Wiretap Channels with Random States Non-Causally Available at the Encoder

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Joint work with Paul Cuff and Haim Permuter

Ben-Gurion University

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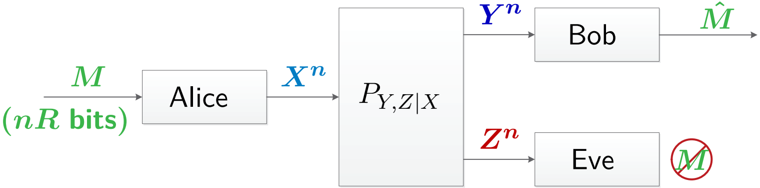
November 18th, 2016

The Wiretap Channel

Degraded [Wyner 1975], General [Csiszár-Körner 1978]



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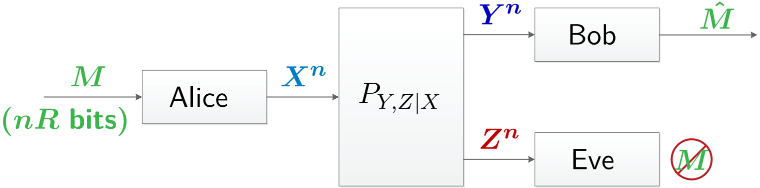

Secrecy-Capacity:

The Wiretap Channel

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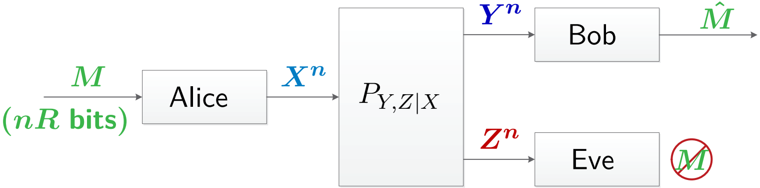

Secrecy-Capacity:

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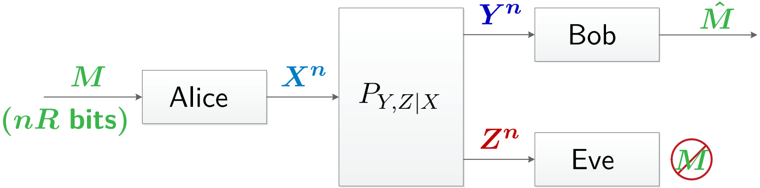

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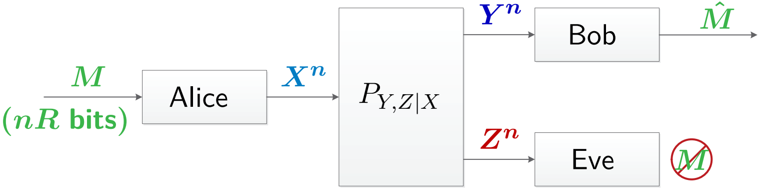

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- Pad nR **message bits** with $n\tilde{R}$ **random garbage bits**.

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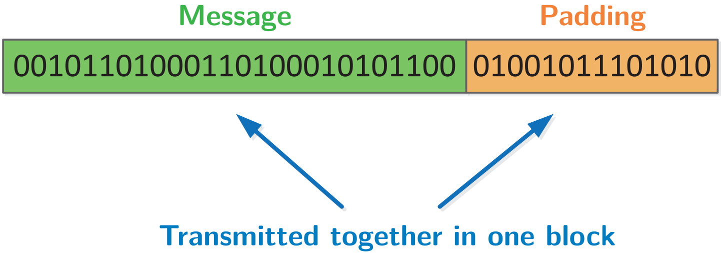

Message

Padding

00101101000110100010101100

01001011101010

Transmitted together in one block

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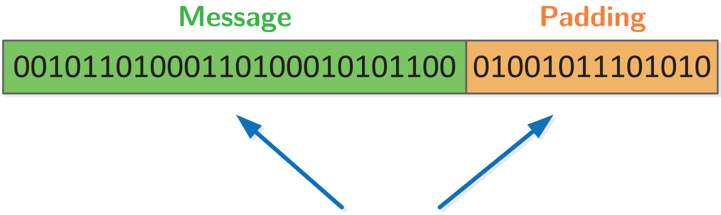

