Information Storage in the Stochastic Ising Model at Zero Temperature

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MIT

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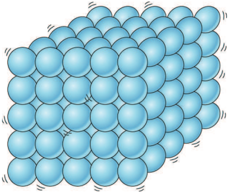
Storing Information Inside Matter



Storing Information Inside Matter

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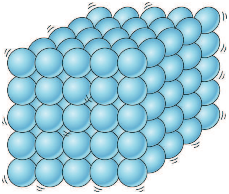

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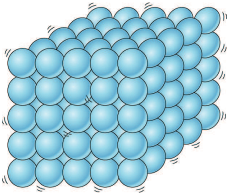

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1. Writing data $\Longrightarrow$ Perturb local state of particles
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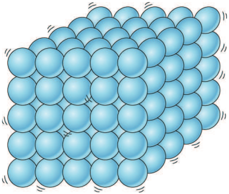

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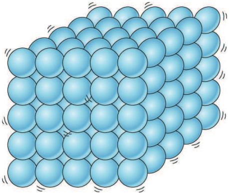

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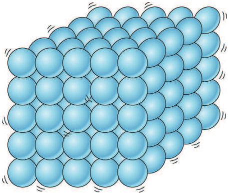

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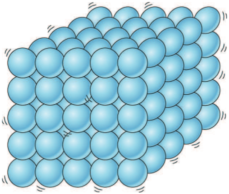

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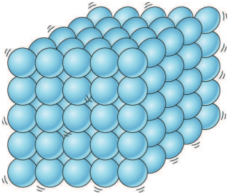

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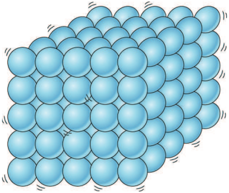

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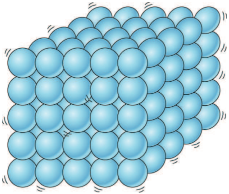
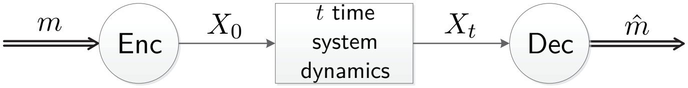


Goals:

- Distill notion of storage from particular technology
- Capture interparticle interaction and system's dynamics
- How much data can be stored and for how long?

Operational Framework



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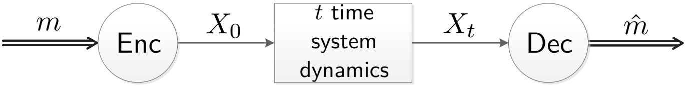

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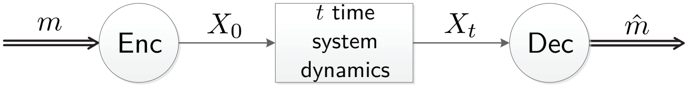

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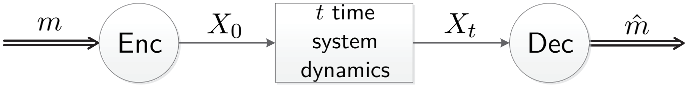

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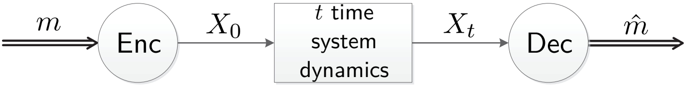

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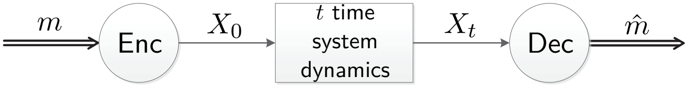

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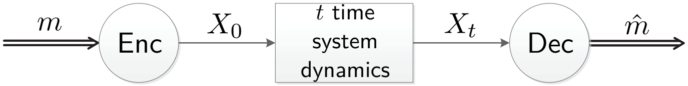

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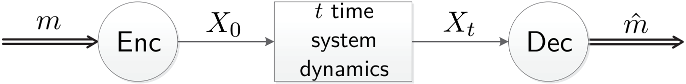

