Killing Death Spiral Softly with a Small Connection Charge

Tao Sun† and Lang Tong‡

***Abstract*—The death spiral hypothesis points to the possibility that, with increasing integration of behind-the-meter renewables, the revenue of a regulated utility declines, which forces the utility to increase the price of electricity to maintain revenue adequacy. This in turn drives more consumers to adopt renewable technology, which further erodes the financial standing of the utility.**

We analyze the interactions between a regulated utility who sets the retail tariff and its price-elastic customers whose decisions to adopt renewable technology are influenced by the retail tariff and the cost of the technology. We establish conditions for the existence of death spiral and the stable diffusion of renewable technologies. We show in particular that linear tariffs always induce death spiral when the fixed operating cost of the utility rises beyond a certain threshold. For two-part tariffs with connection and volumetric charges, the Ramsey pricing that optimizes myopically social welfare subject to the revenue adequacy constraint induces a stable equilibrium. The Ramsey pricing, however, inhibits renewable adoption with a high connection charge. In contrast, a two-part tariff with a small connection charge results in a stable adoption process with a higher level of renewable adoption and greater long-term social welfare. Market data are used to illustrate various solar adoption scenarios.

***Index Terms*—Diffusion dynamics, equilibrium, retail tariff, renewable integration, distributed energy resources.**

I. Introduction

Death spiral for electric utilities stands for a positive feedback scenario in which, when the utility raises its price to cover its cost, consumers reduce consumptions. This forces the utility to increase further its price, which lowers the consumption even further.

The possibilities of death spiral for electric utilities have been raised several times since 1960's [1], and this topic has attracted considerable attention recently, thanks to the rapid deployment of the behind-the-meter solar photovoltaic (PV) and other distributed energy resources such as storage. A main difference this time is the role of disruptive technology such as solar PV and residential storage. Both technologies have direct impacts on the revenue of the utility.

There is some evidence supporting the underlying assumptions of the death spiral hypothesis. Recent reports issued by the California Public Utility Commission (CPUC) [2], [3] state that "From 2012 to 2016, system average rates (SAR) across the three IOUs has increased at an annual average of approximately 3.44%, which is well above the average annual inflation rate of 1.3% over the same time period." Meanwhile, "all three utilities have experienced declines in kWh sales, which also lead to increased rates when revenue requirement remains flat or rises." Data in [4] further show that "the flattening or declining trend in kWh sales is driven by a changing economy, growth in the customer (so called behind-the-meter) solar industry, increasing availability of demand side management (DSM) programs such as energy efficiency, and the incremental proliferation of retail choice."

The above snapshot statistics are consistent with the more general trend discussed in one of the earliest work on death spiral hypothesis in solar PV adoption by Cai, Adlakha, Low, Martini, and Chandy [5]. Using data from an investor-owned regulated utility, the results in [5] shows, empirically, the effects of positive feedback loop involving PV adoption, the loss of revenue, and rate changes. The empirical analysis also shows that high connection charges slow the rate of solar adoption. A more recent empirical study [6] using nationwide data by Darghouth, Wiser, Barbose, and Mills, besides confirming the general feedback phenomenon and the negative impact of connection charges on PV adoption, shows more nuanced effects of dynamic pricing on PV adoption.

While empirical studies suggest the potential of death spiral, they lack the predictive power on the dynamics of the feedback loop of PV adoption and its policy implications. With decreasing costs of solar PV, there is a need for a fundamental understanding of the PV adoption dynamics and impacts of key parameters in the adoption process. Such parameters include the cost of solar, tax incentives, and the fixed operating cost of the utility.

A. Summary of Results

This paper complements existing empirical studies such as [5], [6] with an analytical study on the dynamics of PV adoption. In particular, we aim to shed lights on the following questions:

- Can death spiral happen under the current tariff?
- What are the conditions and pricing mechanisms for a stable diffusion of renewable technology?
- What is the maximum installation capacity (referred to as the limiting capacity) achievable by a stable diffusion process.
- Does a higher level of renewable penetration implies greater social welfare?

The main contribution of this work is an analytical framework that allows us to study the PV adoption process as a nonlinear dynamical system. This model captures interactions between a regulated utility and its price-elastic and net-metered consumers who maximize the consumer surplus and make PV

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adoption decisions based on the payback time of the solar investments. Such decisions are influenced by the tariff set by the regulator and the cost of solar PV.

By analyzing the nonlinear system with the tariff and the installed solar capacity as its states, we establish conditions for the existence of equilibria, death spiral, and stable equilibria. These conditions are applied to benchmark tariff policies. A main conclusion is that linear tariffs are prone to induce death spiral; so are two-part tariffs with fixed connection charge. On the other hand, the Ramsey pricing with the optimized electricity price and connection charge, guarantees a stable diffusion. The high connection charge of the Ramsey price, however, has a negative impact on PV adoption. We show, in fact, that Ramsey pricing stalls PV adoption. In contrast, a mechanism that adds a small connection charge to the Ramsey linear tariff induces a stable adoption process that achieves a higher level of PV adoption. We demonstrate in addition that, while maximizing the immediate overall social welfare, Ramsey pricing may generate less social welfare in the long run.

B. Related Work

The literature is limited on the *dynamics* of PV adoption beyond the empirical studies in [5], [6] and economic analysis [1]. To our best knowledge, this work appears to be the first to pursue an analytical characterization of the PV adoption dynamics in the framework of nonlinear dynamical feedback systems.

In many ways, whereas our results corroborate conclusions in [1], [5], [6], we provide deeper analytical insights on various aspects of the death spiral phenomena. For instance, there is a consensus that, although the possibility of death spiral is real in the era of greater DER, the likelihood of a death spiral occurring is small, especially if the regulator and the utility set the tariff policy proactively, including the proper use of connection charges [1]. Our analysis is consistent with these conclusions. We provide, however, qualitatively and quantitative answers on how such proactive measures can be applied dynamically.

