A Useful Analogy Between Wiretap and Gelfand-Pinsker Channels

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MIT and Ben Gurion University

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Dualities and Analogies in Information Theory

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- **Gelfand-Pinsker and Wiretap Channel** [Liang-Poor-Shamai 2009]

The Wiretap Channel & The GP Channel

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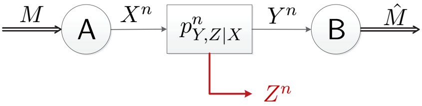

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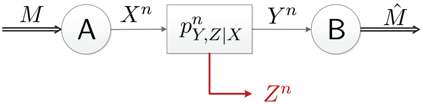

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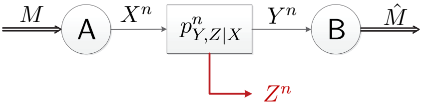

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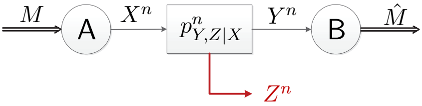

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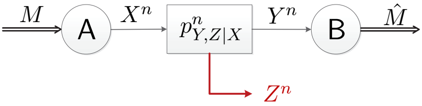

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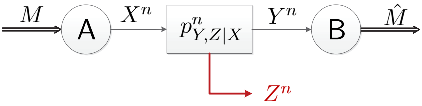

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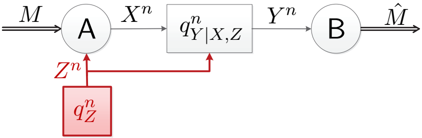

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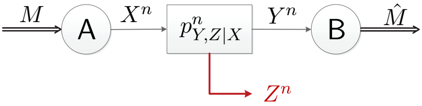

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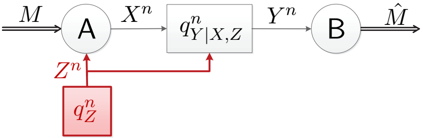

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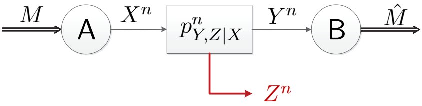

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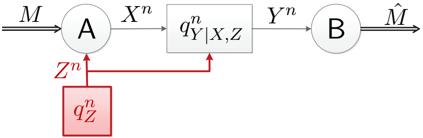

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Gelfand-Pinsker Channel vs. Wiretap Channel

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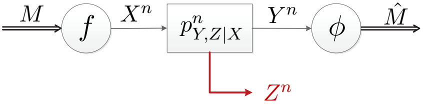

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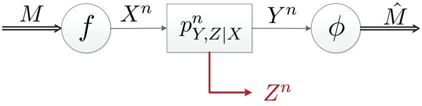

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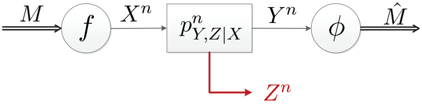

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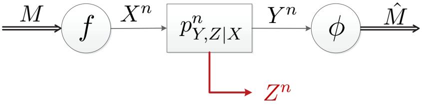

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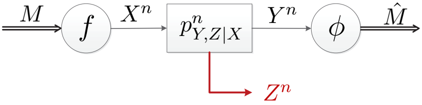

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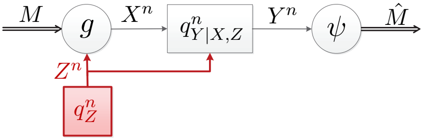

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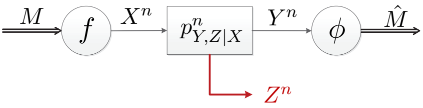

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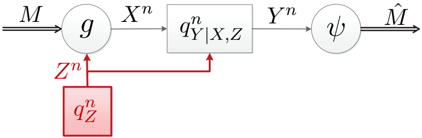

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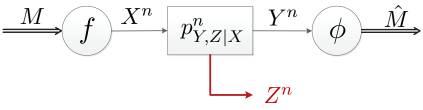