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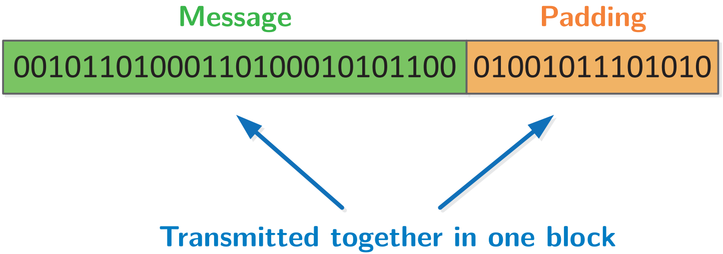

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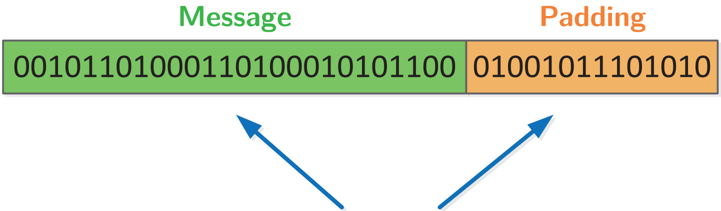

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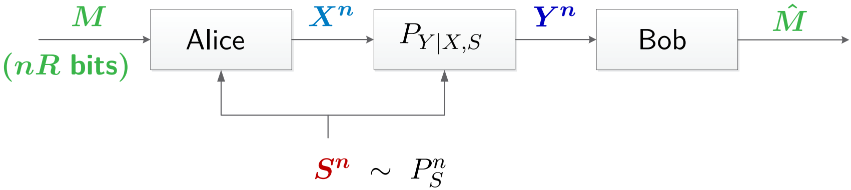
- **Security:** $\tilde{R} > I(U;Z)$.

The Gelfand-Pinsker Channel

[Pelfand-Pinsker 1980]



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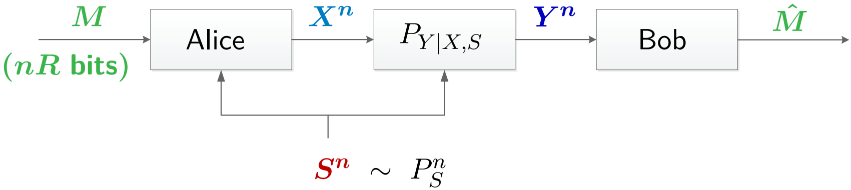

Capacity:

The Gelfand-Pinsker Channel

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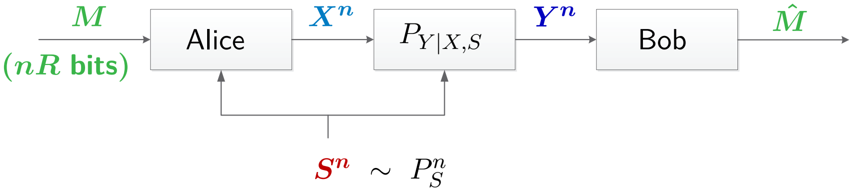

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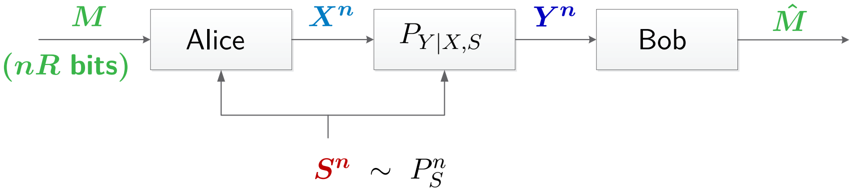

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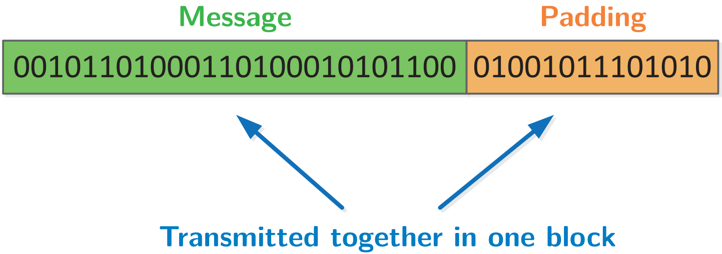