In summarizing relevant work in the literature, we highlight works that are relevant to key parts of our model and analysis. For the retailer model, it comes down to the classical problem of tariff design for a regulated monopoly [7]. In approving a proposed tariff, the regulator takes into account the impact of the tariff on overall social welfare, fairness, and societal concerns. In such a setting, the classical Ramsey pricing [7], [8] aims to maximize the social welfare subject to the break-even constraint for the utility. In this context, we consider both the linear and a nonlinear (two-part) tariffs; the latter consists of a linear volumetric charge and a connection charge. Originally studied by Oi in his seminal work [9], the two-part tariff is now widely adopted by utilities for residential customers in the United States where nearly 87% of the residential customers face some form of connection charges [10].

Tariff models for electricity markets with stochastic demand are extensively studied. See *e.g.,* [11] and references therein. With the increasing presence of distributed energy resources (DER), there is heightened attention on different types of tariff [12]. In such settings, the Ramsey pricing problem for the retail utility in distribution systems with stochastic distributed energy resources is considered in [13]–[16]. Our dynamic model builds upon the analysis in [15], [16].

A key component of our analysis is to incorporate a solar PV diffusion model in our analysis. To this end, we adopt a widely used S-curve model for the aggregated consumer behavior [17]–[19], under an implicit assumption of successful PV diffusion.

II. Consumer, Retailer, and Diffusion Models

A. Retail Tariff Model

In this paper, we consider retail tariffs uniformly applied to all consumers. We assume that the retailer sets tariff T ahead of each consumption period. The tariff is approved by the regulator periodically, say, on a yearly basis. In period k, the tariff T_k is fixed until the next period. For simplicity, we restrict ourselves to flat tariff, *i.e.,* the volumetric charge does not vary with time. Most results presented here can be generalized for dynamic tariffs where a consumer is charged based on the time of use (TOU) [20].

Two widely applied tariff classes are considered:

1) *Linear tariff :* $\mathscr{T}_{\rm L} = \{T : T(d) = \pi d\}$ where d is the total consumption in the period. In this case, a consumer is charged at the same rate at all time within the period based on the total consumption.
2) *Two-part tariff:* $\mathscr{T}_{\rm 2P} = \{T : T(d) = A+\pi d\}$ where A is the connection charge independent of the consumption.

Naturally, $\mathscr{T}_L$ is a subclass of $\mathscr{T}_{\rm 2P}$ with connection charges set to zero.

B. Consumer decision model

We assume price elastic demands. Consumer i's demand depends on the local random state ω_i that is fixed within each period. This assumption is made to simplify our presentation; it can be generalized to be time varying following [13], [15].

Knowing the set tariff T, consumer i maximizes his surplus:

$$\mathrm{cs}_i(T,\omega_i) = \max_q \Bigg(u_i(q,\omega_i) - T(q - r_i(\omega_i))\Bigg), \quad (1)$$

where $u_i(q,\omega)$ is the utility of consuming q, and $r_i(\omega_i)$ the realized behind-the-meter renewable for consumer i. Let the solution of (1) be $D_i(T,\omega_i)$, which represents his load profile.

With total M consumers in the service area of the utility, the expected consumer surplus under a two-part tariff is

$$\overline{\mathrm{cs}}(T,R) = \mathbb{E}(U(T,\omega)) - \pi(\mathbb{E}(D(T,\omega) - R\bar{r}_0) - MA, \quad (2)$$

where $\omega = (\omega_1,\cdots,\omega_M)$ is the random state of all customers, $U(T,\omega) = \sum_i u_i(D_i(T,\omega_i),\omega_i)$ and $D(T,\omega) = \sum_i D_i(T,\omega_i)$ the aggregated utility and demand, respectively. The expected total renewable is given by $R\bar{r}_0 = \mathbb{E}(\sum_i r_i(\omega_i))$ where $\bar{r}_0$ the expected renewable generation per unit-capacity installed and R the total installed capacity. The first term on the right hand side is the aggregated consumer utility, the second the total volumetric charge, and the last the total connection charge.

C. Retailer decision model

We model the retail utility as a regulated monopoly, which is the case in most parts of the United States. Here we assume that the retailer imports electricity from the wholesale market to satisfy the aggregated demand of its customers. The retailer is assumed to be a price taker*. This model is a reasonable approximation of the deregulated two-settlement electricity market.

The retailer sets the tariff and seeks its approval by the regulator in each tariff setting period. As a regulated monopoly, the retailer is allowed to break even to satisfy the revenue adequacy constraint. Under that constraint, the retailer's tariff can also be set to benefit the consumers and the society in general in a variety of ways. The revenue adequacy condition is met by setting the retail surplus to zero, which is defined by

$$\overline{\text{rs}}(T,\theta,R) = \mathbb{E}((\pi-\lambda)(D(\pi,\omega)) - Rr_0(\omega)) + MA - \theta. \quad (3)$$

Here λ is the wholesale price of electricity and $(D(\pi,\omega)) - Rr_0(\omega))$ the net consumption†. The first term on the right hand side is the revenue from energy consumption. The second term MA is the revenue from the connection charge. The break-even condition can be satisfied by jointly allocating these two types of revenue to the fixed operating cost of the utility.

We model the retailer's pricing decision by a *tariff policy* μ that maps its expected future operating cost‡ θ and the current level of renewable adoption R to a tariff T in some tariff class in the next period. In particular, at the end of the kth period, the tariff in the next period T_{k+1} is given by

$$\mu: \quad T_{k+1} = \mu(R_k, \theta_k)$$

where R_k the installed capacity at the end of period k and θ_k is the utility's expected fixed cost.

An important type of tariff policy is the *Ramsey pricing* in which the retailer maximizes the social welfare subject to the revenue adequacy constraint. Equivalently, the retailer solves the following constrained optimization to determine T_{k+1} given the current level of renewable installation R_k and the (expected) fixed cost θ_k in the next period:

$$\mu^*: \quad \max_{T\in\mathscr{T}} \overline{\text{cs}}(T,R_k) \quad \text{s.t.} \quad \overline{\text{rs}}(T,\theta_k,R_k) = 0. \quad (4)$$

where $\mathscr{T} \in \{\mathscr{T}_{2P}, \mathscr{T}_L\}$ is the tariff class. Let μ^*_{2P} and μ^*_L be the Ramsey pricing for the two-part tariff and linear tariff classes, respectively.

