Semantic Security in the Presence of Active Adversaries

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Joint work with Paul Cuff and Haim Permuter

Ben Gurion University

Advanced Communications Center Annual Workshop

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Motivation

Information Theoretic Security over Noisy Channels

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1. Security versus **computationally unlimited** eavesdropper.

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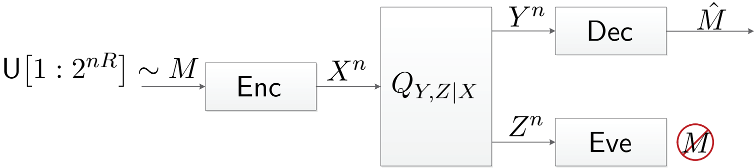
Our Goal: Stronger metric and remove "known channel" assumption.

Wiretap Channels - Security Metrics

Wiretap Channels and Security Metrics

Degraded [Wyner 1975], General [Csiszár-Körner 1978]



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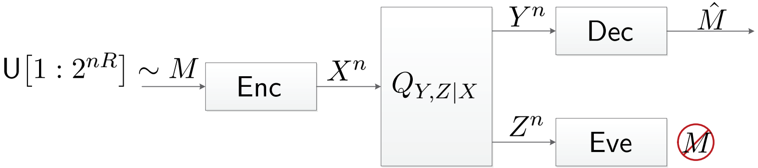

$\{\mathcal{C}_n\}_{n\in\mathbb{N}}$ - a sequence of (n,R)-codes

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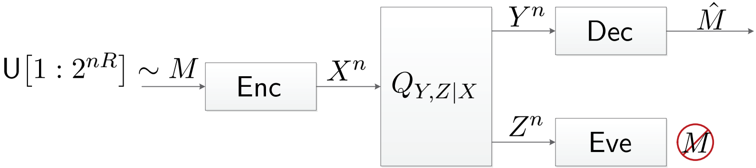

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- **Weak-Secrecy:** $\frac{1}{n} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$

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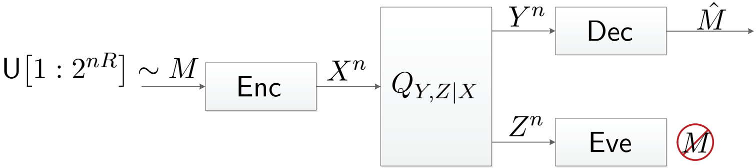

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- **Weak-Secrecy:** $\frac{1}{n} I_{\mathcal{C}_n}(M; Z^n) \xrightarrow[n\to\infty]{} 0.$ Only leakage **rate** vanishes

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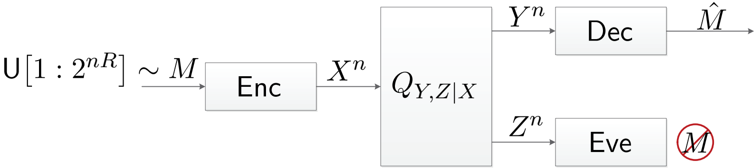

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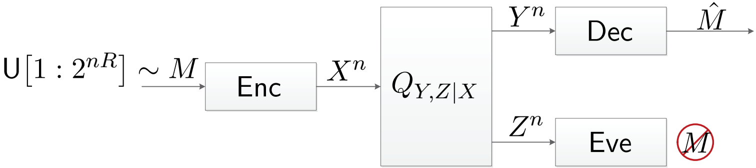

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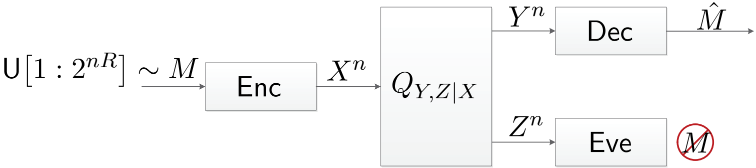

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- **Strong-Secrecy:** $I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$ Security only on **average**

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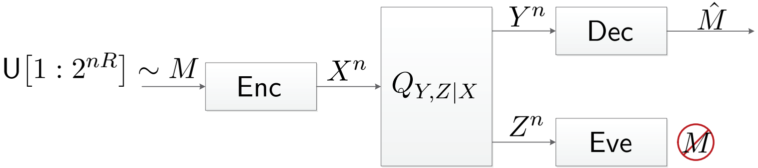