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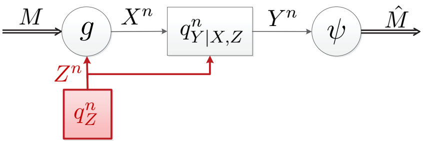

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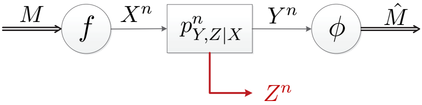

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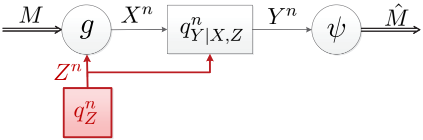

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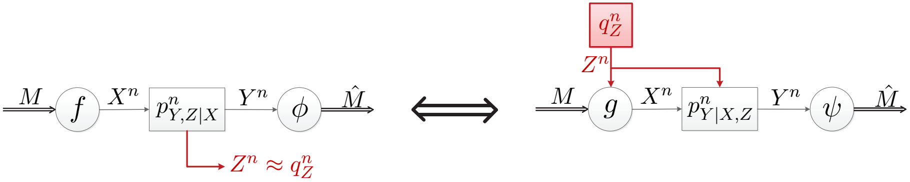

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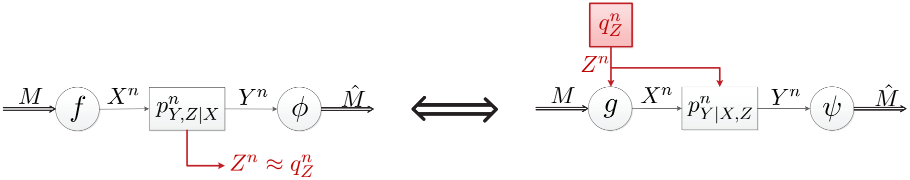

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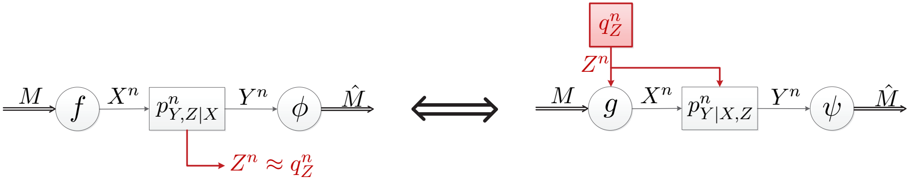

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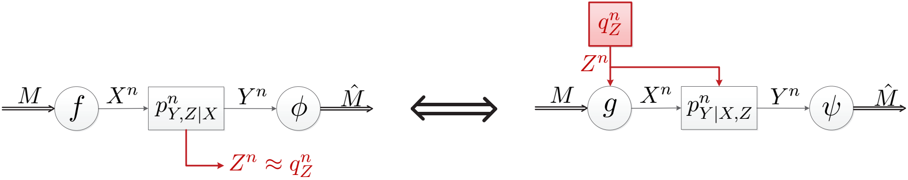

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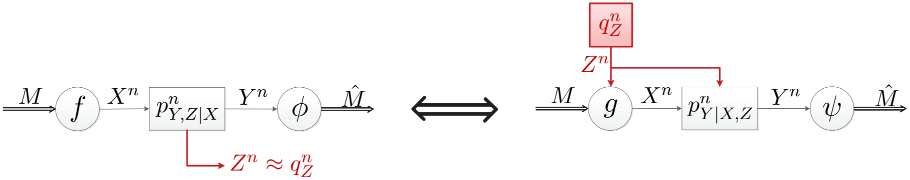

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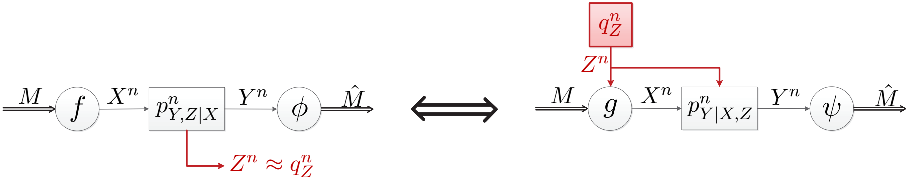