D. PV Diffusion Model

We now present a model for the solar PV adoption as a diffusion process of new technology. We assume that the adoption decision of a residential customer is based on his investment's payback time, which depends on the cost of solar PV and the reduced payment for consumption. Instead of considering individual adoption decisions, we model the diffusion process for the entire customer population.

Specifically, for a given tariff T and per-unit (kWh) PV purchasing cost ξ, let the installed renewable capacity in aggregation be $s(t,T,\xi)$ at time t. Illustrated in Fig. 1, $s(t,T,\xi)$ referred to as the *PV diffusion curve* and is defined by the following equation:

$$s(t,T,\xi) = R_\infty(T,\xi)\cdot\eta(t), \quad (5)$$

where $R_\infty(T,\xi)$ is the *market potential* of the PV diffusion, and the cumulative installed fraction $\eta(t)$ is a sigmoid function satisfying $\eta(0)=0$ and $\lim_{t\to\infty}\eta(t)=1$. This model has been used to model the adoption of renewable technology, and there is a parametric form of $R_\infty(T,\xi)$ that can be used in practice [21]. A well known form of $\eta(t)$ is the Bass diffusion model [22].

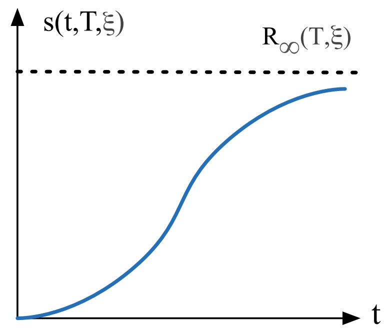

Fig. 1: Renewable diffusion for fixed market potential.

Note that $s(t,T,\xi)$ does not capture the dynamics of the diffusion process; it describes the evolution of the diffusion for *fixed* tariff T and PV cost ξ throughout the diffusion. In reality, the tariff is set by the utility periodically and the cost of PV declines. The evolution of the actual installed PV capacity in each period depends not only on the tariff and cost in that period but also on those in previous periods. In other words, the installed PV capacity has to be calculated using not a single but a collection of such S-curves. The dynamics of PV capacity evolution is presented in Section III.

III. Dynamics and Stability of PV Diffusion

A. Dynamics of PV Diffusion

We now introduce a discrete-time dynamical system model for the PV diffusion process where the time index k corresponds to the decision epoch of the retailer. The state $\sigma_k = (T_k, R_k)$ of the dynamic system includes the tariff T_k set by the retailer at the beginning of the tariff period and the installed PV capacity R_k at the end of the tariff period. The evolution of the system state is governed by the system equation

$$\sigma_{k+1} = f(\sigma_k, \chi_k), \quad (6)$$

where $\chi_k = (\theta_k, \xi_k)$ is the exogenous (input) process containing the expected operating cost θ_k and the per-unit purchasing cost of PV ξ_k. In analyzing the stability of the diffusion process, we set the exogenous input to constant, $\chi_k = \chi$.

*A large retail utility, strictly speaking, can influence the wholesale price of electricity.

†For simplicity, we assume the wholesale price is a scaler random variable for ease of presentation. A more accurate model is to treat the wholesale price as a random process at a minute level time scale. See [20].

‡θ includes only the fixed operating cost.

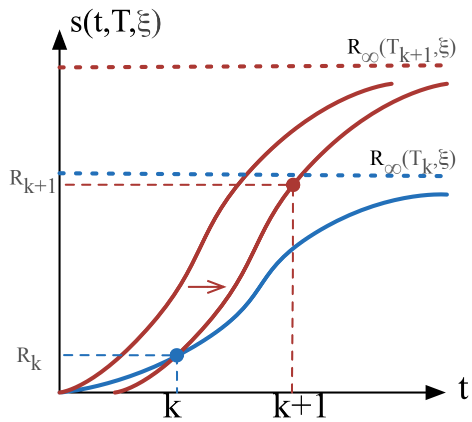

Fig. 2: Dynamics of renewable diffusion when $R_\infty(T_{k+1},\xi) \geq R_k$.

The exogenous input can be time varying when we consider *controlled diffusion* that sets tariff in response to varying costs.

The state evolution is Markovian following $R_k \rightarrow T_{k+1} \rightarrow R_{k+1}$. Specifically, $f(\cdot,\cdot)$ is specified by the tariff policy μ and the PV diffusion curve $s(t,T,\xi)$ as follows:

$$T_{k+1} = \underset{T\in\mathscr{T},\overline{\mathrm{rs}}(T,\theta,R_k)=0}{\arg\max} \overline{\mathrm{cs}}(T,R_k), \tag{7}$$

$$R_{k+1} = \begin{cases} R_k, & \text{if } R_\infty(T_{k+1},\xi_{k+1}) < R_k; \\ s(1+\eta^{-1}(\frac{R_k}{R_\infty(T_{k+1})}),T_{k+1},\xi_{k+1}), & \text{o.w.} \end{cases} \tag{8}$$

Note that, at the beginning of period $k+1$, the installed PV capacity is R_k. The installed capacity R_{k+1} at the end of the period $k+1$ is obtained from the diffusion curve associated with T_{k+1} by $s(t_k+1,T_{k+1},\xi_{k+1})$ where t_k is such that $s(t_k,T_{k+1},\xi_{k+1}) = R_k$. See Fig. 2 for an illustration of (8) when the installed PV capacity is no more than the market potential. The case that $R_k > R_\infty(T_{k+1},\xi_{k+1})$ usually happens only when there is an exogenous shock in the system.

B. Death Spiral and its Existence Conditions

The notion of death spiral is associated with the trajectory of a dynamic system defined through the tariff policy μ and the diffusion curve.

Definition 1 (Death spiral and critical diffusion level)**.** *An orbit of the dynamic system (7-8) starting from σ_0 is a death spiral induced by tariff policy μ if it ends at a state σ_{k_o} for which the optimization (7) to determine T_{k_o+1} is not feasible. The critical diffusion level $R^\sharp_\mu$ is the supremum of R at which a revenue adequate tariff exists*

$$R^\sharp_\mu = \sup\{R : \overline{\mathrm{rs}}(\mu(R,\theta),\theta,R) = 0\}. \tag{9}$$

We now focus on establishing existing conditions of death spiral. In this analysis, we assume exogenous parameters $\chi = (\theta,\xi)$ are fixed. For brevity, we drop notational dependencies on θ,ξ, and χ.

