

The Diagonal Vector Gaussian Finite State MAC with Cooperative Encoders and Delayed CSI

Ziv Goldfeld, Haim H. Permuter, Benjamin M. Zaidel

Ben Gurion University

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Outline

- Motivation

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- Conferencing channel model

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- Common message channel model

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- Common message channel model
- Common message main result

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- Summary

Motivation (delayed state information)

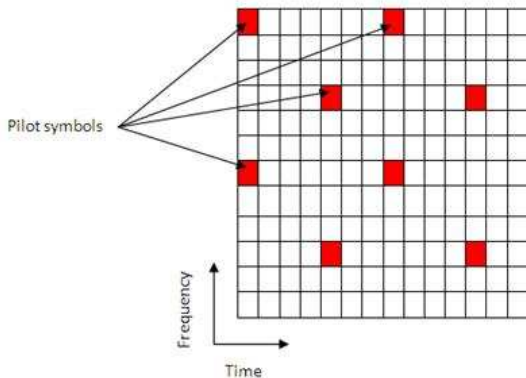
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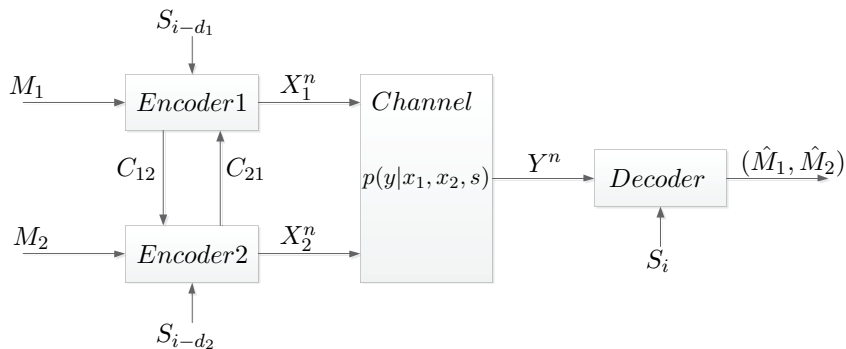
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- Channel state information (CSI) needs to be estimated.

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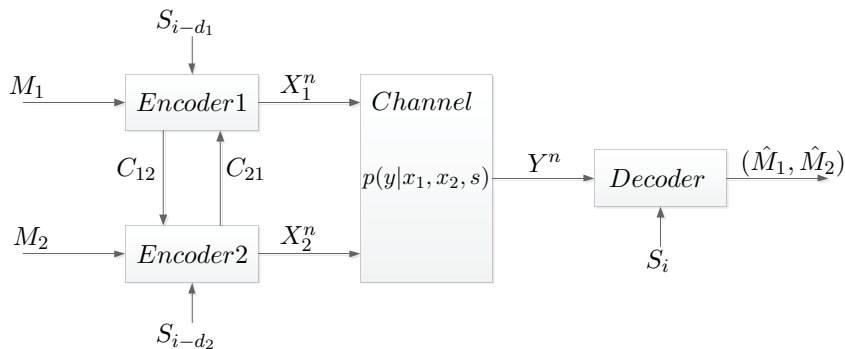
- Channel state models fading, noise and interference of uncontrolled signals.
- Channel state information (CSI) needs to be estimated.
- In LTE uplink standard, pilot signals are sent by the users in



FSM-MAC with Conferencing and Delayed CSI

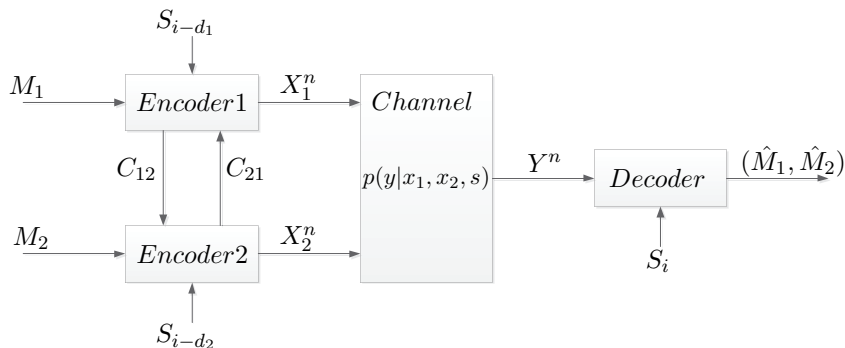


FSM-MAC with Conferencing and Delayed CSI



- CSI known to the Decoder and delayed CSI known to the Encoders.

