Semi-Deterministic Broadcast Channels with Cooperation

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Outline

- Motivation and past work
- AK problem with one-sided encoder cooperation
- SD-BC with one-sided decoder cooperation
- Duality
- Summary

Motivation and Past Work

- The two-encoder multiterminal source coding problem [Berger, 1978], [Tung, 1978].

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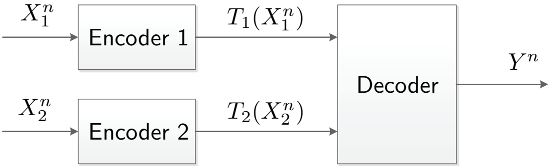

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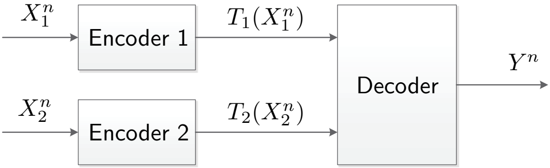

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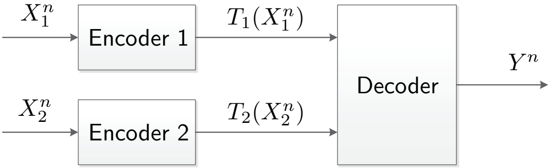

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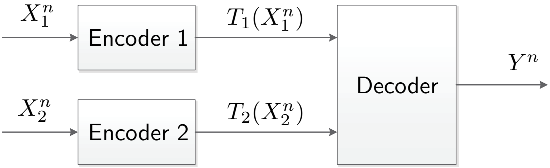

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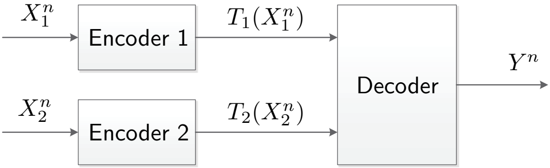

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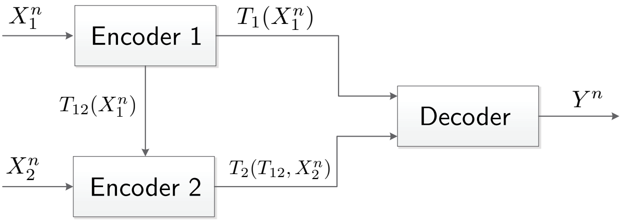
  - Milestone towards multiuser channel-source duality.

AK Problem with Cooperation - Definition

Without cooperation [Ahlswede-Körner, 1975]



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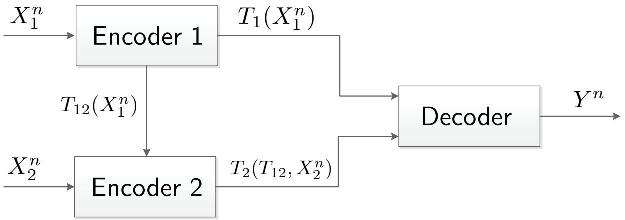

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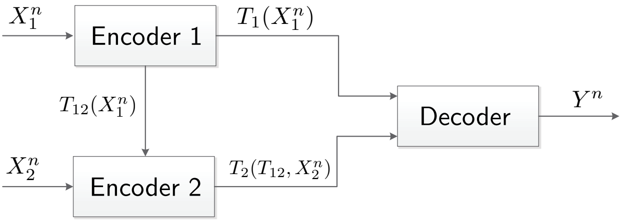

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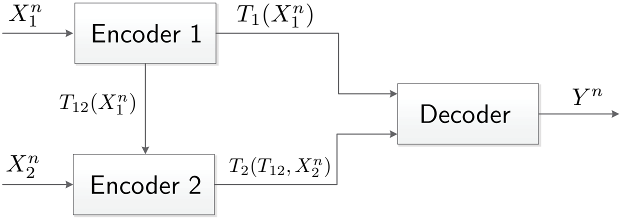

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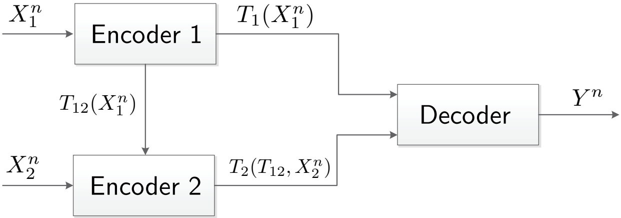

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Theorem (Coordination-Capacity Region)

For a desired coordination distribution $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

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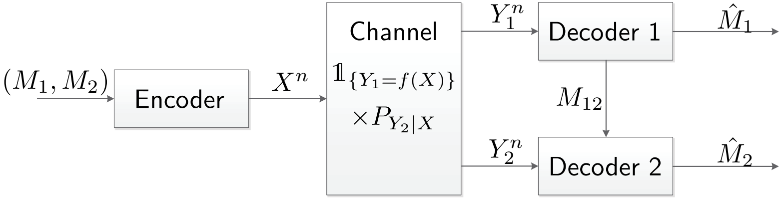