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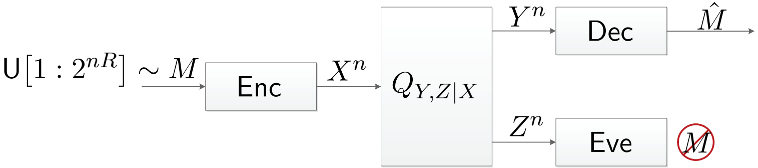

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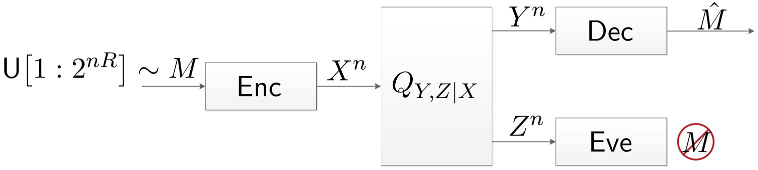

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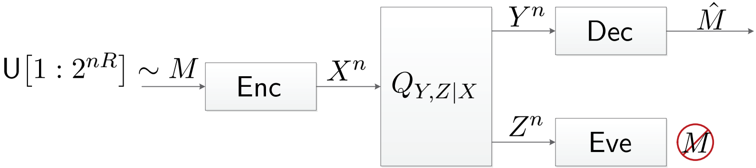

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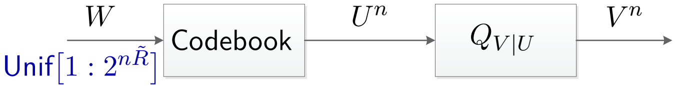
⋆ A single code must work well for all message PMFs ⋆

A Stronger Soft-Covering Lemma

Soft-Covering - Setup



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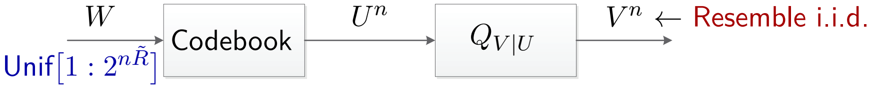

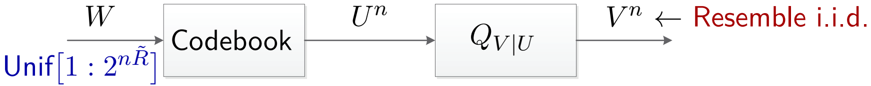

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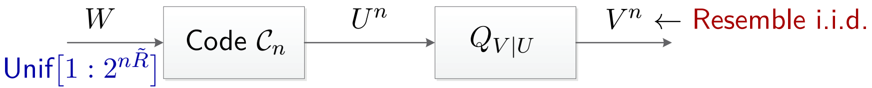

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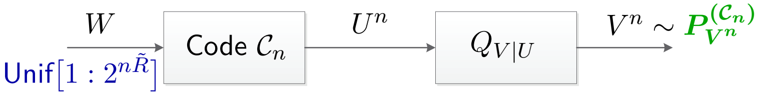

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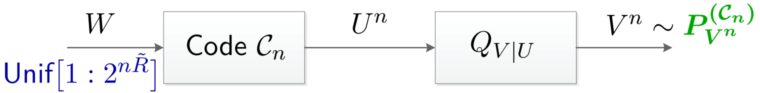

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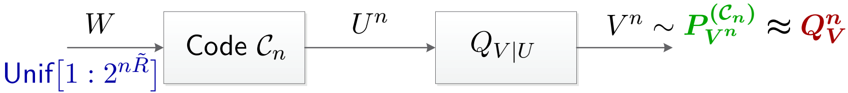

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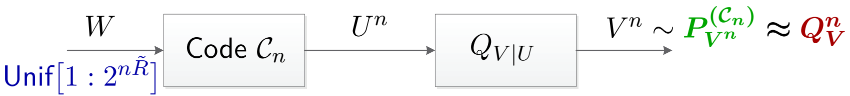

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⋆ **Goal:** Choose $\tilde{R}$ (codebook size) s.t. $P_{V^n}^{(\mathcal{C}_n)} \approx Q_V^n$ ⋆

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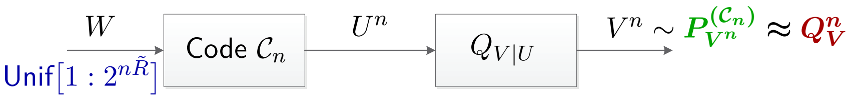

