ & 0 \le P_{it} \le \bar{P}_{it}, \qquad (\underline{\rho}_{it}, \bar{\rho}_{it}) \\ & \underline{r}_i \le P_{i(t+1)} - P_{it} \le \bar{r}_i, \quad (\underline{\mu}_{it}, \bar{\mu}_{it}) \end{array} \tag{1}$$

where P_{0t} is the inelastic demand at time t, $P_{it}, i = 1, \cdots, N$ the decision variables representing the dispatch levels for the N generators, $f_{it}(\cdot)$ the generation cost function assumed to be convex.

There are three types of constraints in (1). The power balance constraint with dual variables (λ_t), the individual generation constraints defined by the capacities of generators with dual variables $(\underline{\rho}_{it}, \bar{\rho}_{it})$, and the individual ramping constraints defined by ramping limits $(\underline{r}_i, \bar{r}_i)$ with dual variables $(\underline{\mu}_{it}, \bar{\mu}_{it})$. The ramping constraints apply to consecutive time intervals, and it is this coupling of decision periods that makes the pricing problem different from the standard LMP model.

Although we do not include a network in the above formulation, the results here can be generalized.

B. The Locational Marginal Price

The locational marginal pricing (LMP) is the prevailing pricing mechanism used in deregulated electricity markets. It is a uniform pricing scheme defined for each t by the marginal cost increase to serve demand P_{0t}.

Mathematically, let the total cost associated with dispatch $P = (P_{it})$ be

$$C(P) = \sum_{i=1}^{N}\sum_{t} f_{it}(P_{it}).$$

Let the primal and dual variable solutions of (1) be $P^* = (P^*_{it})$, and $\lambda^* = (\lambda^*_t)$, etc. The LMP at time t is defined by

$$\pi_t^{\text{LMP}} := \frac{\partial}{\partial P_{0t}} C(P^*) = \lambda^*_t, \tag{2}$$

*In a one-shot multi-period economic dispatch, the dispatch over the entire scheduling horizon is solved together.

where the second equality follows the envelope theorem. Under LMP, the load pays $\pi_t^{\text{LMP}} P_{0t}$ for its consumption in time inverval t and generator i is paid $\pi_t^{\text{LMP}} P_{it}$ for its generation. This scheme was considered earlier in [16].

C. Market Equilibrium

The multi-period economic dispatch and the associated LMP come from a centralized bid-based clearing process: the generators submit their bids $\{f_{it}(\cdot)\}$, the system operator solves (1), computes the dispatch $P^* = (P_{it}^*)$, and sends the dispatch signals to generators. Ideally, generators follow the signal and generate collectively P^*, and the operator settles the market using the LMP (π_t^{LMP}) computed from (2).

The question arises whether, in such a setting, the generators would voluntarily follow the dispatch signal P^*. This question needs to be answered by an equilibrium analysis that considers a decentralized setting in which, given prices from the operator, whether a rational generator will produce the same amount as determined by the operator.

Definition 1 (General equilibrium)**.** *Let $P_i = (P_{it}, t = 1, \cdots, T)$ be a sequence of generations of generator i and $\pi = (\pi_t)$ a sequence of prices over the entire scheduling horizon. We say that price π and generation $\{P_i, i = 1, \cdots, N\}$ form a general equilibrium if they satisfy*

1) *Market clearing condition: $\sum_i P_{it} = P_{0t}$ for all t;*
2) *Individual rationality: for all i,*

$$P_i = \arg \max_{p_t, t=1\cdots T} \{\textstyle\sum_t (\pi_t p_t - f_{it}(p_t)) \\ \qquad | -\underline{r}_i \le p_{t+1} - p_t \le \bar{r}_i, 0 \le p_t \le \bar{P}_{it}\}.$$

We call an equilibrium optimal if P_i is also a solution of the centralized multi-period economic dispatch problem (1).

With this definition, we have the following equilibrium result for LMP [17].