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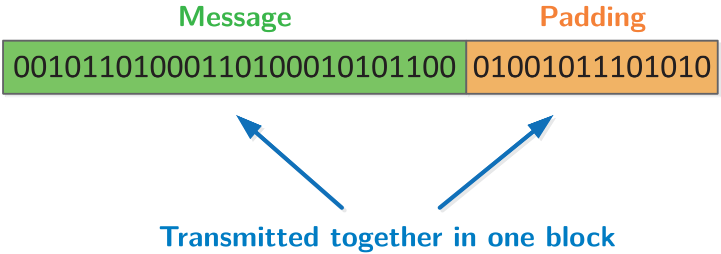

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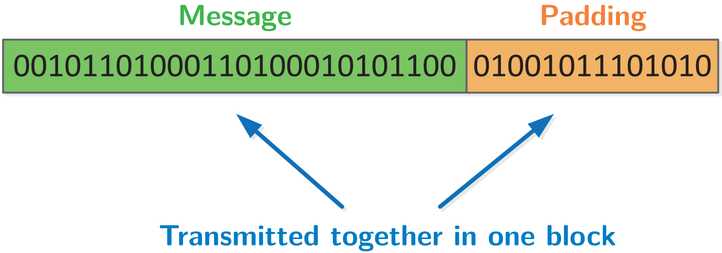

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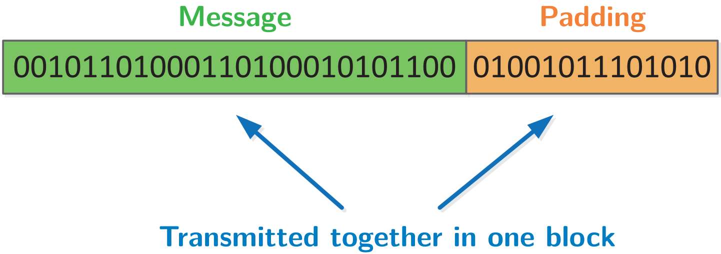

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Gelfand-Pinsker Channel vs. Wiretap Channel

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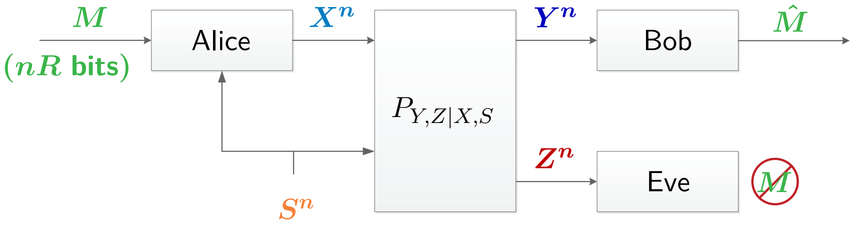
The Gelfand-Pinsker Wiretap Channel



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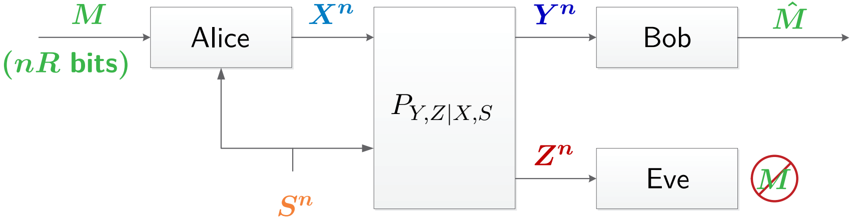

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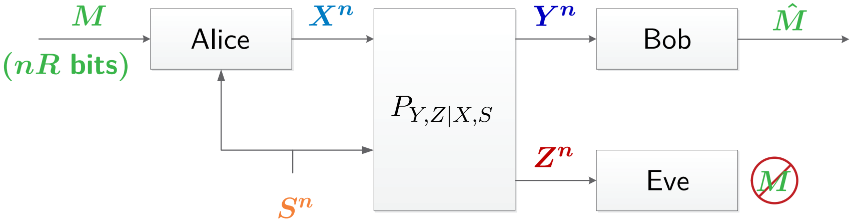

