Semantic Security versus Active Adversaries

Ziv Goldfeld Joint work with Paul Cuff and Haim Permuter

Ben Gurion University

Information Theory and Applications Workshop

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Information Theoretic Security over Noisy Channels

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Security versus computationally unbounded eavesdroppers.

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- Security metrics insufficient for (some) applications.

Information Theoretic Security over Noisy Channels

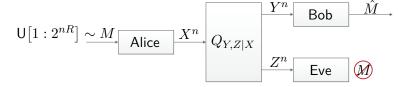
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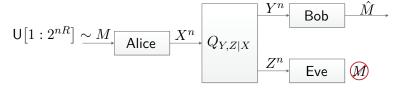
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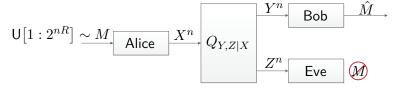
Our Goal: Stronger metrics and remove "known channel" assumption.





$$\left\{\mathcal{C}_{n}\right\}_{n\in\mathbb{N}}$$
 - a sequence of (n,R) -codes

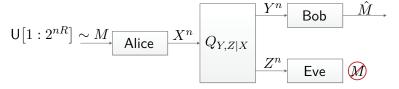
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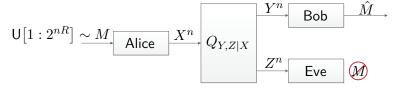
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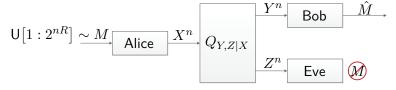
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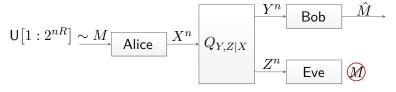
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$$\mathsf{U}[1:2^{nR}] \overset{}{\sim} M \quad \mathsf{Alice} \quad \overset{X^n}{\longrightarrow} Q_{Y,Z|X} \quad \overset{Y^n}{\longrightarrow} \quad \mathsf{Eve} \quad \overset{\hat{M}}{\longrightarrow} \quad \mathsf{Eve} \quad \mathsf{M}$$

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- Strong Secrecy: $I_{\mathcal{C}_n}(M; \mathbb{Z}^n) \xrightarrow[n \to \infty]{} 0$. Security only on <u>average</u>



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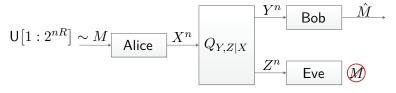
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★ A single code must work well for all message PMFs ★

Strong Soft-Covering Lemmas



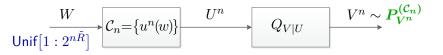




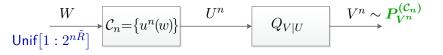
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 - \star Goal: Choose $ilde{R}$ (codebook size) s.t. $P_{V^n}^{(\mathcal{C}_n)} pprox Q_V^n \star$



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Strong Soft-Covering Lemma

Lemma (ZG-Cuff-Permuter 2016)

If $ilde{R}>I_Q(U;V)$, then there exist $\gamma_1,\gamma_2>0$ s.t. for n large enough

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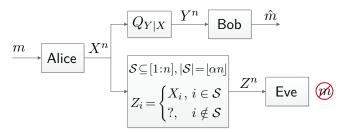
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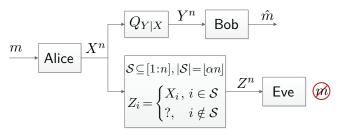
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- Extensions: Heterogeneous version, superposition codes.

Some Applications

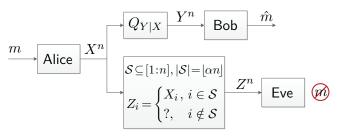


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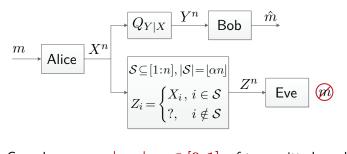


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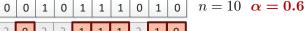


Ziv Goldfeld

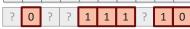
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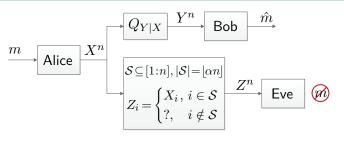


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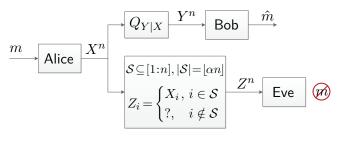


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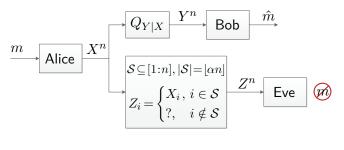


- Observed:
 - ★ Ensure security versus all possible choices of observations ★

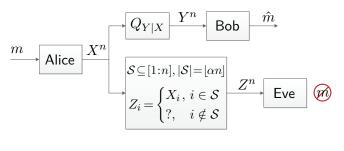
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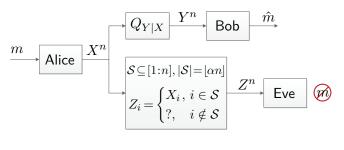
Ozarow-Wyner 1984: Noiseless main channel



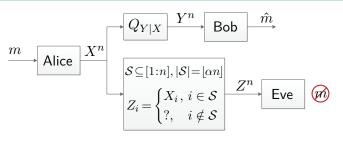
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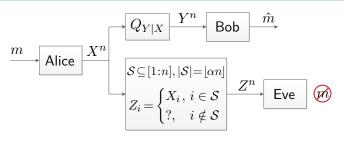
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 - Lower & upper bounds Not match in general.

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For any
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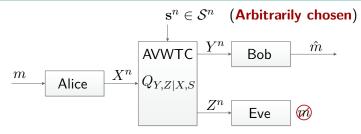
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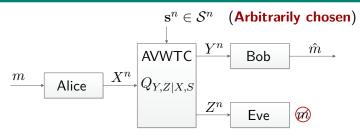
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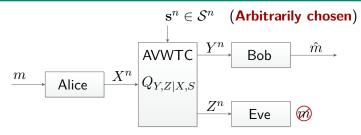
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- Standard (erasure) wiretap code & Stronger tools for analysis.



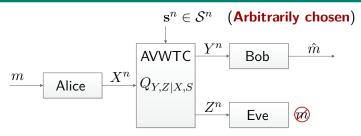
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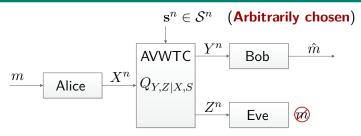
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★ Subsumes WTC II model and result ★

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Thank you!