Photon noise and correlations in semiconductor cascade lasers

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A comprehensive model for photon noise and correlations in semiconductor cascade lasers is presented. Photon emission events in different gain sections of cascade lasers are correlated. These correlations are found to be positive and arise because the gain sections are connected electrically. The scaling of photon correlations and intensity noise with the number of cascade sections is discussed. The model presented in this letter is applicable to a variety of cascade laser structures including bipolar interband cascade lasers and unipolar intersubband cascade lasers. For comparison, photon noise and correlations in parallel lasers arrays are also discussed. © 2000 American Institute of Physics. [S0003-6951(00)03109-0]

In semiconductor lasers, each electron (or hole) injected into the device emits at best a single photon. In semiconductor cascade lasers, electrons (or holes) are recycled from one gain stage to the next such that each carrier injected into the device is capable of producing as many photons as the number of cascaded gain stages. The slope efficiency (W/A) of cascade lasers increases in proportion to the number of cascaded gain sections and can therefore be much in excess of the conventional limit $\hbar \omega/e$. Since the signal-to-noise ratio (SNR) in optical links under direct laser modulation is proportional to the square of the laser slope efficiency, cascade lasers can play an important role in optical communication systems.¹ Various types of semiconductor cascade lasers have been realized. The devices, which will be focused upon in this letter, are shown schematically in Fig. 1. Interconnect coupled cascade lasers (either integrated or discrete)¹⁻³ fall in categories A and B, and unipolar intersubband cascade lasers⁴ and bipolar interband cascade lasers⁵ belong to category A. A parallel laser array⁶ is shown in category C. A property common to all these laser devices is that the gain sections are all connected electrically. As a result, the carrier density fluctuations and photon emission events in different gain sections become correlated.

In this letter, a detailed model for describing photon noise and correlations in semiconductor cascade lasers is presented. Experimental results on photon correlations in series cascades and parallel arrays of light emitting devices have been reported in Ref. 7. A theoretical model is needed to explain these results and also to evaluate the performance of different types of cascade lasers in optical links.^{1–3} The relative intensity noise (RIN) of cascade lasers is found to be influenced by the correlated nature of photon emission from the different gain sections. These correlations in photon emission are found to be positive in cascade lasers and negative in parallel laser arrays. The scaling of photon correlations, relative intensity noise (RIN), and the Fano factor (F) with the number of cascade sections is also discussed.

The model presented in this paper consists of a system of coupled Langevin rate equations for fluctuations in the photon density and in the carrier densities in different energy levels of a gain section. Fluctuations in the carrier densities are also coupled to the fluctuations in the current pumping the gain section. These current fluctuations are responsible for correlating fluctuations in the carrier density and also in the photon emission events in different gain sections. Although in this letter equations will be presented only for interband lasers, all the essential ideas are easily carried over to intersubband quantum cascade lasers.⁸ For interband lasers with *N* different gain sections, using the notation of Ref. 9, photon density fluctuations δN_P^{α} in the α th optical cavity, carrier density fluctuations δN_W^{β} in the barriers and cladding regions in the *j*th gain section can be described by linearized coupled Langevin rate equations,

$$\frac{d \,\delta N_B^j}{d \,t} = \frac{\delta I^j}{q \,V_B} - \delta N_B^j \left(\frac{1}{\tau_C} + \frac{1}{\tau_L}\right) + \frac{\delta N_W^j}{\tau_E} \left(\frac{V_W}{V_B}\right) + F_E^j \left(\frac{V_W}{V_B}\right) - F_C^j - F_L^j, \tag{1}$$



FIG. 1. Cascade laser devices and parallel laser arrays. Category A has cascaded gain sections inside a single optical cavity. Category B has cascaded gain sections inside separate optical cavities. Category C is a parallel array of lasers.

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$$\frac{d \,\delta N_W^j}{d \,t} = \frac{\delta N_B^j}{\tau_C} \left(\frac{V_B}{V_W}\right) - \frac{\delta N_W^j}{\tau_E} - \gamma_{NN} \,\delta N_W^j - \gamma_{NP} \,\delta N_P^\alpha + F_C^j \left(\frac{V_B}{V_W}\right) - F_E^j + F_N^j, \tag{2}$$

$$\frac{d \,\delta N_P^{\alpha}}{d t} = \gamma_{PN} \sum_{j=1}^N C^{\alpha j} \delta N_W^j - \gamma_{PP} \,\delta N_P^{\alpha} + F_P^{\alpha} \,. \tag{3}$$