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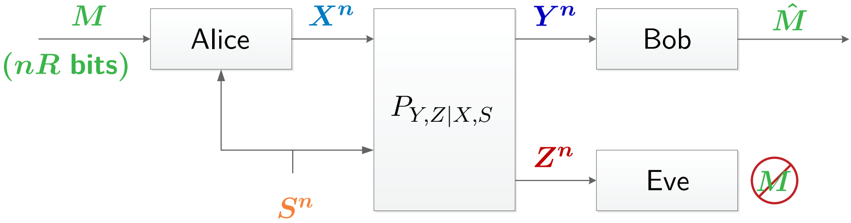

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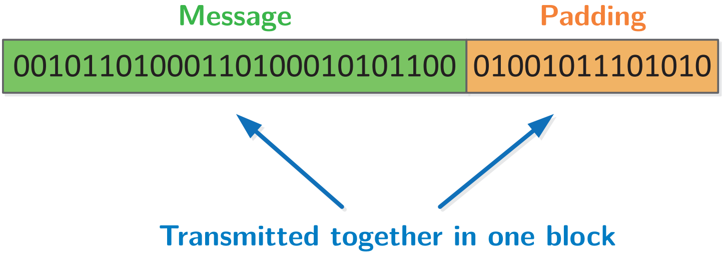
Naive Approach: Combining **wiretap coding** with **GP coding**.



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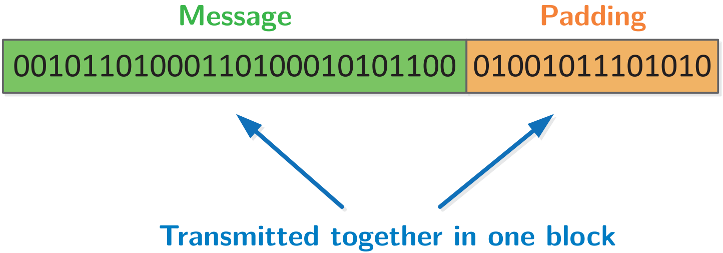

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Wiretap Channels with Encoder and Decoder CSI

Key Extraction Scheme [Chia-El Gamal 2012]

Assume $\boldsymbol{S}^n$ is know to Receiver $\boldsymbol{Y} = (\boldsymbol{Y}, \boldsymbol{S})$.

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Note: They consider causal state information.
This region is adapted to take advantage of non-causal state information.

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Better than previous scheme!

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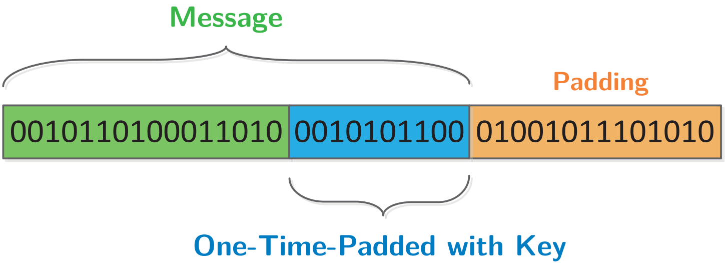

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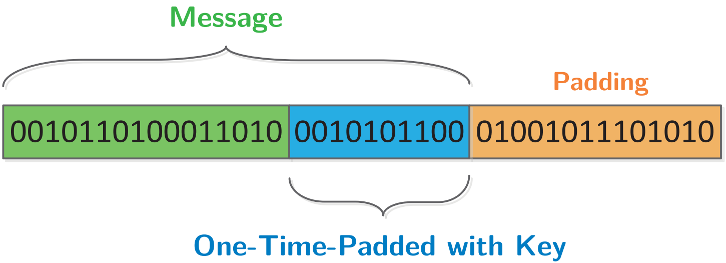