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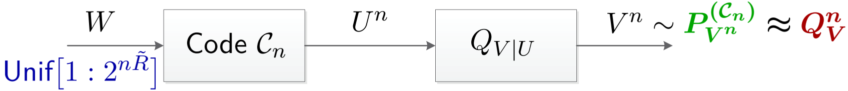

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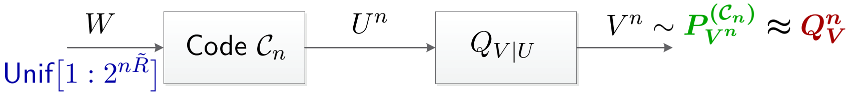

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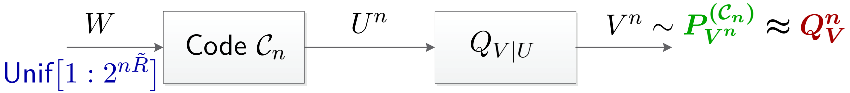

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  - Also provided converse.

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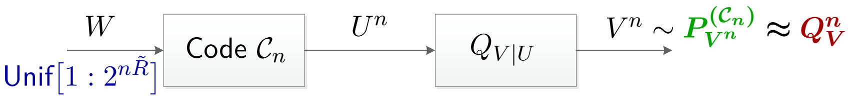

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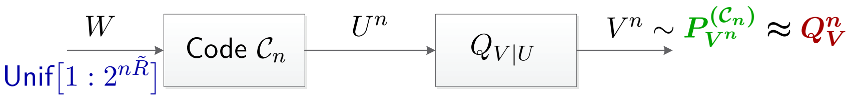
A Stronger Soft-Covering Lemma



Theorem

If $\tilde{R} > I_Q(U;V)$ *and* $|\mathcal{V}| < \infty$*, then there exists* $\gamma_1, \gamma_2 > 0$ *s.t.*

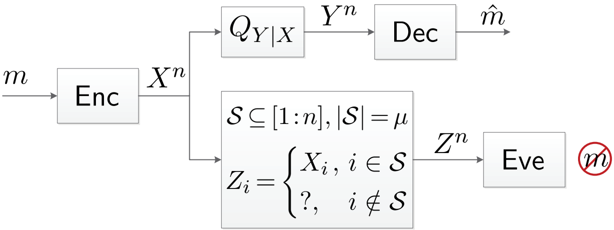
$$\mathbb{P}_{\mathbb{C}_n}\left(D\left(P_{V^n}^{(\mathbb{C}_n)} \middle\| Q_V^n \right) > e^{-n\gamma_1} \right) \leq e^{-e^{n\gamma_2}}$$

for n *sufficiently large.*

Wiretap Channels of Type II

Wiretap Channels of Type II - Definition

[Ozarow-Wyner 1984]



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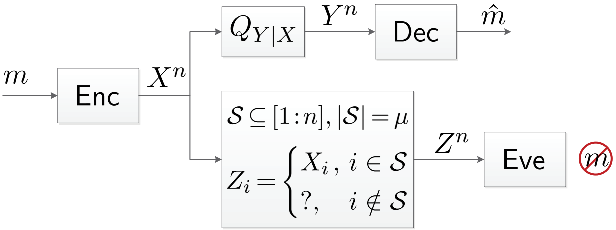

- **Eavesdropper:** Can observe a subset $\boldsymbol{\mathcal{S}} \subseteq [1:n]$ of size $\boldsymbol{\mu = \lfloor \alpha n \rfloor}$, $\boldsymbol{\alpha \in [0,1]}$, of transmitted symbols.

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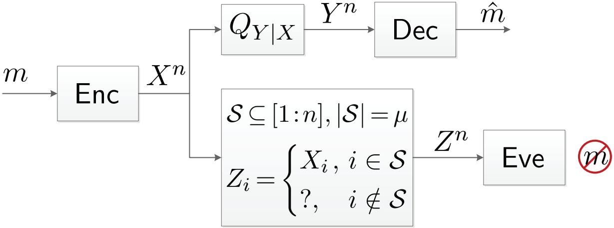

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- **Transmitted:**

0	0	1	0	1	1	1	0	1	0

$n = 10$ $\boldsymbol{\alpha = 0.63}$

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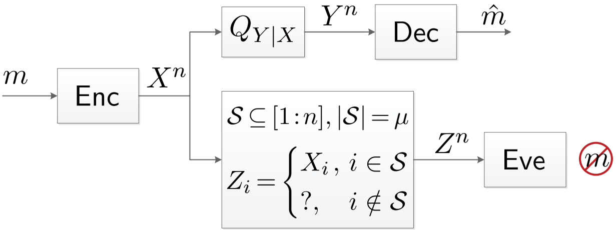