 τ_C and τ_E are quantum well capture and escape times, and τ_L is the lifetime associated with carrier leakage. V_W and V_B are the volumes of the well and cladding regions. Coefficients γ_{NN} , γ_{PP} , γ_{NP} , and γ_{PN} are described in detail in Ref. 9. F_C^j , F_E^j , and F_L^j are Langevin sources which describe the noise associated with carrier capture, escape, and leakage processes, respectively. F_N^j is the Langevin noise source for radiative and nonradiative carrier generation and recombination in the quantum wells. F_P^{α} is the Langevin source which models the noise in stimulated and spontaneous photon emission and absorption and photon loss from the cavity. In cascade devices, the same current flows through all the gain sections and index j is not needed for δI^{j} in Eq. (1). For single cavity devices, index α is not needed in Eqs. (2) and (3), and $C^{\alpha j} = 1$. For multiple cavity devices, $C^{\alpha j} = 1$ only if the *j*th gain section is inside the α th optical cavity. The fluctuations in current δI^{j} , and voltage δV^{j} , for the *j*th gain section are related,

$$\delta I^{j} = q V_{B} \left(F_{inj}^{j} - \frac{\delta N_{B}^{j}}{\tau_{G}} \right) + G \ \delta V^{j}. \tag{4}$$

 F_{inj}^{j} is the Langevin noise source associated with carrier injection into the active region. The second term inside the parenthesis in the above equation is the decrease in carrier injection rate as a result of increase in the carrier density δN_B^j . The last term is the increase in injection current with the increase in voltage across the active region. The fluctuations δP in the total output power are related to δN_P^{α} ,

$$\delta P = \sum_{\alpha} \delta P^{\alpha} = \sum_{\alpha} \left(\eta_o \hbar \omega \frac{V_P \delta N_P^{\alpha}}{\tau_P} + F_o^{\alpha} \right).$$
 (5)

 V_P is the volume of the optical cavity. τ_p is the photon lifetime inside the optical cavity, and η_o is the output coupling efficiency.⁹ The Langevin source F_o^{α} is included to take into account photon partition noise at the output facet.9 Nonzero correlations for all the Langevin noise sources introduced in Eqs. (1)-(5) can be deduced from the methods described in Ref. 9.

For all numerical simulations, we have used 1.55 μ m InP Fabry-Perot lasers in which each gain section has five quantum wells. Each laser cavity is 400 μ m long and 2 μ m wide with reflectivities of 0.3 and 0.9 at the two facets, and internal loss of 5 cm^{-1} . It is assumed that each gain section has a series contact resistance of 3 Ω . The power supply is assumed to be a noiseless current source with a parallel resistance R_S (=50 Ω unless stated otherwise). All circuit resistances generate thermal noise. The value of G in Eq. (4) is chosen such that the differential resistance R_D of a gain section at high bias is close to 0.5 Ω . For bipolar cascade lasers, in which gain sections are coupled via back diodes,⁵ the differential resistance of a back diode is assumed to be 1 Ω .⁵ A



FIG. 2. Normalized photon intensity correlation function (C_p) as a function of bias for multiple cavity cascade lasers (MCCL) and parallel laser arrays (PLA).

back diode is assumed to generate full shot noise. Values assumed for τ_C , τ_E , τ_G , and τ_L are 50, 100, 50, and 150 ps, respectively.

Figure 2 shows the low frequency normalized power cross correlation $C_P(N) = \langle \delta P^{\alpha} \delta P^{\beta} \rangle / \sqrt{\langle \delta P^{\alpha^2} \rangle \langle \delta P^{\beta^2} \rangle}$ between light output from two different cavities in multiple cavity cascade lasers (category B) and parallel laser arrays (category C). C_P is found to be positive for cascade lasers and negative for parallel laser arrays in agreement with the experimental results in Ref. 7. In cascade lasers, a positive value of C_P results from the relaxation currents which flow in the external circuit in response to fluctuations in the carrier density inside the gain sections and also from the thermal noise currents originating in circuit resistances. For example, a photon emission event in one section of the cascade will cause the carrier density in that section to fall below the average value. In order to restore the carrier population, a relaxation current will flow in the external circuit. This relaxation current will tend to increase the photon generation rate in other sections. Thus, a photon emission in one section of the cascade will increase the probability of photon emission in other sections. Similarly, noise currents from thermal sources will also tend to positively correlate the photon emission events in different sections since the same noise current will flow through all the sections in series. In a parallel array of lasers C_P is negative. In this case, the relaxation current, which flows in the external circuit following a photon emission event in one section of the array, decreases the carrier density and, consequently, the photon generation rate in other sections of the array. Therefore, photon emission in one section of the array inhibits photon emission in other sections of the array. Thermal noise sources behave a little differently in a parallel array. Thermal noise originating in the contact resistances give a negative contribution to C_P . Thermal noise from the source resistance R_S tend to make C_P positive. In Fig. 2, R_S is set to 0 and ∞ for cascade lasers and parallel arrays, respectively, as these values of R_S give the largest cross correlations that can be measured experimentally for each device. Figure 2 also shows that cross correlations decrease with the increase in number of sections. Downloaded 04 Mar 2008 to 128.84.87.77. Redistribution subject to AIP license or copyright; see http://apl.aip.org/apl/copyright.jsp