Message

Padding

0010110100011010 0010101100 01001011101010

One-Time-Padded with Key

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The Gelfand-Pinsker Wiretap Channel - Our Scheme

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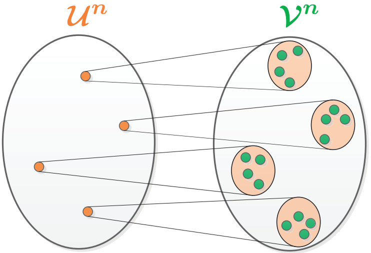

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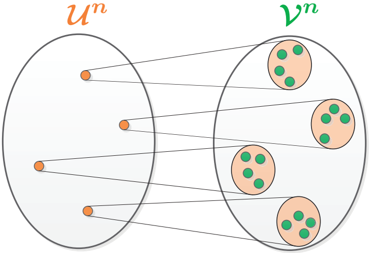

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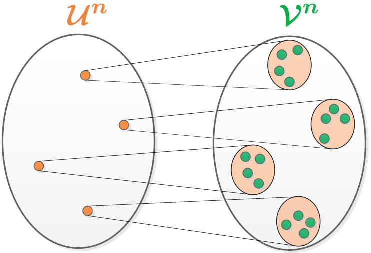

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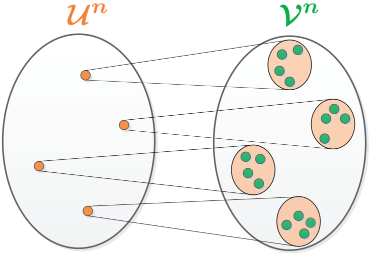

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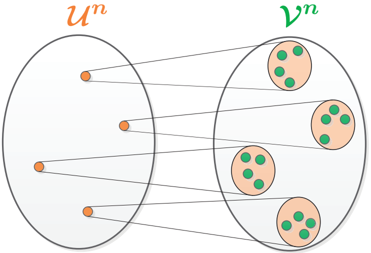

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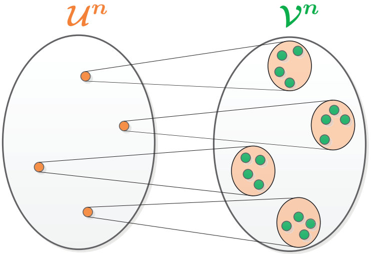

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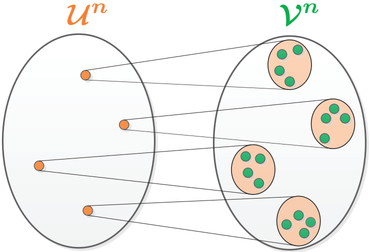

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★ **Analysis:** Likelihood Encoder & Strong Soft-Covering Lemma ★

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Theorem (ZG-Cuff-Permuter 2016)

$$\mathsf{C}_{\mathsf{GP-WTC}} \geq \max_{\substack{P_{U,V,X|S}: \\ I(U;Y)-I(U;S)\geq 0}} \min \left\{ \begin{array}{c} I(V;Y|U) - I(V;Z|U), \\ I(U,V;Y) - I(U,V;S) \end{array} \right\}$$

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- **Total communication** rate of entire superposition codebook.

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Relation to Previous Schemes:

- Upgrade from **weak-secrecy** to **semantic-security**.

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Theorem (ZG-Cuff-Permuter 2016)

$$\mathsf{C}_{\mathsf{GP-WTC}} \geq \max_{\substack{P_{U,V,X|S}: \\ I(U;Y)-I(U;S)\geq 0}} \min \left\{ \begin{array}{c} I(V;Y|U) - I(V;Z|U), \\ I(U,V;Y) - I(U,V;S) \end{array} \right\}$$

Joint distribution: $P_S P_{U,V,X|S} P_{Y,Z|X,S}$.