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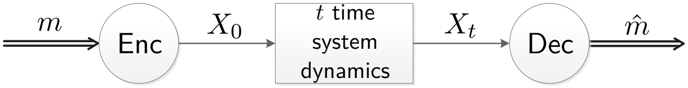

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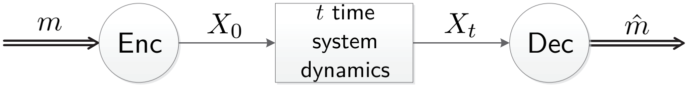

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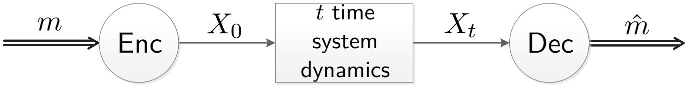

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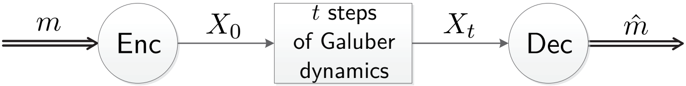
Warm (β small) $\Longrightarrow$ Weak interactions

Cold (β large) $\Longrightarrow$ Strong interactions

Measuring Information Storage



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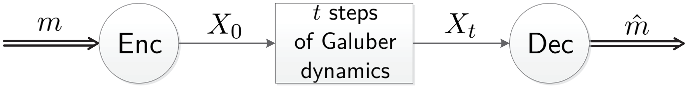

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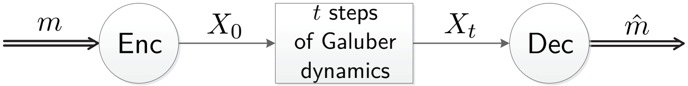

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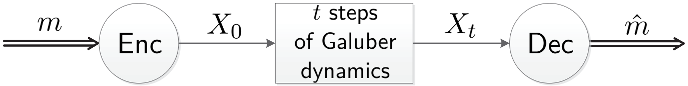

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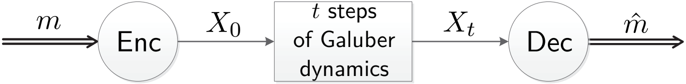

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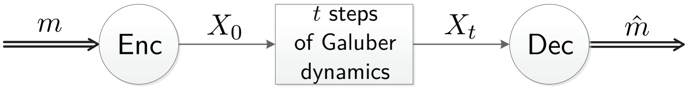

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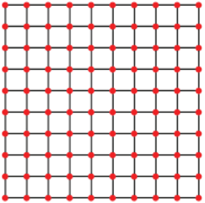

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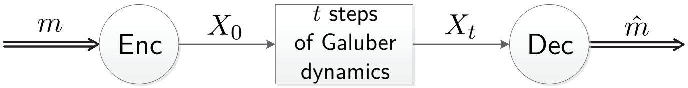

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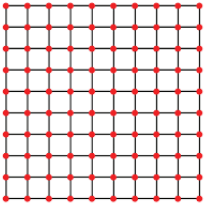

⊛ **Warm:** n-fold DM BSC $\left(\frac{1}{2} + o(1)\right)$ after $t = O(n)$.

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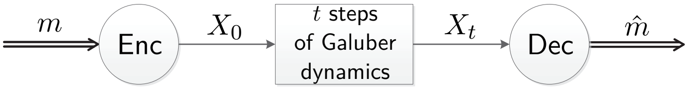

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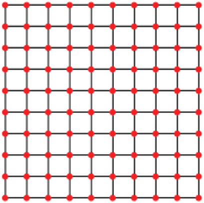

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⊛ **Cold:** Can interactions (memory) help?