Our analysis rely on the characterization of the *potential function* defined as follows.

Definition 2 (Potential function)**.** *Given a tariff policy μ, The potential function at diffusion level R is defined as*

$$p_\mu(R) = R_\infty(\mu(R)). \tag{10}$$

The potential function serves as a surrogate for the more complicated iterative map f. Being the limiting installation capacity on the diffusion curve, $p_\mu(R)$ measures the headroom beyond the current installation capacity R.

The existence condition for death spiral is illustrated in Fig. 3. It states that the gap between $p(R)$ and R is strictly positive in the left neighborhood of the critical diffusion $R^\sharp$.

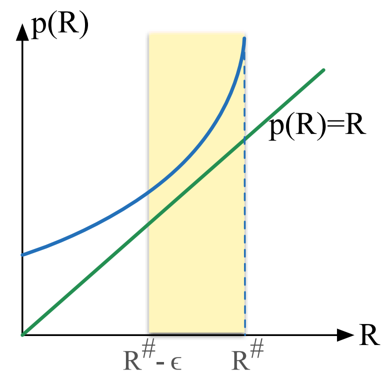

Fig. 3: Condition for death spiral.

Theorem 1 (Existence condition of death spiral)**.** *Given an initial state σ_0 with $R_0 < R^\sharp$, a tariff policy μ generates a death spiral if there exists an $\epsilon > 0$ such that*

- *$R_{k_0} \in (R^\sharp - \epsilon, R^\sharp]$ for some $k_0 \geq 0$;*
- *$p(R) > R$ for all $R \in (R^\sharp - \epsilon, R^\sharp]$.*

The condition is necessary if $p(R)$ is monotonically increasing.

Theorem 1 provides a way to check, at least numerically, the possibility of death spiral. The following theorem gives the precise condition for the Ramsey linear tariff to induce death spiral.

Theorem 2 (Death spiral condition for Ramsey tariff)**.** *For the Ramsey linear tariff μ^*_{L}, there exists a threshold $\theta^\dagger$ such that a retailer cost $\theta > \theta^\dagger$ induces a death spiral. In particular, if consumers' demand function is affine with negative slop and random disturbance, i.e., $D(\pi,\omega) = B(\omega) - G\pi$, where $B(\omega)$ is the additive disturbance and G positive, then*

$$\begin{aligned}\theta^\dagger = \tfrac{1}{4G}[&b(R^\dagger)^2 - 4G\mathbb{E}[\lambda(B(\omega) - R^\dagger r_0(\omega))] \\ &- (b(R^\dagger) + 2GR^{-1}_\infty(R^\dagger))^2],\end{aligned} \tag{11}$$

where $R^\dagger$ is characterized by

$$-\frac{dR^{-1}_\infty(R^\dagger)}{dR} = \frac{R^{-1}_\infty(R^\dagger)\bar{r}_0 - \mathbb{E}[\lambda r_0(\omega)]}{b(R^\dagger) + 2GR^{-1}_\infty(R^\dagger)}, \tag{12}$$

and $b(R) = -G\bar{\lambda} - (\mathbb{E}[B(\omega)] - R\bar{r}_0)$.

C. Stable Diffusion

Death spiral is a form of instability. We now consider conditions for *stable diffusion*. The exogenous parameters are again fixed and ignored in our notations in this subsection.

We begin with standard definitions of the equilibrium and stable equilibrium.

Definition 3 (Stable equilibrium and stable diffusion)**.**

1) *A state σ^* is an equilibrium if $\sigma^* = f(\sigma^*)$.*
2) *An equilibrium σ^* is Lyapunov stable if, for each $\epsilon > 0$, there exists a $\delta = \delta(\epsilon)$ such that, for every trajectory $(\sigma_0, \sigma_1, \cdots)$ that is not a death spiral, $\|\sigma_0 - \sigma^*\| < \delta$ implies $\|\sigma_k - \sigma^*\| < \epsilon$ for all $k > 0$.*
3) *A trajectory $(\sigma_0, \sigma_1, \cdots)$ is a stable diffusion if it converges to a stable equilibrium.*

Lemma 1 (Existence of equilibrium)**.** *Given a tariff policy μ, if there exists an R^* such that $p(R^*) = R^*$, then $\sigma^* = (\mu(R^*), R^*)$ is an equilibrium.*

This condition is intuitive; it states the case when current level of installed PV capacity R already reaches $R_\infty(\mu(R))$.

Theorem 3 (Stability condition and convergence)**.** *Given a tariff policy μ, an equilibrium $\sigma^* = (T^*, R^*)$ is Lyapunov stable if there exists an ϵ such that $R < p(R) \leq R^*$ for all $R \in (R^* - \epsilon, R^*)$, and $p(R) \leq R$ for all $R \in (R^*, R^* + \epsilon)$. If in addition that $R_0 \in (R^* - \epsilon, R^*)$ for an initial state $\sigma_0 = (T_0, R_0)$, then $\lim_{k\to\infty} \sigma_k = \sigma^*$.*

A graphical illustration of Theorem 3 is given in Fig. 4.

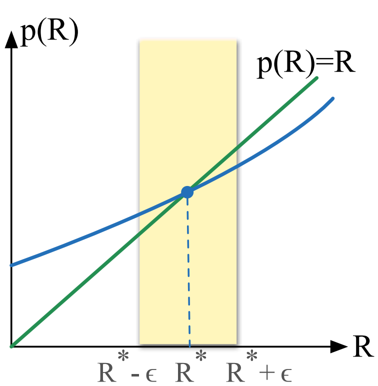

Fig. 4: Condition for stability.