Proposition 1 (LMP as an equilibrium pricing)**.** *The LMP π^{LMP} and the multi-period economic dispatch P^* forms an optimal general equilibrium.*

Note that, for the general equilibrium model, the individual generator uses prices for the entire scheduling horizon to determine its all the generations in one-shot. This model, however, is not suitable for dispatch problems under load uncertainty when it is not possible to provide the generators with the price vector over the entire horizon. For a rolling window dispatch, it is only reasonable to assume that only the price for the current interval is available. We therefore need a condition that applies to each individual time interval rather than over the entire scheduling horizon.

Definition 2 (Partial equilibrium)**.** *Let π_t be the price at time t and $P_t = (P_{1t}, \cdots, P_{Nt})$ a vector of generations from all generators at time t. We say that price π_t and generation P_t form a partial equilibrium if they satisfy*

1) *Market clearing condition: $\sum_i P_{it} = P_{0t}$ for all t;*
2) *Individual rationality: for all i and t*

$$P_{it} = \arg \max_{0 \le p \le \bar{P}_{it}} (\pi_t p - f_{it}(p)).$$

Note that, for the partial equilibrium model, the individual generator uses only the current price π_t to determine its generation at time t, ignoring the ramping constraints. Given a set of price-dispatch pairs $\{(\pi_t, P_t)\}$, if each (π_t, P_t) is a partial equilibrium, jointly they don't necessarily form a general equilibrium. It turns out that one can say the same for the converse: if $(\pi = (\pi_t), P = (P_t))$ forms a general equilibrium, it is not necessary that every (π_t, P_t) forms a partial equilibrium. The following result shows a severe limitation of the single period pricing assumed in the partial equilibrium condition.

Theorem 1 (Disequilibrium of LMP)**.** *Let $P^* = (P_t^*)$ be the solution of the multi-period economic dispatch of (1). Assume that the dual variables $(\underline{\mu}_t, \bar{\mu}_t)$ are not all zero. Then there does not exist a uniform price $\pi = (\pi_t)$ such that, for all t, (π_t, P_t^*) form a partial equilibrium.*

An example of such disequlibrium is given in Example 1.

The disequilibrium result highlights a fundamental limitation in uniform pricing of multi-period dispatch under uncertainty. Uniform pricing has been a pillar of deregulated wholesale market design; it symbolizes the notion of market transparency. Yet, if a uniform pricing is used *one interval at a time*, generators have incentive to deviate and market efficiency cannot be achieved.

III. Temporal Locational Marginal Price

We now present the derivation of TLMP and its properties. As a generalization of LMP, the TLMP model prices each resource separately, resulting in a nonuniform pricing scheme. A related but different scheme was presented in [16].

Consider a system with one inelastic load and N generation resources. At each time t, TLMP defines a vector $\pi_t = (\pi_{it}, i = 0, \cdots, N)$ of prices with π_{0t} being the price of demand and π_{it} the price of the ith generator.

Under the multi-period economic dispatch model (1), let $C(p)$ be the cost of dispatch $P = p$. As in LMP, the TLMP for resource i is defined by the marginal cost of serving or receiving resources i.

Definition 3. *The TLMP of resource i at time t for resource i to receive (as a load) or provide (as a generator) $p_{it} = P_{it}^*$ is defined by*

$$\pi_{it} = \frac{\partial}{\partial p_{it}} C(P^*), \quad i = 0, 1, \cdots, N.$$

Here we use the convention that the price of consuming power is positive and the price for providing power is negative.

Treating the p_{it} as a *parameter* in the optimization set at $p_{it} = P_{it}^*$, it follows immediately from the envelop theorem that TLMP can be computed as follows.