Relation to Previous Schemes:

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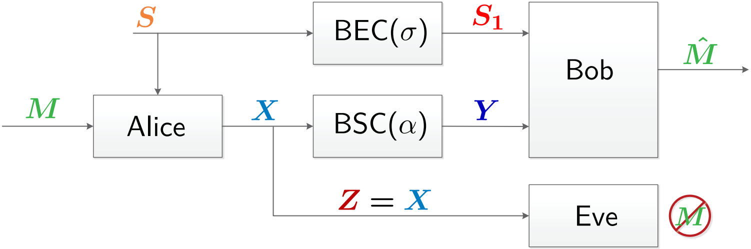

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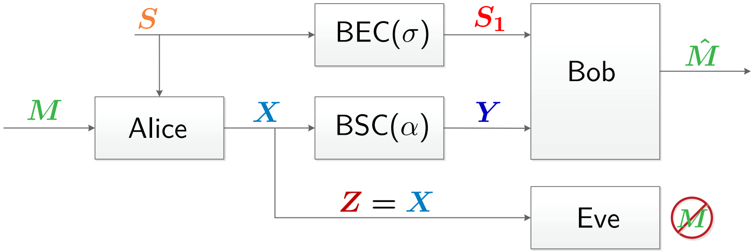

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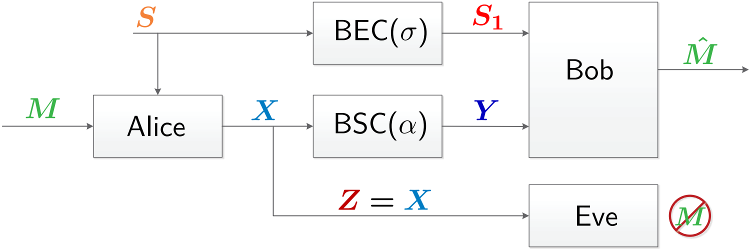

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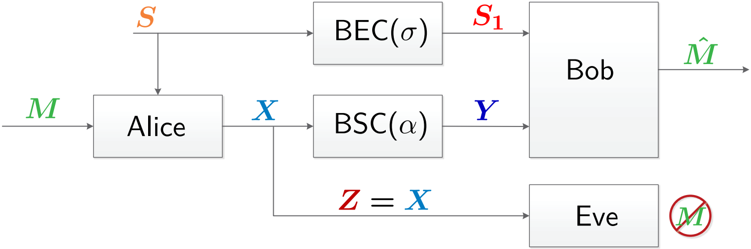

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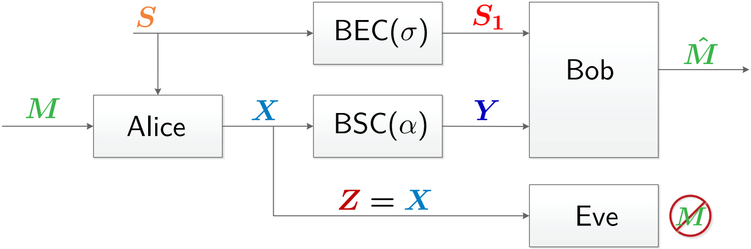

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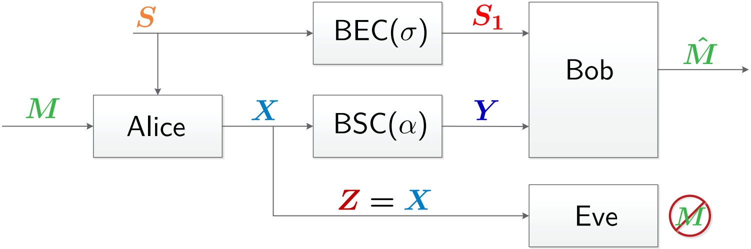

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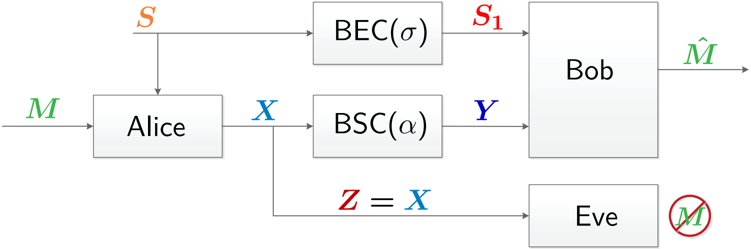

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  - Only one auxiliary - lacks flexibility to play both roles!

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Thank you!