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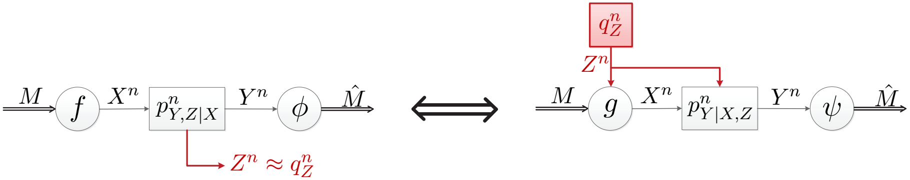

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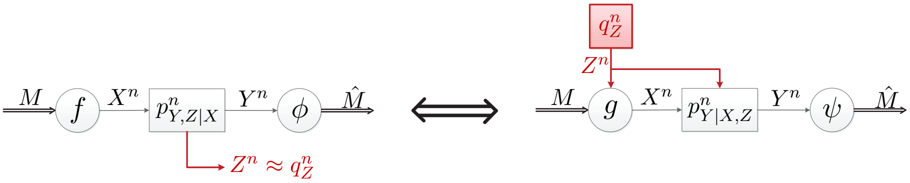
4. Set GP channel transition prob. to $p_{Y|X,Z}$.

Main Analogy Result (Point-to-Point)



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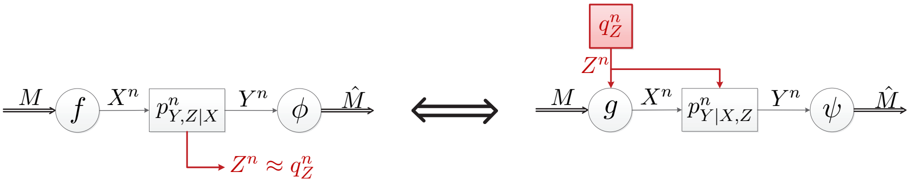

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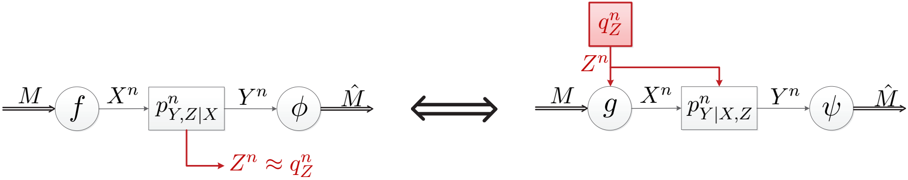

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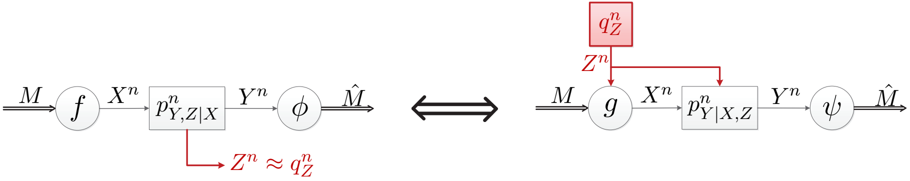

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  $\Longrightarrow$ Calculate $g_n \triangleq P_{X^n|M,Z^n}$ and set $\psi_n = \phi_n$.

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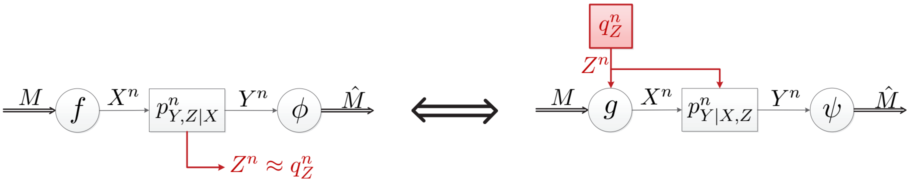