Theorem 4 (Stable diffusion via Ramsey two-part tariff)**.** *For an initial state $\sigma_0 = (T_0, R_0)$ with $0 \leq R_0 \leq p(R_0)$, the Ramsey two-part tariff μ^*_{2P} induces a diffusion approaching to the unique stable equilibrium $(\mu(p(R_0)), p(R_0))$.*

D. Limiting Diffusion Capacity

In this subsection, we are interested in finding the highest level of PV diffusion $R^\dagger$ achievable by a stable diffusion. The following definition formalizes the notion of limiting diffusion capacity.

Definition 4 (Limiting diffusion capacity)**.** *The limiting diffusion capacity is the supremum of the equilibria achievable by stable diffusions with initial installation $R_0 = 0$.*

The following theorem provides a tariff policy that achieves the limiting diffusion capacity.

Theorem 5 (Achieving limiting capacity)**.** *If the Ramsey linear tariff induces a death spiral, the limiting capacity is achieved by the two-part tariff that adds the minimum (fixed) connection charge so that there is a stable diffusion.*

For the linear demand model defined in Theorem 2, it can be shown that the limiting capacity is $R^\dagger$ and the fixed connection charge that achieves the limiting diffusion capacity is given by $A^\dagger = (\theta - \theta^\dagger)/M$.

IV. Numerical Examples

In this section, we analyze renewable diffusion dynamics in both short-run and long-run cases within a hypothetical distribution utility facing the wholesale price in New York city and its residential demand. The same setting of demand model, consumption profile, revenue estimation, and solar PV data is used as in [15], [16].

The default tariff of the Consolidated Edison Company of New York (ConEd) in 2015 for its 2.2 million residential customers is a two-part tariff T^{CE} with a flat rate $\pi^{\text{CE}} = \$0.172/\text{kWh}$ and a connection charge $A^{\text{CE}} = \$0.52/\text{day}$. We use this tariff to compute the utility's fixed costs, which amount to $\theta^{\text{CE}} = \$6.03\text{M}$. A consumer surplus of $\overline{\text{cs}}_0(T^{\text{CE}}) = \$9.54\text{M}$ is assumed.

The integration of solar PV is modeled based on a simulated 5kW-DC-capacity rooftop system in NYC. The market potential R_∞ is computed based on the expected payback years t^{PB} at the time of purchasing, with $t^{\text{PB}} = \xi/\mathbb{E}[\pi^\top r_0(\omega)]$. We take the solar PV cost of NYC in 2015 as the initial solar cost $\xi_0 = \$4250/\text{kW}^\$$. An exponential fit in [21] is adopted in calculating market potential: $R_\infty = R^{\text{MS}} \cdot e^{-0.3t^{\text{PB}}}$. As in [23], the total market size R^{MS} is set to be 90% of all customers installing, and $\eta(t)$ is set to model a medium-rate adoption using the Bass-diffusion model.

A. Short-run Analysis

This is the case where exogenous parameters including the retailer's cost and the solar cost are fixed when considering one trajectory of dynamics.

We illustrate in Fig. 5 the curves of potential functions under different tariff policies. For each tariff class, the potential function is increasing on solar capacity (The potential function of Ramsey two-part tariff is horizontal). The diffusion equilibrium of the Ramsey two-part tariff $\mu^*_{2\text{P}}$ is almost at 0, which stalls the solar diffusion (green curve). This stalling diffusion is due to the low retail rate under such tariff policy, leading to a long payback time. The Ramsey two-part tariff with a fixed connection charge A^{CE} (as currently used by ConEd), induces a stable equilibrium at 97.7MW (brown curve).

If we increase the retailer cost to \$6.65M (around 10%) and take the Ramsey linear tariff, the new tariff policy μ^*_{L} induces a death spiral (blue curve). If a connection charge $A \geq \$0.088/\text{day}$ is introduced, however, the diffusion can stay off death spiral and achieve a stable equilibrium. Moreover, if we adopt the critical connection charge $A^\dagger = \$0.088/\text{day}$, the limiting capacity $R^\dagger = 698.5\text{MW}$ is achieved (magenta curve).

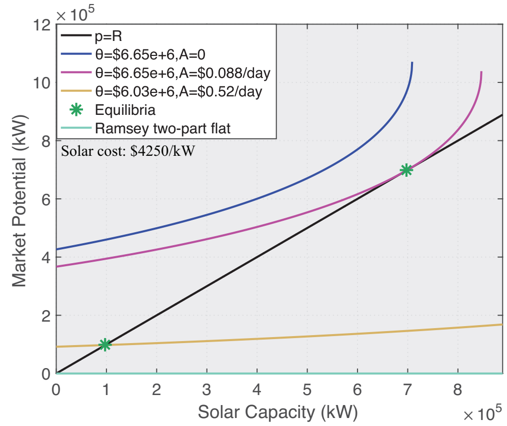

Fig. 5: Potential function in short run analysis.

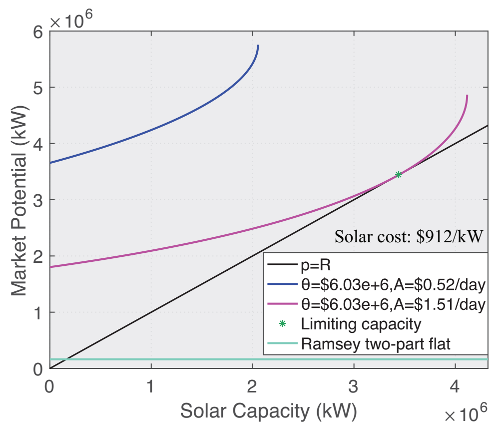

Fig. 6: Potential functions of Ramsey two-part tariff $\mu_{2\mathrm{P}}^*$, linear tariff μ_{L}^*, and Ramsey two-part tariff with fixed connection charge

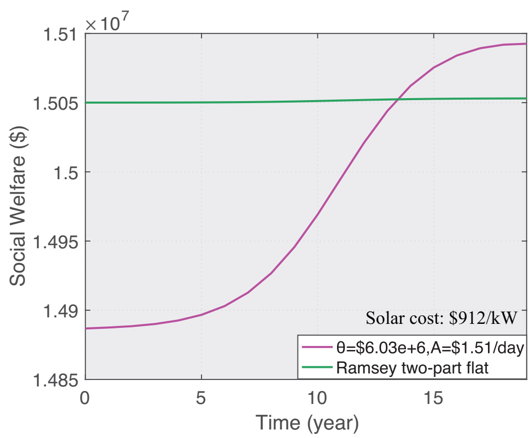

Fig. 7: Social welfare induced by Ramsey two-part tariff $\mu_{2\mathrm{P}}^*$ and two-part tariff with fixed connection charge