FSM-MAC with Conferencing and Delayed CSI



- CSI known to the Decoder and delayed CSI known to the Encoders.
- Conferencing between the Encoders is possible through limited links.

Channel Model and Notation

- Finite number of states $|\mathcal{S}| < \infty$.

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- Finite number of states $|\mathcal{S}| < \infty$.
- Channel state is a stationary Markov process independent of the messages.
- The random variables S_i, S_{i-d} denote the channel state at time i , and $i - d$, respectively.
- The (S_i, S_{i-d}) joint distribution is stationary and is given by

$$P(S_i = s_l, S_{i-d} = s_j) = \pi(s_j)K^d(s_l, s_j).$$

- The conferencing takes place prior to the transmission throughout the channel.

Channel Model - Partial Cooperation [Willems82]

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- The state process is independent of the conference communications.
- The conference is held using two communication links with finite capacities C_{12} and C_{21} .

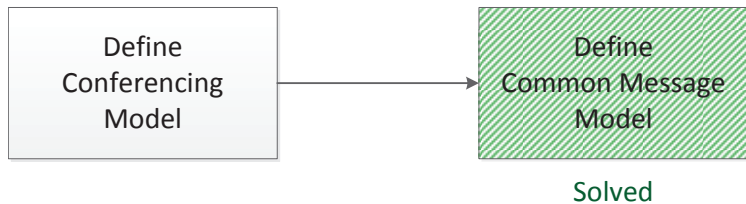
Capacity Proof Chronology

Define
Conferencing
Model

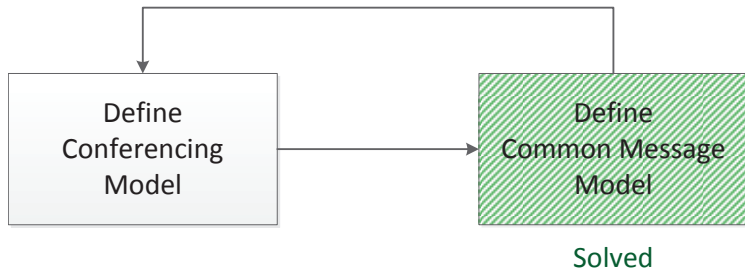
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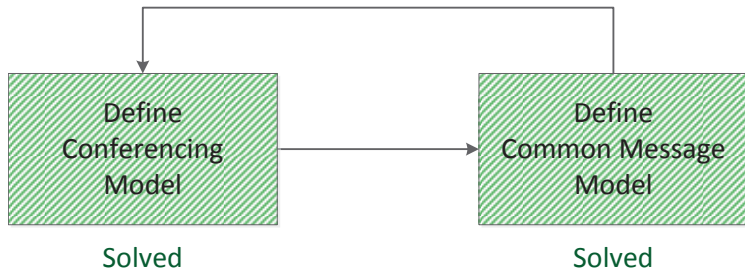
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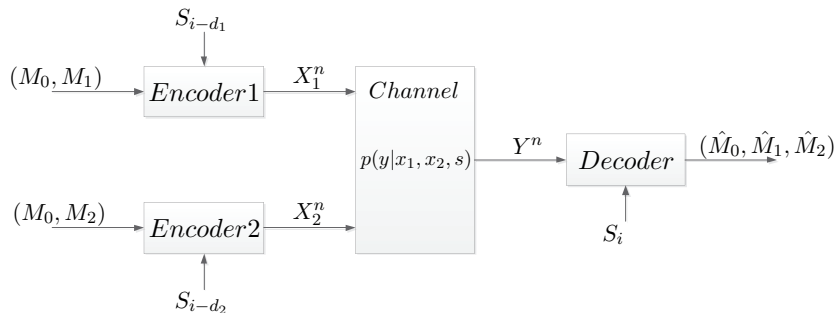
Capacity Proof Chronology