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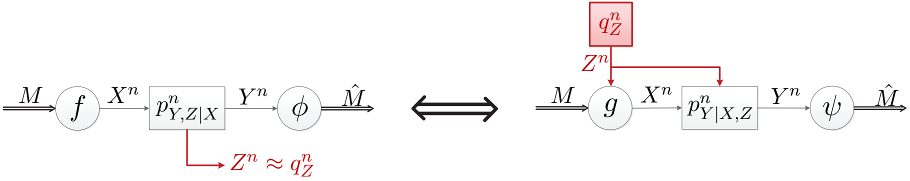
Main Analogy Result (Point-to-Point)



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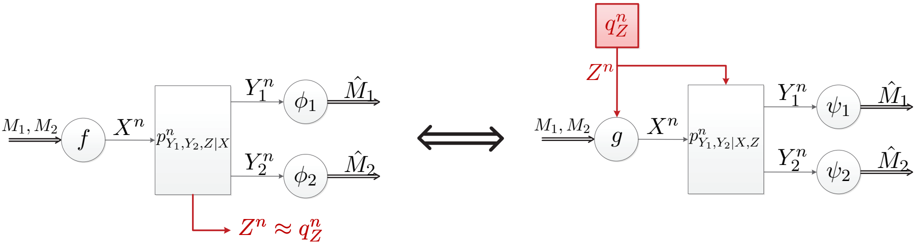

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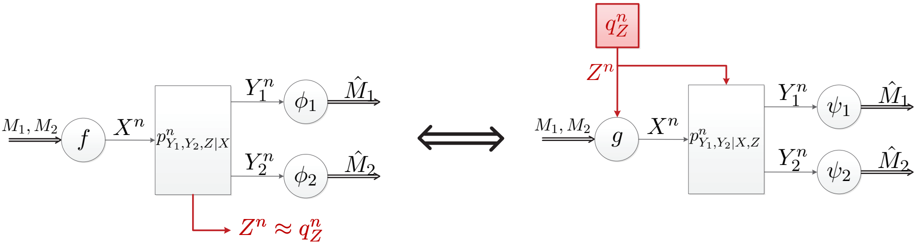

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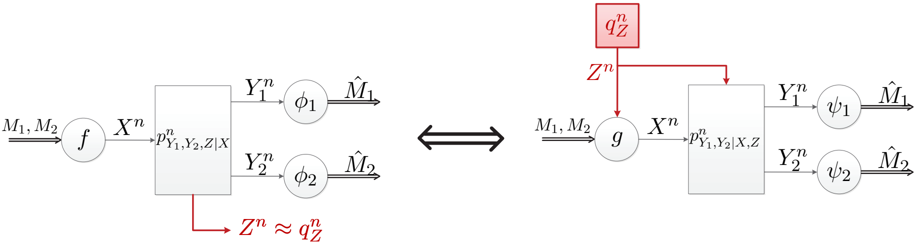

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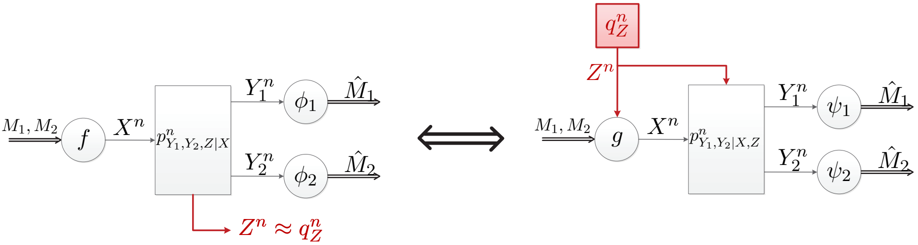

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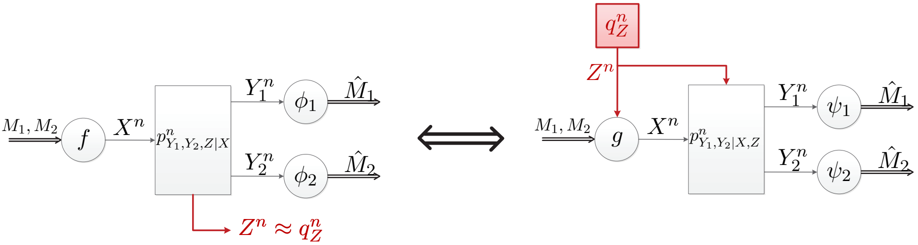