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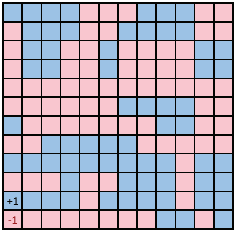

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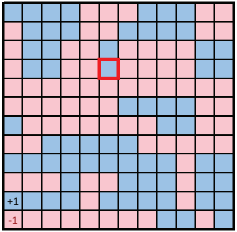

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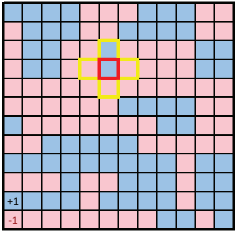

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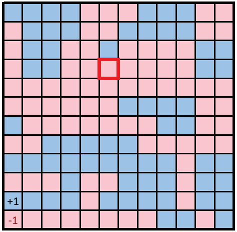

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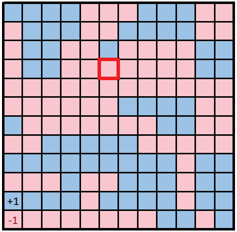

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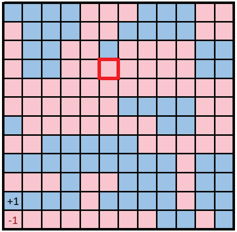

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- **Achievability:** Linear codes (Gilbert-Varshamov)

Q1: What (if anything) can be stored for infinite time?

Storing for Infinite Time

Theorem

For the zero-temp. SIM on $\sqrt{n}\times\sqrt{n}$ *grid* $\quad I_n^{(\infty)} \triangleq \lim_{t\to\infty} I_n(t) = \Theta(\sqrt{n})$

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- **Stable Config.:** $\sigma \in \Omega$ is *stable* if $P(\sigma,\sigma) = 1$ (ground states).
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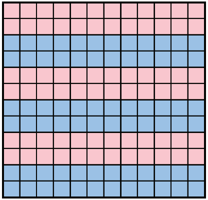

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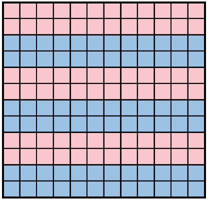
⊛ # Stripes $= 2^{\Theta(\sqrt{n})}$



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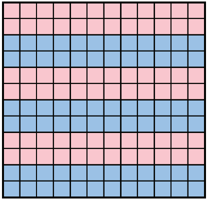

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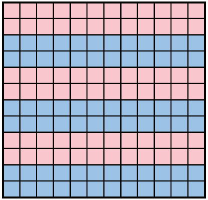

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- **Lemma:** Zero-temp. SIM is absorb. MC & Stripes are absorb. set

Storing for Infinite Time

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$$\implies \lim_{t\to\infty} \mathbb{P}\Big(X_t \in \{\text{Stripes}\}\Big) = 1$$

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$$\implies \lim_{t\to\infty}\mathbb{P}\Big(X_t\in\{\text{Stripes}\}\Big)=1 \quad\implies\quad I_n^{(\infty)}=O(\sqrt{n})$$

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Q2: Can we do better than $\sqrt{n}$ for finite superlinear t?

Storing $\omega(\sqrt{n})$ Bits for Superlinear Time

Theorem

Let $a(n) = o(n)$*. Then* $\exists c > 0$ *s.t.* $I_n(t) = \Omega\left(\frac{n}{a(n)}\right)$, $\forall t \leq c \cdot a(n) \cdot n$.

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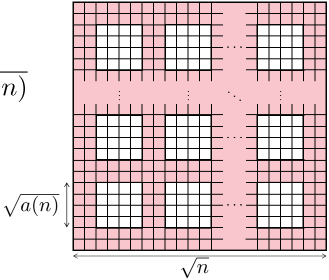
- **Codebook Construction:**
  - Tile grid with mono. sub-squares of side $\sqrt{a(n)}$



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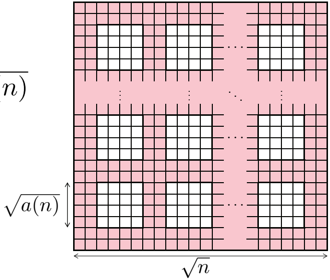

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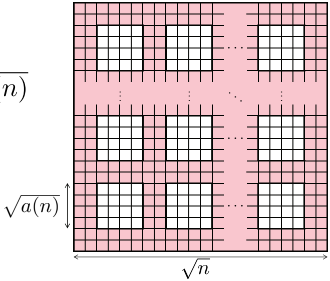