FIG. 3. Scaling of the relative intensity noise (RIN) and the Fano factor (F) for multiple cavity cascade lasers (MCCL), split waveguide cascade lasers (SWCL), bipolar cascade lasers (BCL), and parallel laser arrays (PLA) with the number of cascade/array sections (N).

In cascade lasers, increase in the circuit resistance with the increase in number of cascade sections suppresses current fluctuations and, therefore, reduces cross correlation. In parallel laser, arrays current fluctuations get distributed between the parallel sections and increase in the number of sections reduces the correlation between any two sections.

Figure 3 shows the scaling of low frequency relative intensity noise (RIN= $\langle \delta P^2 \rangle / P_{avg}^2$) and the Fano factor (F $=\langle \delta P^2 \rangle / 2\hbar \omega P_{avg}$ in cascade lasers and in parallel laser arrays with the number of sections N. The data are shown at a current bias of four times the threshold. For multiple cavity lasers, it is assumed that the power from all cavities is collected before the noise is measured.¹ In single cavity cascade lasers, scaling to higher N can be performed in two ways. In the split waveguide cascade laser, the 400 μ m long optical cavity is partitioned electrically into smaller sections which are connected in series.^{2,3} By doing so, the total device resistance is scaled up by N^2 and the threshold current is scaled down by N. In the bipolar cascade laser,⁵ the differential resistance is scaled up by N and the threshold current does not change with N. Comparisons between different cascade lasers can be made in a variety of ways and we have chosen to keep photon density and modulation bandwidth the same for all the lasers. As a result, the total average output power from a multiple cavity cascade laser (or a parallel array) is N times that from a single cavity laser. The same power scaling also holds for the bipolar cascade laser with Ncascade sections. In the split waveguide cascade laser, the output power does not change with N. Figure 3 shows that the RIN and the Fano factor are larger in multiple cavity cascade lasers than in parallel laser arrays. Since the noise is measured after the power from all cavities is collected, negative power cross correlations in parallel laser arrays result in lower noise as compared to that in cascade lasers in which the power cross correlations are positive. In all cascade lasers, the Fano factor increases with the increase in N, although the relative intensity noise in multiple cavity cascade lasers and bipolar cascade lasers decreases with the increase in N. This is because the average output power in multiple cavity cascade lasers and bipolar cascade lasers also increases (almost linearly) with the increase in N. Bipolar cascade lasers exhibit almost the same RIN and Fano factor as multiple cavity cascade lasers. In the split waveguide cascade laser, the increase in RIN and Fano factor with N is caused predominantly by the increase in current noise because of the thermal noise generated in the contact resistances.

A theoretical model for photon noise and correlations in cascade lasers and parallel laser arrays has been presented. The emission of photons in different gain sections connected electrically are correlated. These correlations are positive for cascade lasers and negative for parallel laser arrays. In multiple cavity cascade lasers (category B) and bipolar cascade lasers (category A), for a fixed average photon density, RIN decreases with the increase in number of cascade sections but the Fano factor increases. In the split waveguide cascade lasers (category A), for a fixed average photon density inside the cavity, both RIN and the Fano factor are found to increase with the increase in number of cascade sections. For all types of cascade lasers considered in this paper the increase in intensity noise with N is found to be relatively small. Therefore, we expect that in cascade lasers the signalto-noise ratio (SNR) under direct laser modulation can be significantly improved over single section lasers sincethe increase in signal level resulting from the high slope efficiency of cascade lasers can more than compensate for the increase in intensity noise. However, the increase in device resistance with the increase in cascade sections may set the upper limit for the maximum SNR that can be achieved.

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