Similar potential functions figures with a different solar cost can be obtained, as shown in Fig. 6. If we still adopt fixed connection charge A^{CE} in this case, the death spiral is induced because the payback time becomes shorter with a lower solar

[§]The solar cost data in New York State starting from 2009 can be found at https://www.nysolarmap.com/

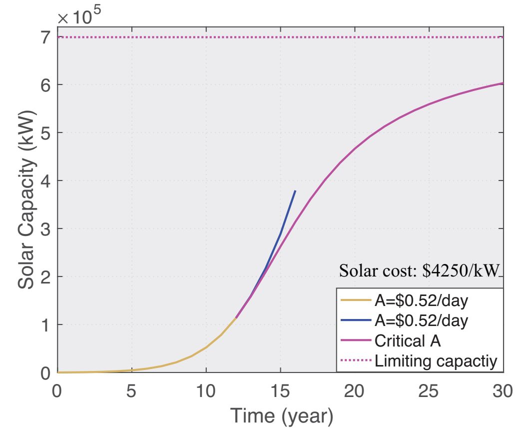

Fig. 8: Long-run solar diffusion with retailer cost increasing by 2% every year from θ^{CE}

cost. By adding a connection charge, the limiting capacity diffusion can be achieved as well (magenta curve).

Fig. 7 compares the dynamics of social welfare induced by a Ramsey two-part tariff with fixed connection charge (A=$1.51/day) and $\mu_{2\mathrm{P}}^*$ facing fixed solar cost $912/kW and retailer cost θ^{CE}. $\mu_{2\mathrm{P}}^*$ optimizes social welfare at each period, but almost stalls the solar diffusion. The social welfare induced by $\mu_{2\mathrm{P}}^*$ thus has a slow growth. Under the Ramsey two-part tariff with fixed connection charge, the social welfare is low at first but eventually becomes higher than under $\mu_{2\mathrm{P}}^*$ due to a higher solar installation. This comparison indicates that there exists some trade-off between adding connection charges and integrating more renewables for the long-run social welfare optimization. The Ramsey two-part tariff $\mu_{2\mathrm{P}}^*$, which maximizes the social welfare greedily, is not the optimal choice for social welfare maximization in the long run.

B. Long-run Analysis

We plot in Fig. 8 and Fig. 9 the long-run solar diffusion dynamics under an increasing process of retailer cost and a decreasing process of solar cost respectively. It is shown that both exogenous processes induce death spiral (brown→blue). Adding critical connection charges, however, can stay off the death spiral and achieve a stable diffusion (brown→magenta). Moreover, while introducing critical connection charges lowers the speed of solar integration, the diffusion capacity in long run is higher compared with the fixed connection charge case, which generates a death spiral and then stalls solar diffusion.

V. Conclusion

In addressing the death spiral hypothesis, we have proposed an analytical framework based on a dynamical system model for the PV diffusion process. One conclusion is that linear tariffs in general are prone to death spiral when the fixed cost of the utility rises beyond a certain level. More importantly, our model allows one to estimate the time when critical installation level is reached and death spiral is imminent. Another conclusion is that adding a small connection charge not only can stop death spiral but also stimulates PV adoption. In contrast, the Ramsey pricing, although guaranteeing a stable

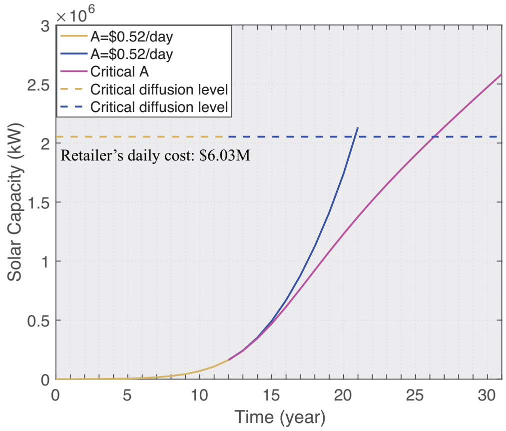

Fig. 9: Long-run solar diffusion with solar cost decreasing by 5% every year from ξ_0

PV diffusion and higher short run social welfare, stalls PV adoption and has lower long run social welfare. Our model also suggests a simple strategy that achieves the limiting PV adoption and a high level of long run social welfare.

We have assumed a simple flat tariff model. A more relevant tariff structure is the dynamic tariff that has the volumetric charge varying with the time of use. Many of these results can be extended. See [20].

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APPENDIX

Proposition 1. *For a trajectory* $(\sigma_0, \sigma_1, \cdots)$, *if* $p(R_k) > R_k$, *we have* $R_k < R_{k+1} = h(R_k, \mu(R_k, \theta)) < p(R_k)$.

Proof:. It directly holds from Equation (8). □

Proposition 2. *If there exists an* ϵ *such that* $R < p(R) \leq R^*$ *for all* $R \in (R^* - \epsilon, R^*)$ *with* $p(R^*) = R^*$, *for each* $R_0 \in (R^* - \epsilon, R^*)$, *we have* $\lim\limits_{k\to\infty} R_k = R^*$.

Proof:. Leveraging Proposition 1, $\{R_t\}$ is strictly increasing and bounded by R^*. Suppose $\{R_t\}$ converges to $R' \in (R_0, R^*)$. It can be induced that $h(R', \mu(R', \theta)) = R'$. As $p(R') > R'$, there is a contradiction with Proposition 1. Hence $\{R_k\}$ must converge to R^* (Monotone convergence theorem). □

Proof of Theorem 1. Sufficiency $\Rightarrow$: Leveraging Proposition 1, $\{R_k\}$ is monotonically increasing. Suppose $R^\sharp$ is an upper bound of $\{R_k\}$. Thus there exists an $R' \in (R_{k0}, R^\sharp]$ such that $\{R_k\}$ converges to R' (Monotone convergence theorem). Hence, $h(R', \mu(R', \theta)) = R'$. As $p(R') > R'$, there is a contradiction with Proposition 1. Thus $R^\sharp$ is not an upper bound of $\{R_k\}$, indicating that the death spiral occurs.