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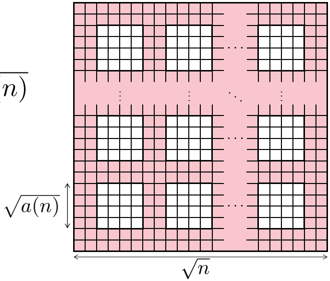

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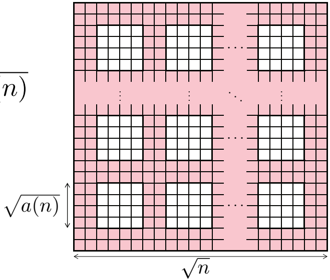

- **Continuous-Time:** Updates according to i.i.d. Poiss$(1/n)$ clocks.

$$I_n(t) \approx I_n^{(c)}((1+o(1))t), \qquad t \sim \text{suplog}(n)$$

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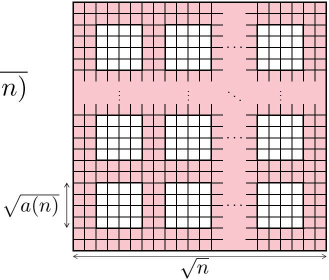

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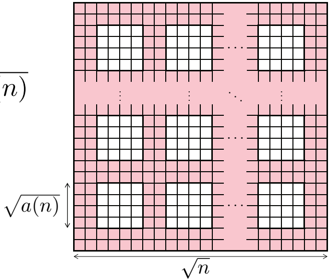

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- **Tensorization:** $I_n^{(c)}(t) \geq K \cdot \max_{p_1} I\left(\left[X_0^{(c)}\right]_1 ; \left[X_t^{(c)}\right]_1\right)$

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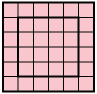
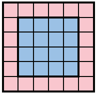
Sub-Square:

0



1



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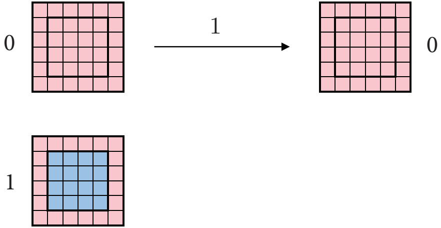

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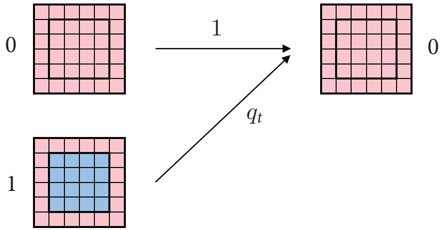

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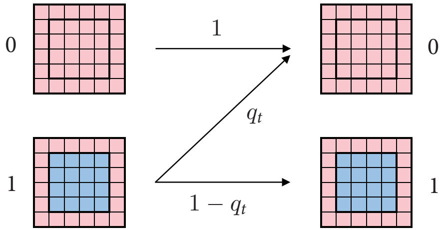

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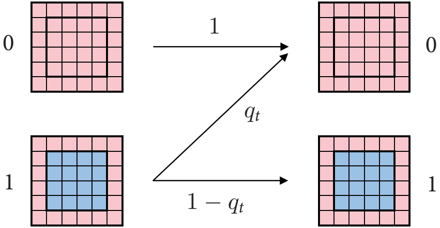

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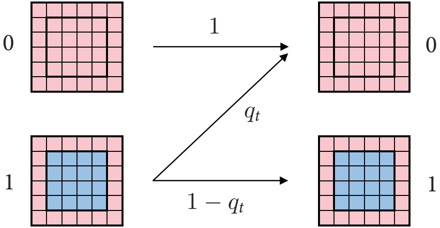

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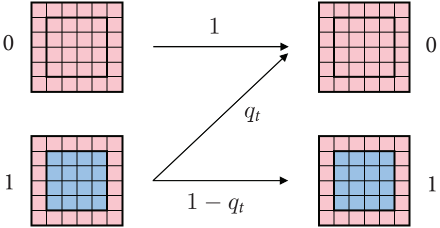