Capacity Proof Chronology



Common Message Model



Main Results Common Message with Delayed CSI

$$(d_1 \geq d_2)$$

Theorem

The capacity region of FSM-MAC with a common message, CSI at the decoder and delayed CSI at the encoders with delays d_1 and d_2 , is

$$R_1 < I(X_1; Y | X_2, U, S, \tilde{S}_1, \tilde{S}_2),$$

$$R_2 < I(X_2; Y | X_1, U, S, \tilde{S}_1, \tilde{S}_2),$$

$$R_1 + R_2 < I(X_1, X_2; Y | U, S, \tilde{S}_1, \tilde{S}_2),$$

$$R_0 + R_1 + R_2 < I(X_1, X_2; Y | S, \tilde{S}_1, \tilde{S}_2),$$

for some joint distribution of the form:

$$P(u | \tilde{s}_1) P(x_1 | \tilde{s}_1, u) P(x_2 | \tilde{s}_1, \tilde{s}_2, u).$$

The joint distribution $(S, \tilde{S}_1, \tilde{S}_2)$ is the same joint distribution as $(S_i, S_{i-d_1}, S_{i-d_2})$.

- Coding scheme: Encode using MUX, decode simultaneously using joint-typicality.

Achievability - Ideas and Discussion

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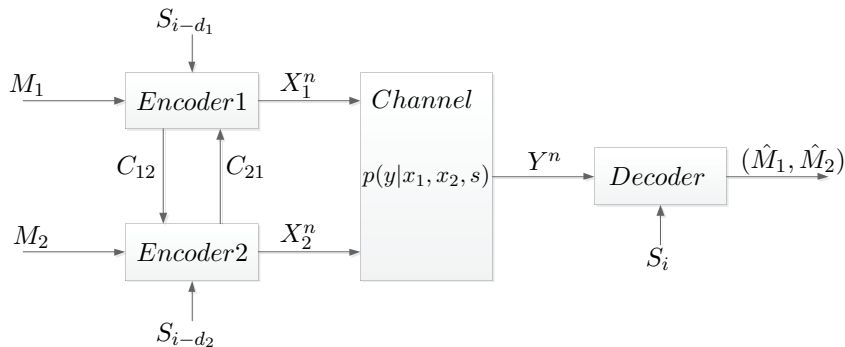
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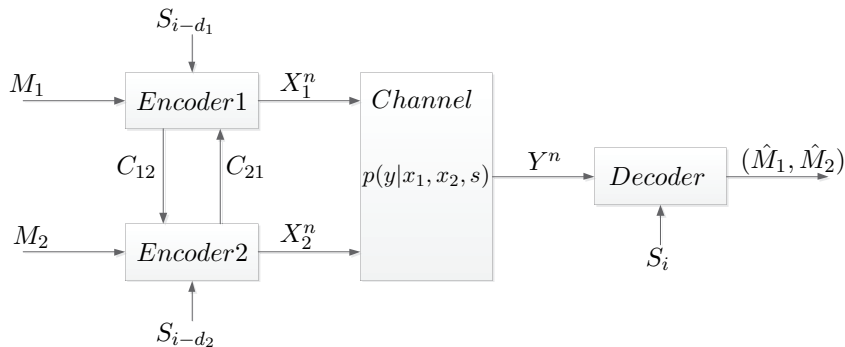
Achievability - Ideas and Discussion

- Coding scheme: Encode using MUX, decode simultaneously using joint-typicality.
 - Achieves every possible point in the region.
 - Can be easily extended to multiple users.
- Generalizes the result for the FSM-MAC with delayed CSI and no common message [Basher/Shirazy/P.11].