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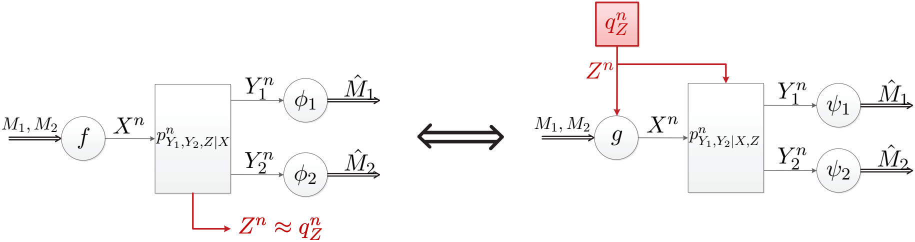

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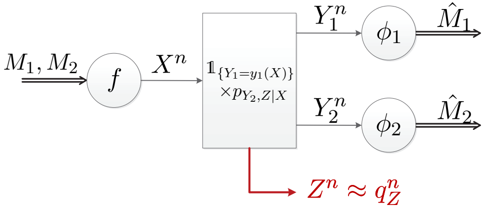

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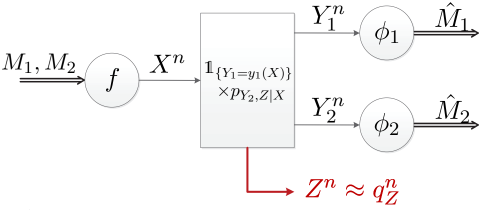

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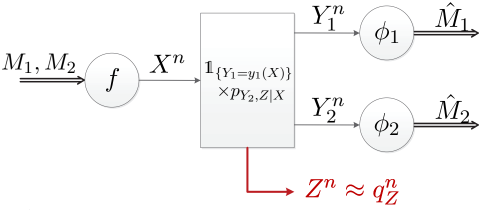

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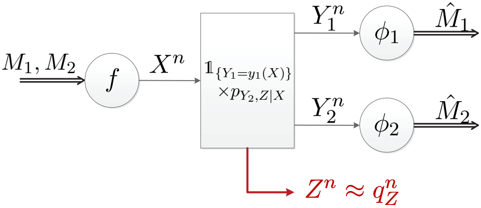

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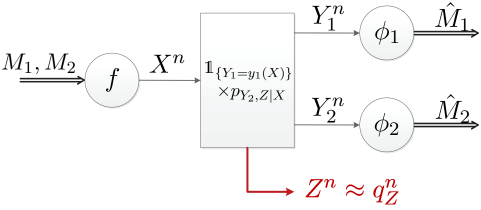

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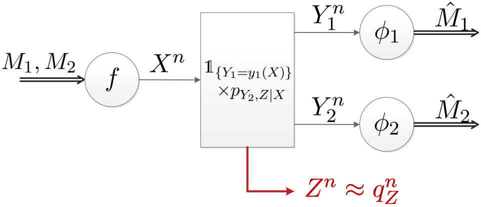

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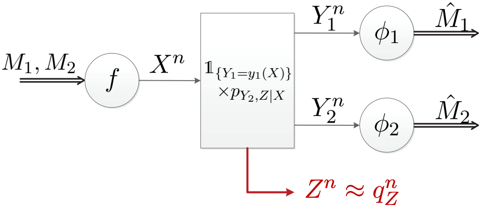

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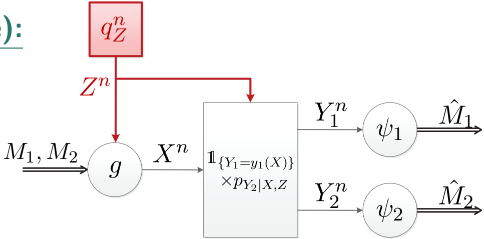
  - **Converse:** Needs both - memory in Z^n & correlation with (M_1, M_2).