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- **Transmitted:** 0 0 1 0 1 1 1 0 1 0 $n = 10$ $\alpha = 0.63$
- **Observed:** ? 0 ? ? 1 1 1 ? 1 0

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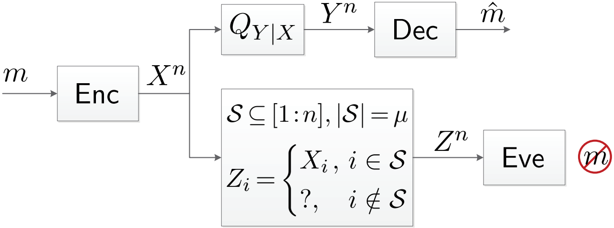

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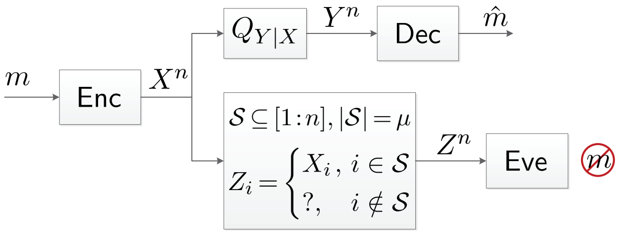
Observed:	?	0	?	?	1	1	1	?	1	0

$n = 10$ $\boldsymbol{\alpha = 0.63}$

⋆ Ensure security versus all possible choices of $\boldsymbol{\mathcal{S}}$ ⋆

Wiretap Channels of Type II - Past Results

[Ozarow-Wyner 1984]



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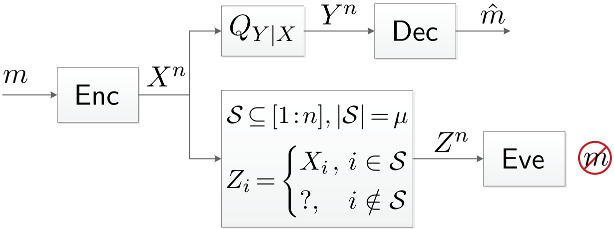

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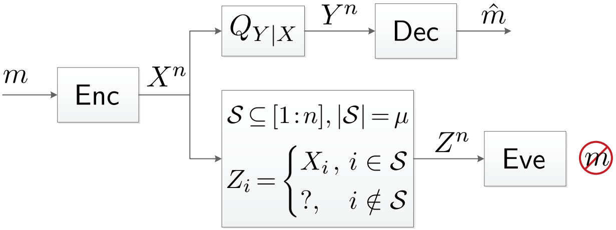

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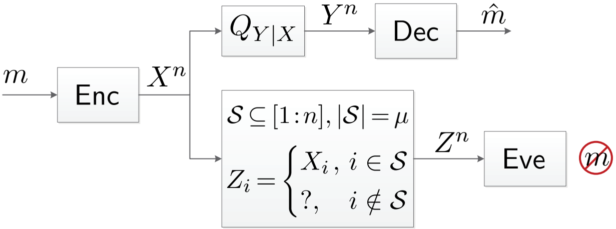

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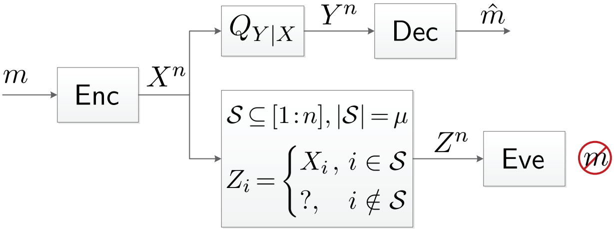

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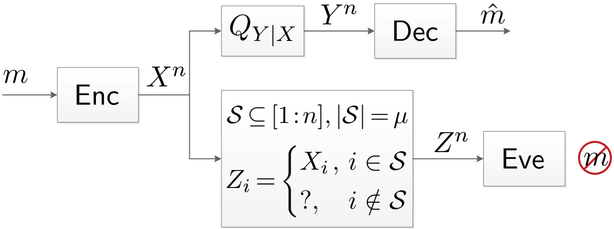

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  - Lower & upper bounds - Not match in general.