MAC with Conferencing and Delayed CSI



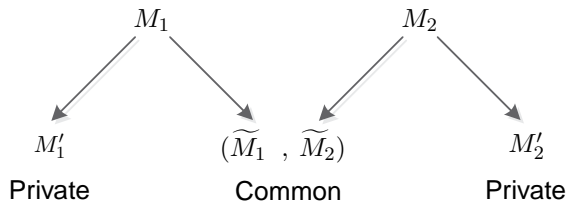
MAC with Conferencing and Delayed CSI



Share as much as possible of the messages through the conferencing links.

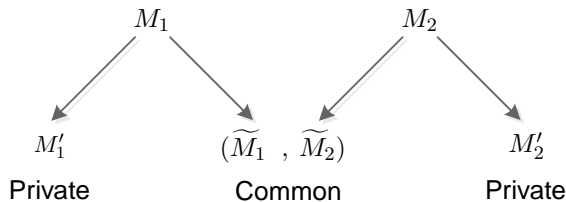
Conferencing Setting - Achievability Outline

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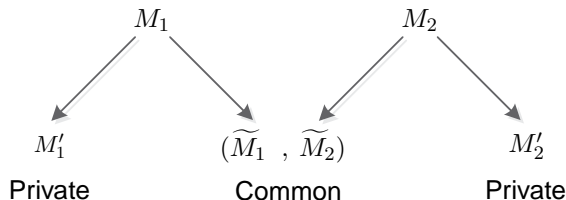
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| Message | Rate |
|---|-------------------|
| $M'_0 = (\widetilde{M}_1, \widetilde{M}_2)$ | $C_{12} + C_{21}$ |
| M'_1 | $R_1 - C_{12}$ |
| M'_2 | $R_2 - C_{21}$ |

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Using common message result:

Conferencing Setting - Achievability Outline

Using common message result:

$$(R_1 - C_{12}) \leq I(X_1; Y | X_2, U, S, \tilde{S}_1, \tilde{S}_2),$$

$$(R_2 - C_{21}) \leq I(X_2; Y | X_1, U, S, \tilde{S}_1, \tilde{S}_2),$$

$$(R_1 - C_{12}) + (R_2 - C_{21}) \leq I(X_1, X_2; Y | U, S, \tilde{S}_1, \tilde{S}_2),$$

$$(C_{12} + C_{21}) + (R_1 - C_{12}) + (R_2 - C_{21}) \leq I(X_1, X_2; Y | S, \tilde{S}_1, \tilde{S}_2).$$

Main Results with Conferencing and Delayed CSI

$$(d_1 \geq d_2)$$

Theorem

The capacity region of FSM-MAC with partially cooperative encoders, CSI at the decoder and CSI at the encoders with delays d_1 and d_2 , is

$$R_1 < I(X_1; Y | X_2, U, S, \tilde{S}_1, \tilde{S}_2) + C_{12},$$

$$R_2 < I(X_2; Y | X_1, U, S, \tilde{S}_1, \tilde{S}_2) + C_{21},$$

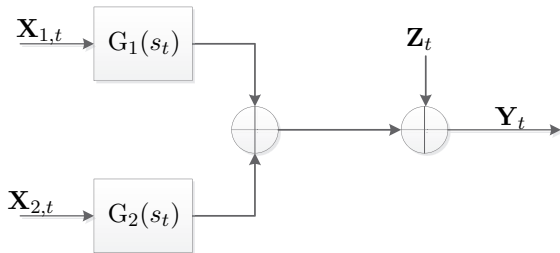
$$R_1 + R_2 < \min \left\{ \begin{array}{l} I(X_1, X_2; Y | U, S, \tilde{S}_1, \tilde{S}_2) + C_{12} + C_{21}, \\ I(X_1, X_2; Y | S, \tilde{S}_1, \tilde{S}_2) \end{array} \right\},$$

for some joint distribution of the form:

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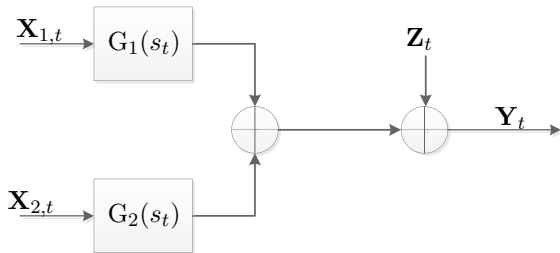
Vector Diagonal Gaussian FSM-MAC - Channel Model

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The channel model,

$$\underline{\mathbf{Y}}_t = G_1(s_t)\underline{\mathbf{X}}_{1,t} + G_2(s_t)\underline{\mathbf{X}}_{2,t} + \underline{\mathbf{Z}}_t,$$

Vector Diagonal Gaussian FSM-MAC - Channel Model

- $\{G_1(s)\}_{s \in \mathcal{S}}$ and $\{G_2(s)\}_{s \in \mathcal{S}}$ are real diagonal channel transition matrices of dimension $N \times N$.

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- \mathbf{Z} is independent of \mathbf{X}_1 and \mathbf{X}_2 .
- All vectors are real and of dimension $N \times 1$.
- The inputs are bounded by the following power constraints,

$$\text{tr}(\Sigma_{X_1 X_1}) \leq \mathcal{P}_1 \quad ; \quad \text{tr}(\Sigma_{X_2 X_2}) \leq \mathcal{P}_2.$$

Vector Diagonal Gaussian FSM-MAC - Proof Outline

- The main difficulty is to that a Gaussian triplet $(\mathbf{X}_1, \mathbf{U}, \mathbf{X}_2)$ satisfying

$$\mathbf{U} - \tilde{S}_1 - (S, \tilde{S}_2),$$

$$\mathbf{X}_1 - (\mathbf{U}, \tilde{S}_1) - (S, \tilde{S}_2),$$

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- Use an extension of the idea of [Lapidoth/Bross/Wigger08] and [Lapidoth/Venkatesan07].

Vector Diagonal Gaussian FSM-MAC - Main Result

$$R_1 < \frac{1}{2} \sum_{\tilde{s}_1} \pi(\tilde{s}_1) \sum_{\tilde{s}_2} K^{d_1-d_2}(\tilde{s}_2, \tilde{s}_1) \sum_s K^{d_2}(s, \tilde{s}_2) \sum_{i=1}^N \log \left(1 + (g_1^i(s))^2 \gamma_1^i(\tilde{s}_1) \right) + C_{12},$$

$$R_2 < \frac{1}{2} \sum_{\tilde{s}_1} \pi(\tilde{s}_1) \sum_{\tilde{s}_2} K^{d_1-d_2}(\tilde{s}_2, \tilde{s}_1) \sum_s K^{d_2}(s, \tilde{s}_2) \sum_{i=1}^N \log \left(1 + (g_2^i(s))^2 \gamma_2^i(\tilde{s}_1, \tilde{s}_2) \right) + C_{21},$$

$$R_1 + R_2 < \frac{1}{2} \sum_{\tilde{s}_1} \pi(\tilde{s}_1) \sum_{\tilde{s}_2} K^{d_1-d_2}(\tilde{s}_2, \tilde{s}_1) \sum_s K^{d_2}(s, \tilde{s}_2) \sum_{i=1}^N \log \left(1 + (g_1^i(s))^2 \gamma_1^i(\tilde{s}_1) \right. \\ \left. + (g_2^i(s))^2 \gamma_2^i(\tilde{s}_1, \tilde{s}_2) \right) + C_{12} + C_{21},$$

$$R_1 + R_2 < \frac{1}{2} \sum_{\tilde{s}_1} \pi(\tilde{s}_1) \sum_{\tilde{s}_2} K^{d_1-d_2}(\tilde{s}_2, \tilde{s}_1) \sum_s K^{d_2}(s, \tilde{s}_2) \sum_{i=1}^N \log \left(1 + (g_1^i(s))^2 P_1^i(\tilde{s}_1) \right. \\ \left. + (g_2^i(s))^2 P_2^i(\tilde{s}_1, \tilde{s}_2) + 2g_1^i(s)g_2^i(s) \sqrt{(P_1^i(\tilde{s}_1) - \gamma_1^i(\tilde{s}_1))(P_2^i(\tilde{s}_1, \tilde{s}_2) - \gamma_2^i(\tilde{s}_1, \tilde{s}_2))} \right),$$