If $p(R)$ is monotonically increasing,

Necessity $\Leftarrow$: Since a death spiral is induced, there must exist $R_0 \leq R_{k1} < R^\sharp$ such that $p(R_{k1}) > R^\sharp$ (Otherwise $R_{k+1} < p(R_k) \leq R^\sharp$ for all k, indicating there is no death spiral). Moreover, as $p(R)$ is monotonically increasing, $p(R) > p(R_{k1}) > R^\sharp > R$ holds for $R \in (R_{k1}, R^\sharp]$. Thus the necessity is proved. □

Assumption 1. The Ramsey linear tariff π^* is monotonically increasing on the retailer cost θ and on the PV capacity R. The market potential $R_\infty(\pi)$ is strictly increasing convex on π.

Proposition 3. *The critical diffusion level* $R^\sharp$ *is such that, for Ramsey linear tariff* $\pi^* = \mu^*(R^\sharp, \theta)$, $\pi^* = \arg\max\limits_{\pi} \overline{\text{rs}}(T, \theta, R^\sharp)$. *Denote* $\overline{\text{rs}}^{\text{M}}(T, \theta, R^\sharp) = \max\limits_{\pi} \overline{\text{rs}}(T, \theta, R^\sharp)$.

Proof:. If there exists a π_0 such that $\overline{\mathrm{rs}}(T_0,\theta,R^\sharp) > \overline{\mathrm{rs}}(T^*,\theta,R^\sharp) = 0$, there must exist $R' > R^\sharp$ satisfying $\overline{\mathrm{rs}}(T_0,\theta,R') > 0$ due to the continuity. Thus a contradiction is induced with the definition of critical diffusion level. □

Proposition 4. *The critical diffusion level $R^\sharp$ is monotonically decreasing on the retailer cost θ.*

Proof:. Leveraging Proposition 3, for a retailer cost θ_1 and the corresponding critical diffusion level $R_1^\sharp$, we have $\overline{\mathrm{rs}}^{\mathrm{M}}(T,\theta_1,R_1^\sharp) = 0$. Hence, with the expression of $\overline{\mathrm{rs}}$ in (3), we have $\overline{\mathrm{rs}}^{\mathrm{M}}(T,\theta_2,R_1^\sharp) < 0$ for all $\theta_2 > \theta_1$. Thus $R_2^\sharp < R_1^\sharp$. □

Assumption 2. The optimized linear tariff policy is such that $\pi^*(0,\theta)\bar{r}_0 - \mathbb{E}[\lambda r_0(\omega)] > 0$.

Proposition 5. *For Ramsey linear tariff with the linear demand model $D(\pi,\omega) = B(\omega) - G\pi$, The potential function $p(R,\theta)$ is strictly increasing and convex on θ and on R.*

Proof:. Solving (4) yields

$$\pi^*(R,\theta) = \frac{-b(R) - \sqrt{b(R)^2 - 4ac(R,\theta)}}{2a} \tag{13}$$

where $a = G$, $b(R) = -\overline{\lambda}G - (\mathbb{E}[B(\omega)] - R\bar{r}_0)$, and $c(R,\theta) = \theta + \mathbb{E}[\lambda(B(\omega) - Rr_0(\omega))]$.

a) On θ: $p(R,\theta) = R_\infty(\pi^*(R,\theta))$. Since we have assumed $R_\infty(\pi)$ to be strictly increasing and convex in Assumption (1), we only need to prove $\pi^*(R,\theta)$ is strictly increasing and convex on θ. Differentiating twice $\pi^*(R,\theta)$ with respect to θ we have

$$\begin{aligned} \tfrac{d\pi^*}{d\theta} &= \tfrac{-1}{2a}(-4a)\cdot\tfrac{1}{\sqrt{b(R)^2-4ac(R,\theta)}} \\ &= \tfrac{2}{\sqrt{b^2-4ac}} > 0 \end{aligned} \tag{14}$$

$$\frac{d\pi^*}{d\theta^2} = \frac{2a}{\sqrt{b^2-4ac}(b^2-4ac)} \tag{15}$$

Since $a = G$ and G positive, $\frac{d\pi^*}{d\theta^2} \geq 0$. Since we have assumed $R_\infty(\pi)$ to be strictly increasing and convex, $p(R) = R_\infty(\pi^*(R))$ is strictly increasing and convex on θ.

b) On R: differentiating $\pi^*(R,\theta)$ with respect to R we have

$$\begin{aligned} \pi^*(R)' &= \tfrac{1}{2a}[-b' - (bb' - 2ac')\cdot\tfrac{1}{\sqrt{b^2-4ac}}] \\ &= \tfrac{1}{2a\sqrt{b^2-4ac}}[2ac' - bb' - b'\sqrt{b^2-4ac}] \\ &= \tfrac{1}{\sqrt{b^2-4ac}}[\tfrac{(-b-\sqrt{b^2-4ac})}{2a}b' + c'] = \tfrac{1}{\sqrt{b^2-4ac}}(\pi^*(R)b' + c') \end{aligned} \tag{16}$$

where: $b' = \bar{r}_0$ and $c' = -\mathbb{E}[\lambda r_0(\omega)]$.

Since $\pi^*(0)\bar{r}_0 - \mathbb{E}[\lambda r_0(\omega)] > 0$ (Assumption 2), one can iteratively induct that $\pi^*(R)b' + c' = \pi^*(R)\bar{r}_0 - \mathbb{E}[\lambda r_0(\omega)] > 0$. Thus $\pi^*(R)' > 0$ holds. We differentiate twice $\pi^*(R)$

$$\pi^*(R)'' = \frac{1}{2a}\frac{(b'b - 2ac')^2 - (b^2-4ac)b'^2}{\sqrt{b^2-4ac}(b^2-4ac)} \tag{17}$$

With $\pi^*(R)' > 0$, we have $2ac' - bb' - b'\sqrt{b^2-4ac} > 0$, which yields $(b'b-2ac')^2 - (b^2-4ac)b'^2 > 0$. Thus $\pi^*(R)'' > 0$ holds, which means $\pi^*(R)$ is increasing and convex on $R \in [0, R^\sharp]$.