Proposition 2 (TLMP)**.** *The TLMP associated with the multi-period economic dispatch (1) is given by*

$$\pi_{it} = \begin{cases} \lambda_t & i = 0, \\ -\lambda_t - \Delta_{it} & i = 1, \cdots N. \end{cases} \quad (3)$$

where λ_t is the LMP at time t for all resources and Δ_{it} the ramping price for generator i defined by

$$\Delta_{it} = \bar{\mu}_{it} - \bar{\mu}_{i(t-1)} - \underline{\mu}_{it} + \underline{\mu}_{i(t-1)}. \quad (4)$$

TLMP in (3) has a natural decomposition entirely analogues to the decomposition of LMP into energy and congestion prices. This expression also offers an insight into how TLMP discriminates generators based on their ramping capabilities. All resources that do not have binding ramping constraints are priced uniformly by λ_t whereas resources with ramping constraints are priced based on their offered ramping limits $(\underline{r}_i, \bar{r}_i)$. Note that the shadow prices of ramping limits $(\underline{r}_{it}, \bar{r}_{it})$ is precisely $(\underline{\mu}_{it}, \bar{\mu}_{it})$.

The following theorem summarizes properties of TLMP by allowing non-uniform pricing in the equilibrium analysis.

Theorem 2 (Efficient market equilibrium)**.** *The TLMP π defined in (3) and the optimal dispatch $P^* = (P^*_{it})$ from (1) satisfy the following conditions:*

1) *Optimal market clearing:*

$$\sum_i P^*_{it} = P_{0t}, \quad t = 1, \cdots, T.$$

2) *Individual rationality: for all t and i, P^*_{it} is the solution of resource i's individual profit optimization*

$$P^*_{it} = \arg \max_{0 \le p \le \bar{P}_{it}} \{|\pi_{it}|p - f_{it}(p)\}. \quad (5)$$

3) *Revenue adequacy of the system operator:*

$$\sum_t \pi_{0t} P_{0t} \ge \sum_{i,t} |\pi_{it}| P^*_{it}.$$

4) *Price support: The TLMP of a generator is higher than its marginal cost of generation, i.e.,*

$$|\pi_{it}| \ge \frac{d}{dp} f_i(P^*_{it}), \forall i > 0, \forall t.$$

The notion of market equilibrium used above is one of *partial equilibrium* but allowing discriminative pricing. This is significant in that, if generator i is given π_{it}, not knowing what is ahead, it is optimal for generator i not to deviate from dispatch P^*_{it}.

Incidentally, because we have relaxed the uniform pricing requirement, a direct consequence of the above is that (π, P^*) also forms a *general equilibrium* for the entire scheduling horizon because $(P^*_{i1}, \cdots, P^*_{iT})$ is the solution of the individual *multi-period* profit maximization:

$$\max_{(p_{it}) \in \mathcal{P}_{it}} \sum_t \Big(|\pi_{it}| p_{it} - f_i(p_{it}) \Big).$$

Resource	Capacity (MW)	Marginal cost ($/MW)	Ramp limit
Gen 1	500	25	500
Gen 2	500	30	50

TABLE I: Generation parameters.Generation capacity are in MW. Costs are in $/MW.

Example 1 (Two-generator two-period economic dispatch)**.** Consider the two generator example with parameters shown in TABLE 1.

The dispatch signals and LMP/TLMP prices are listed in TABLE II, assuming the initial dispatch is zero for both generators.

Resource	Dispatch P^*_{i1}	LMP $\pi^{\rm LMP}_{i1}$	TLMP $\pi^{\rm TLMP}_{i1}$	Dispatch P^*_{i2}	LMP $\pi^{\rm LMP}_{i2}$	TLMP $\pi^{\rm TLMP}_{i2}$
Gen 1	380	25	25	500	35	35
Gen 2	40	25	30	90	35	30
Load	425	25	25	590	35	35

TABLE II: TLMP for the two generator two period example. Dispatches are in MW. Prices are in $/MW.

The difference between LMP and TLMP manifests itself in the pricing of generator 2. For LMP, the generator is under-paid in the first interval and over-paid in the second. For TLMP, the generator is paid at its marginal cost for both periods. Even without knowing the price at $t = 2$, there is no incentive for it to deviate from the dispatch of 40.