subject to the constraints,

$$\sum_{\tilde{s}_1} \pi(\tilde{s}_1) \sum_{i=1}^N P_1^i(\tilde{s}_1) \leq \mathcal{P}_1, \quad ; \quad \sum_{\tilde{s}_1} \pi(\tilde{s}_1) \sum_{\tilde{s}_2} K^{d_1-d_2}(\tilde{s}_2, \tilde{s}_1) \sum_{i=1}^N P_2^i(\tilde{s}_1, \tilde{s}_2) \leq \mathcal{P}_2,$$

$$0 \leq \gamma_1^i(\tilde{s}_1) \leq P_1^i(\tilde{s}_1), \forall i \in \{1, \dots, N\}, \tilde{s}_1 \in \mathcal{S},$$

$$0 \leq \gamma_2^i(\tilde{s}_1, \tilde{s}_2) \leq P_2^i(\tilde{s}_1, \tilde{s}_2), \forall i \in \{1, \dots, N\}, (\tilde{s}_1, \tilde{s}_2) \in \mathcal{S}^2.$$

Example: Gilbert-Elliot Gaussian MAC

- At any given time t the channel is in one of two possible states, *Good* or *Bad*.
- $g_1(G) > g_1(B)$ and $g_2(G) > g_2(B)$.

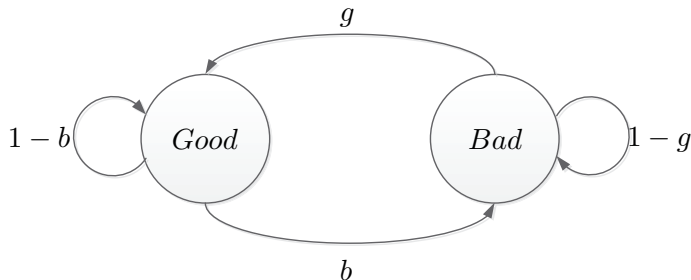
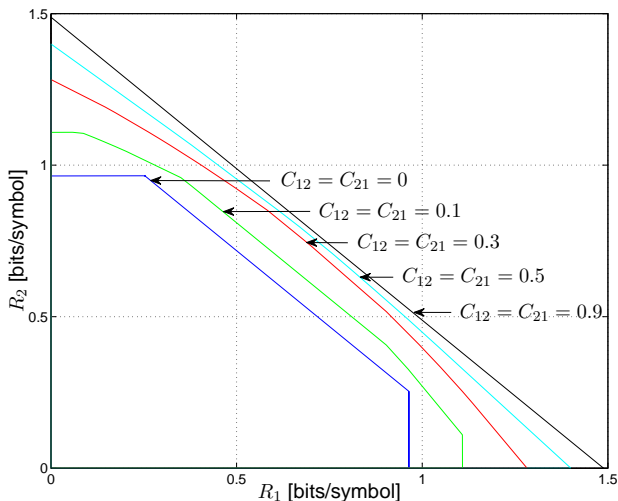