Semantic Security:

Wiretap Channels of Type II - SS-Capacity

Semantic Security: $$\max_{\substack{P_M,\mathcal{S}:\\|\mathcal{S}|=\mu}} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$$

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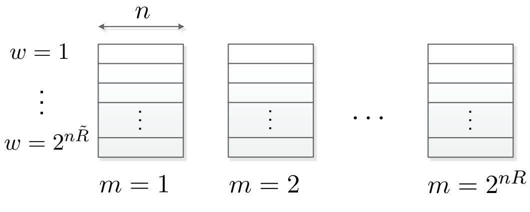

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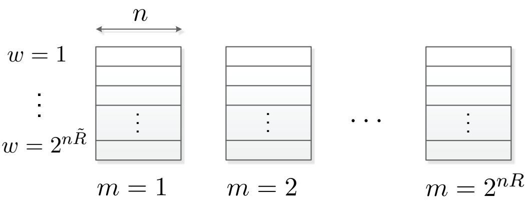

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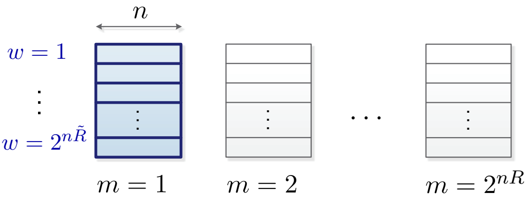

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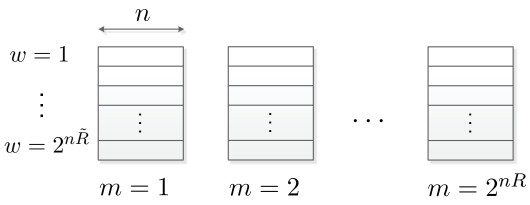

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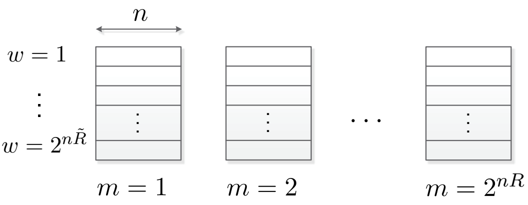

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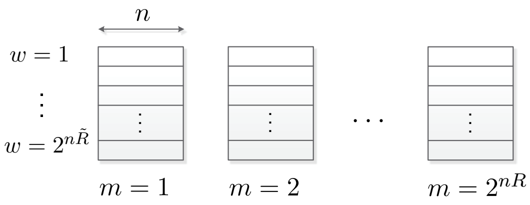

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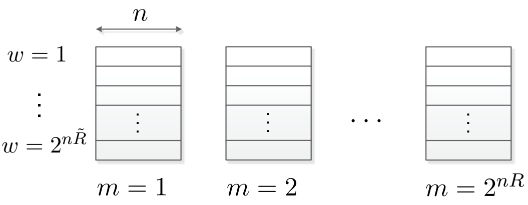

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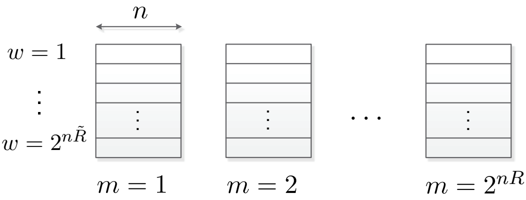

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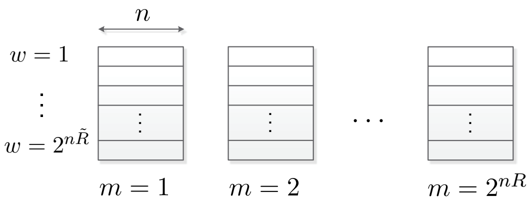

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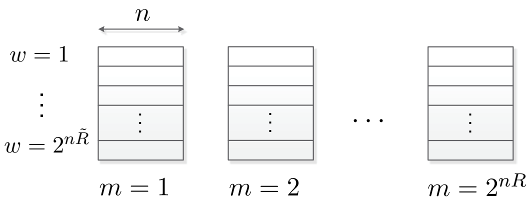

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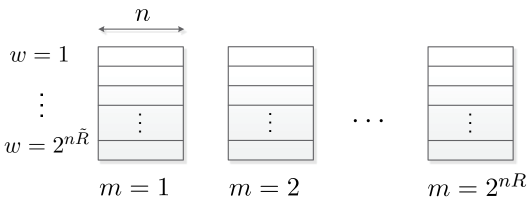

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