IV. Ex Post Temporal Locational Marginal Price

We now extend the TLMP model to ex post pricing. This is motivated by that, when there is significant uncertainty, an implementation of multi-period dispatch is rarely optimal. Ex post pricing aims to price realized dispatches in such a way to provide a level of consistency with the submitted bids. In doing so, ex post pricing has an effect to encourage generators to follow the dispatch signals. Here we adopt the incremental economic dispatch framework used in practice [1] and modify it for the TLMP pricing model.

The incremental dispatch model considers the perturbation around the realized dispatch in the following optimization:

$$\begin{array}{lll} \text{minimize} & \sum_{i,t} c_{it} \Delta P_{it} & \\ \text{subject to} & \sum_i \Delta P_{it} = 0, & (\lambda_t) \\ & \text{and for all } i \text{ and } t = 1, \cdots T & \\ & \underline{r}_i \le P_{i(t+1)} + \Delta P_{i(t+1)} & \\ & \quad -(P_{it} + \Delta P_{it}) \le \bar{r}_i, & (\underline{\mu}_{it}, \bar{\mu}_{it}) \\ & 0 \le P_{it} + \Delta P_{it} \le \bar{P}_{it}, & (\underline{\rho}_{it}, \bar{\rho}_{it}) \\ & -\underline{B}_i \le \Delta P_{it} \le \bar{B}_i, & (\underline{\eta}_{it}, \bar{\eta}_{it}) \end{array} \quad (6)$$

where the decision variables are ΔP_{it}. This of course depends on the sizes of the neighborhood defined by $(\underline{B}, \bar{B})$.

From the above incremental dispatch model, we define the induced (ex post) TLMP (iTLMP) as follows.

Definition 4 (iTLMP)**.** *For a given realized dispatch sequence (P_{it}), the induced (ex post) price (π_{it}) from the incremental dispatch model (6) is defined as*

$$\pi_{it} = \begin{cases} \lambda_t & i = 0, \\ -\lambda_t - \Delta_{it} & i = 1, \cdots N. \end{cases} \quad (7)$$

$$\Delta_{it} = \bar{\mu}_{it} - \bar{\mu}_{i(t-1)} - \underline{\mu}_{it} + \underline{\mu}_{i(t-1)}. \quad (8)$$

where $\lambda_t, \bar{\mu}_{it}, \underline{\mu}_{it}$ are dual variables of (6).

Intuitively, if the realized dispatch (P_{it}) is sufficiently closed to the optimal dispatch (P^*_{it}), then the solution of the incremental dispatch leads to the optimal dispatch, and the resulting multipliers match to those in the optimal dispatch model. Then the P_{it} is priced the same way as in the ex ante model. On the other hand, if some generator deviates significantly from the dispatch signal, its role in setting the ex post price diminishes.

The performance iTLMP depends on the parametric choices of the upper and lower bounds $(\underline{B}_{it}, \bar{B}_{it})$ on ΔP_{it}. We show next that a special choice of $(\underline{B}_{it}, \bar{B}_{it})$ results in some of the desirable properties.

Theorem 3 (Properties of iTLMP)**.** *Consider the incremental dispatch model (1). Let $(\underline{B}_{it}, \bar{B}_{it}) = \frac{1}{2}(\underline{r}_{it}, \bar{r}_{it})$. Then*

1) *Operator's revenue adequacy:*

$$\sum_t \pi_{0t} P_{0t} \geq \sum_{i=1}^{N} \sum_t \pi_{it} P_{it};$$

2) *Price support: for all i and t, if $P_{it} - P^*_{it} < \bar{r}_{it}/2$ and $P^*_{it} > 0$, then resource i receives no less than its bidding price at time t.*

A key feature of iTLMP is that generators with higher ramping capabilities are more likely to be eligible to participate in the price setting and be adequately supported.