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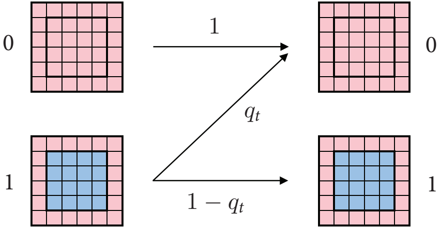

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$$\implies \qquad I_n^{(c)}(t) \geq K \cdot \mathsf{C}_{\mathsf{Z-Channel}}(q_t)$$

$$\implies \qquad I_n^{(c)}(c \cdot a(n) \cdot n) \geq C \cdot K = \Omega\left(\frac{n}{a(n)}\right)$$

Minor Tweaks - Major Impact

Grid with External Field:

Minor Tweaks - Major Impact

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- **Hamiltonian:** $\mathcal{H}(\sigma) = -\left(\sum_{\{u,v\} \in \mathcal{E}} \sigma(u)\sigma(v) + \boldsymbol{h} \sum_{\boldsymbol{v} \in \boldsymbol{\mathcal{V}}} \boldsymbol{\sigma(v)} \right)$

Minor Tweaks - Major Impact

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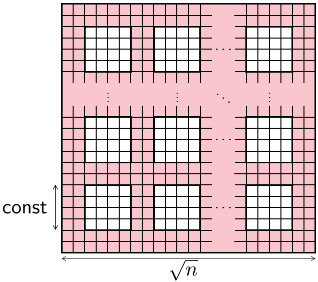
$\implies$ All square-tilings with $a(n) =$ const are stable.



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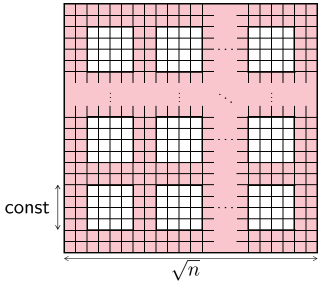
$\implies$ All square-tilings with $a(n) = \text{const}$ are stable.

Theorem

For zero-temp. SIM on $\sqrt{n} \times \sqrt{n}$ grid with external field: $I_n(t) = \Theta(n),\ \forall t$



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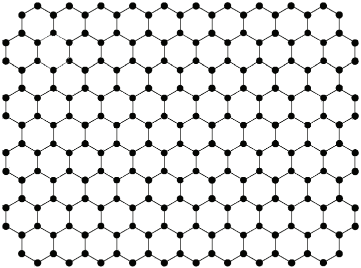

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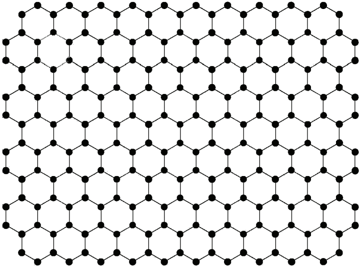

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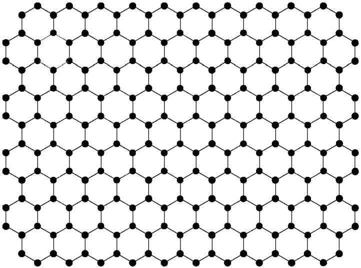

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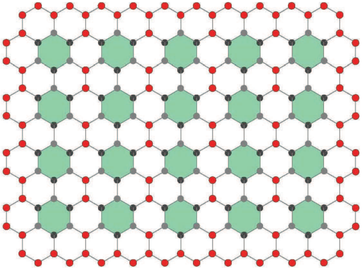

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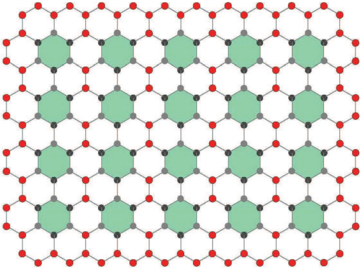

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