Since we have assumed $R_\infty(\pi)$ to be strictly increasing and convex, $p(R) = R_\infty(\pi^*(R))$ is strictly increasing and convex on R.

□

Proof of Theorem 2. With Assumption 1, for $R_0 = 0$, A θ' inducing a death spiral means $p(R,\theta') > R$ for all $R \in [R_0, R^\sharp|_{\theta'}]$. It can be also inferred from Assumption 1 that $p(R,\theta)$ monotonically increasing on π. Hence, for all $\theta^\diamond > \theta'$, leveraging Proposition 4, we have $p(R,\theta^\diamond) > R$ for all $R \in [R_0, R^\sharp|_{\theta^\diamond}]$. According to Theorem 1, a death spiral still occurs.

For a linear demand model:

We look for the infimum of such θ that induces a death spiral, denoted by $\theta^\dagger$. With Proposition 2 and 5, this $\theta^\dagger$ is specified when potential function $p(R,\theta)$ is tangent to $p = R$, or when $\pi^*(R)$ tangent to $R_\infty^{-1}(R)$. Thus the tangent point can be specified by

$$\begin{cases} \pi^*(R)' - R_\infty^{-1}(R)' = 0 \\ \pi^*(R) - R_\infty^{-1}(R) = 0 \end{cases} \tag{18}$$

Further deduction of the first equation yields

$$\begin{aligned} &\pi^*(R)' - R_\infty^{-1}(R)' = \tfrac{1}{\sqrt{b^2-4ac}}(\pi^*(R)b' + c') - R_\infty^{-1}(R)' \\ &= \tfrac{1}{\sqrt{b^2-4ac}}(R_\infty^{-1}(R)b' + c') - R_\infty^{-1}(R)' \end{aligned} \tag{19}$$

Reformulate (19) as

$$\theta = \frac{1}{4a}[b^2 - 4ac_0 - (\frac{R_\infty^{-1}(R)b' + c'}{R_\infty^{-1}(R)'})^2] \tag{20}$$

where $c_0 = \mathbb{E}[\lambda(B(\omega) - Rr_0(\omega))]$. Reformulating the second equation in (18) yields

$$\theta = \frac{1}{4a}[b^2 - 4ac_0 - (b + 2aR_\infty^{-1}(R))^2] \tag{21}$$

With (20) and (21), we can solve $R^\dagger$ which is characterized by

$$-\frac{dR_\infty^{-1}(R^\dagger)}{dR} = \frac{R_\infty^{-1}(R^\dagger)\bar{r}_0 - \mathbb{E}[\lambda r_0(\omega)]}{b + 2aR_\infty^{-1}(R^\dagger)} \tag{22}$$

Substituting $R^\dagger$ into (21) we have

$$\begin{aligned} \theta^\dagger = &\tfrac{1}{4a}[b(R^\dagger)^2 - 4a\mathbb{E}[\lambda(B(\omega) - R^\dagger r_0(\omega))] \\ &-(b(R^\dagger) + 2aR_\infty^{-1}(R^\dagger))^2], \end{aligned} \tag{23}$$

□

Proof of Lemma 1. $f(\sigma^*,\chi) = \sigma^*$ directly holds by computing the dynamics in Equation 7 and Equation (8) □

Proof of Theorem 3. We prove the convergence and stability respectively:

a) convergence: leveraging Proposition 2, it is clear that $\lim\limits_{k\to\infty} \sigma_k = \sigma^*$

b) Lyapunov stable:

For $\|\sigma_0 - \sigma^*\| < \delta$ with $\delta > 0$, we first look for an upper bound of deviations from σ^* induced by all trajectories $(\sigma_0, \sigma_1, \cdots)$. Denote $\Delta R_{\mathrm{M}}(\delta) = \delta$, and $\Delta T_{\mathrm{M}}(\delta) = \max\limits_{R^*-\delta\leq R\leq R^*+\delta} \|\mu(R) - T^*\|$. Thus $g(\delta) = \|(\Delta R_M(\delta), \Delta T_M(\delta))\|$ is one of such upper bounds. Note that $g(\delta)$ is monotonically increasing for $\delta \in (0,\epsilon]$ and $\lim\limits_{\delta\to 0} g(\delta) = 0$.

Hence for all $0 < \epsilon' \leq g(\epsilon)$, there exists $\delta_1 = g^{-1}(\epsilon')$ such that $\|\sigma_0 - \sigma^*\| < \delta_1$ implies $\|\sigma_k - \sigma^*\| < g(\delta_1) = \epsilon'$ for all $k > 0$. For $\epsilon' > g(\epsilon)$, it is clear.

□

Assumption 3. *$\nabla_\pi(\pi, \omega)$ and λ are uncorrelated.*

Proof of Theorem 4. With Assumption 3, for Ramsey two-part tariff, the solution of (4) has the following expression for volumetric charge

$$\pi^\dagger(R, \theta) = \frac{\overline{\lambda}\mathbb{E}[\partial D/\partial \pi]}{\mathbb{E}[\partial D/\partial \pi]} \tag{24}$$

Expression (24) reveals that the flat rate of Ramsey two-part tariff only depend on the wholesale market prices and the demand function, thus stays unchanged with renewable diffusion. The potential function $p(R)$ thus also has the same value for different R. Utilizing Theorem 3, this tariff policy always induces a stable equilibrium. The equilibrium capacity is determined by the market potential facing tariff $\pi^\dagger$. □

Proof of Theorem 5. Leveraging the expression of the retailer surplus in 3, for the Ramsey tariff design problem in 4, the solution for decreasing the retailer cost by $\Delta\theta$ is the same as increasing the connection charge by $\Delta A = \Delta\theta / M$.

Then this theorem directly follows from Theorem 2.

□