V. Conclusion

We consider in this paper the problem of pricing multi-period dispatch when there is load uncertainty. Our goal here is to highlight a fundamental conflict among achieving efficient dispatch, market transparency, and individual rationality. The root of this conflict is that, in the presence of uncertainty, only partial information can be used in dispatch. Pratical tradeoffs have to be made. To this end, we have considered a form of discriminative pricing—TLMP and its ex post pricing extension iTLMP. The advantage of TLMP is that it achieves efficiency and market equilibrium simultaneously. The price paid is the use of discriminative pricing that has market transparency issues.

References

[1] A. L. Ott, "Experience with PJM market operation, system design, and implementation," *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 528–534, May 2003.

[2] "Fast facts - california ISO," January 2018, [ONLINE], Available: https://www.caiso.com/documents/flexibleresourceshelprenewables_fastfacts.pdf.

[3] "The california duck curve is real, and bigger than expected," November 2016, [ONLINE], Available: https://www.greentechmedia.com/articles/read/the-california-duck-curve-is-real-and-bigger-than-expected#gs.0QCAebA.

[4] Y. M. Al-Abdullah, M. Abdi-Khorsand, and K. W. Hedman, "The role of out-of-market corrections in day-ahead scheduling," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 1937–1946, September 2014.

[5] "Flexible ramping product revised draft final proposal," December 2015, [ONLINE], Available: https://www.caiso.com/Documents/RevisedDraftFinalProposal-FlexibleRampingProduct-2015.pdf.

[6] T. Zheng and E. Litvinov, "On ex post pricing in the real-time electricity market," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 153–164, February 2011.

[7] "MISO report: Ramp capability integration," June 2016, [ONLINE], Available: https://www.ferc.gov/CalendarFiles/20160629114652-1%20-%2020160621%20FERC%20Technical%20Conference_MISO%20Ramp%20Product.pdf.

[8] "MISO report: Ramp capability integration check later," November 2016, [ONLINE], Available: https://cdn.misoenergy.org/20161129%20MSC%20Item%2005f%20Ramp%20Capability%20Post%20Implementation%20Analysis74816.pdf.

[9] D. A. Schiro, "ISO-NE report: Procurement and pricing of ramping capability," September 2017, [ONLINE], Available: https://www.iso-ne.com/static-assets/documents/2017/09/20170920-procurement-pricing-of-ramping-capability.pdf.

[10] N. Parker, "SPP report: Ramping product design," August 2015, [ONLINE], Available: https://www.iso-ne.com/static-assets/documents/2017/09/20170920-procurement-pricing-of-ramping-capability.pdf.

[11] Q. Wang and B.-M. Hodge, "Enhancing power system operational flexibility with flexible ramping products: A review," *IEEE Transactions on Industrial Informatics*, vol. 13, no. 4, pp. 1652–1664, August 2017.

[12] D. S. Kirschen and G. Strbac, *Fundamentals of Power System Economics (2nd Edition)*. Wiley, 2018, pp. 148–198.

[13] G. Debreu, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. Yale University Press, 1972.

[14] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory*. Oxford University Press, 1995.

[15] F. Li, Y. Wei, and S. Adhikari, "Improving an unjustified common practice in ex post LMP calculation," *IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 25, NO. 2, MAY 2010*, vol. 25, no. 2, pp. 1195–1197, May 2010.

[16] J. Zhao, T. Zheng, E. Litvinov, and F. Zhao, "Pricing schemes for two-stage market clearing models," in *FERC Software Conference*, 2013, [ONLINE], Available: https://www.ferc.gov/CalendarFiles/20140411125647-M3%20-%20Zhao.pdf.

[17] F. Wu, P. Varaiya, P. Spiller, and S. Oren, "Folk theorems on transmission access: Proofs and counter examples," *Journal of Regulatory Economics*, vol. 10, no. 1, pp. 5–23, July 1996.