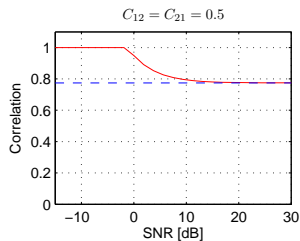
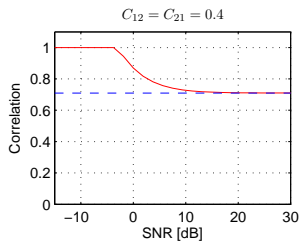
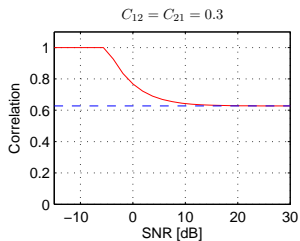
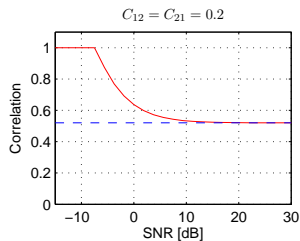
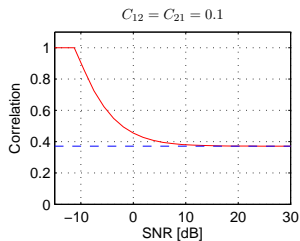
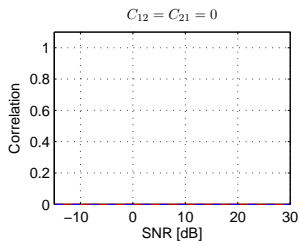
Figure : Two-state AGN channel.

Capacity region of Two-State AGN MAC Example

Fixed delays $d_1 = d_2 = 2$ and symmetrical con. $C_{12} = C_{21}$



Correlation versus SNR



Summary

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Thank you!

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- Channel encoder,

$$X_{1,i} = \left\{ \begin{array}{ll} f_{1,i}(M_1, V_2^\ell), & 1 \leq i \leq d_1 \\ f_{1,i}(M_1, V_2^\ell, S^{i-d_1}), & d_1 + 1 \leq i \leq n \end{array} \right\},$$

$$X_{2,i} = \left\{ \begin{array}{ll} f_{2,i}(M_2, V_1^\ell), & 1 \leq i \leq d_2 \\ f_{2,i}(M_2, V_1^\ell, S^{i-d_2}), & d_2 + 1 \leq i \leq n \end{array} \right\}.$$

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- Solution: Encode using MUX, decode simultaneously using joint-typicality.
- Error analysis yield many inequalities.

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- If both, the encoder and decoder, know the state (with or without delay) one can use MUX-DEMUX scheme. [Goldsmith/varaiya97] [Viswanathan99]
- Problem 1: Here there is an asymmetry between the encoders and the decoder.
- Solution: Can be solved by working on the corner points and using successive decoding. [Basher/Shirazy/P.11]
- Problem 2: Common message generates many corner-points.
- Solution: Encode using MUX, decode simultaneously using joint-typicality.
- Error analysis yield many inequalities.
- The inequalities are reduced using induction and the Fourier-Motzkin elimination.

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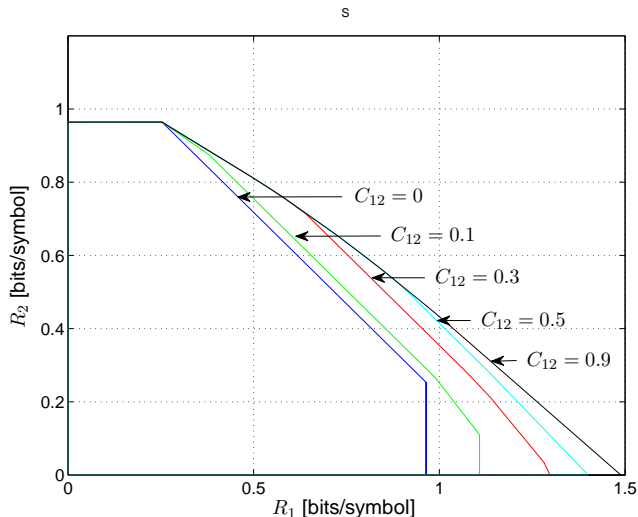
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- MAC with delayed state need one auxiliary.
- Auxiliaries can be combined.
- Identification of the auxiliary random variable U as the **common knowledge** of the two encoders,
 $U_i = (M_0, S^{i-d_1-1})$.

Capacity region of Two-State AGN MAC Example

Fixed delays $d_1 = d_2 = 2$ and asymmetrical con. $C_{12} \geq C_{21} = 0$



Capacity region of Two-State AGN MAC Example

Fixed delays $d_1 = d_2 = 2$ and infinite con. $C_{12} \leq C_{21} = \infty$