Analogous Semi-Deterministic Gelfand-Pinsker BCs
[Lapidoth-Wang 2013]

Semi-Deterministic (Special Case):

$$q_{Y_1,Y_2|X,Z} = \mathbb{1}_{\{Y_1=y_1(X)\}} p_{Y_2|X,Z}$$



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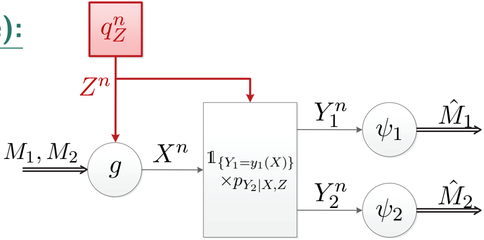

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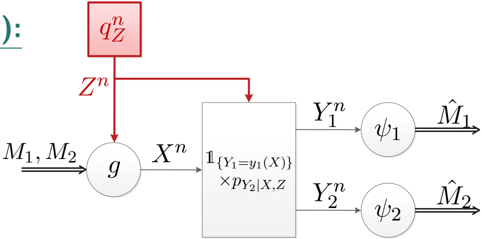

Theorem (Lapidoth-Wang 2013)

The capacity region of $(q_Z, \mathbb{1}_{\{Y_1=y_1(X)\}} p_{Y_2|X,Z})$ *GP-BC is union of*

$$R_1 \le H(Y_1|Z)$$
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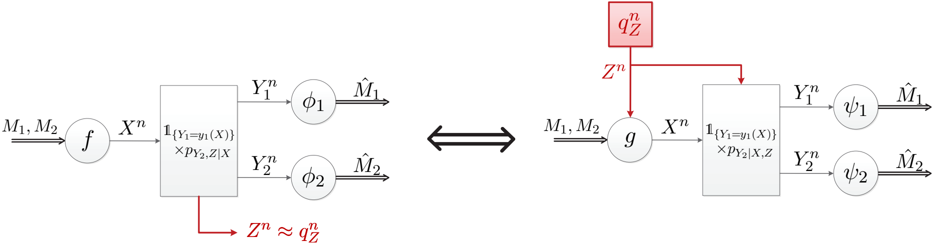
Semi-Deterministic WT-BC - Capacity via Analogy



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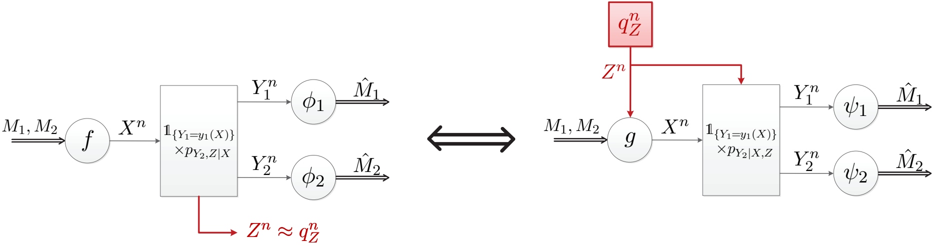

Theorem

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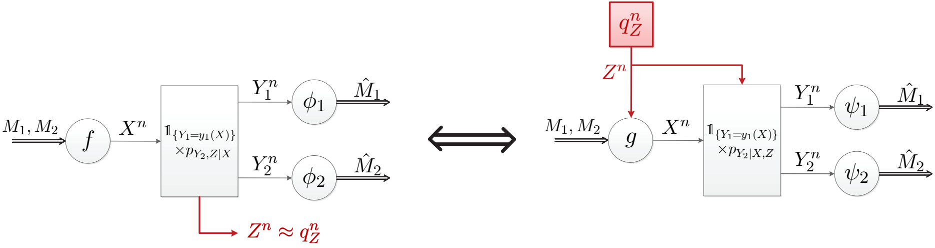

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⊛ Single auxiliary & no less-noisy assumption.

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Thank you!