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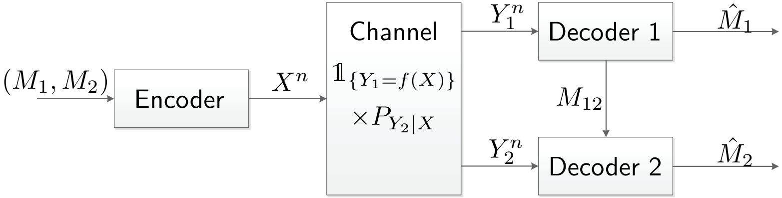

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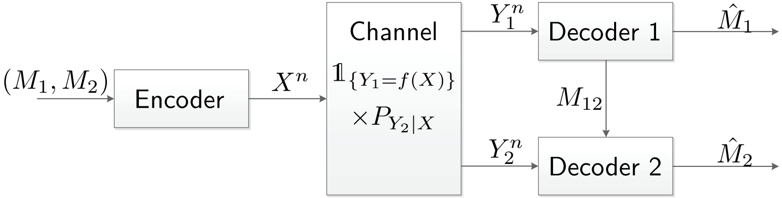

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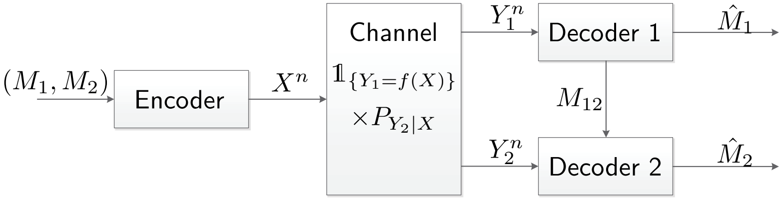

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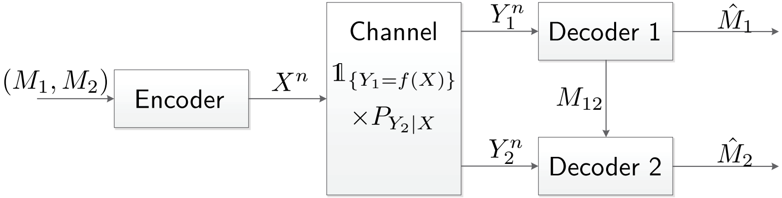

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- **Decoders' Output:** $\hat{M}_1(Y_1^n)$ and $\hat{M}_2(M_{12}, Y_2^n)$.

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Theorem (Capacity Region)

The capacity region is:

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Achievability via rate splitting, Marton coding and Wyner-Ziv-like coding for cooperation protocol.

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Generalization of [Lapidoth and Wang, 2013].

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- Solving one problem $\Longrightarrow$ Valuable insight into solving dual.

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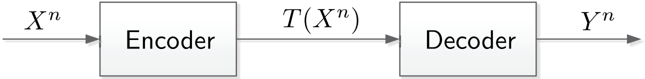

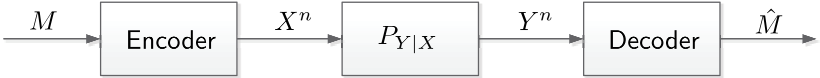

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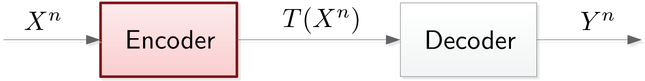

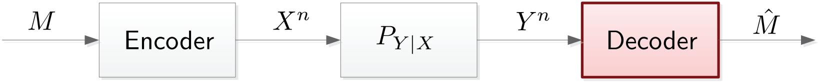

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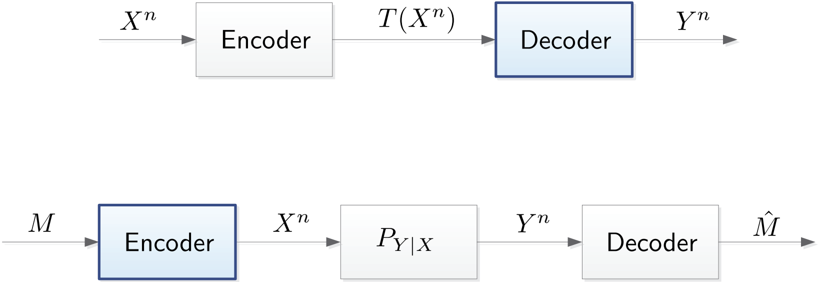

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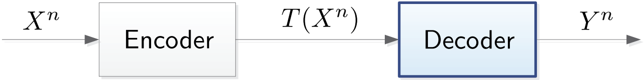
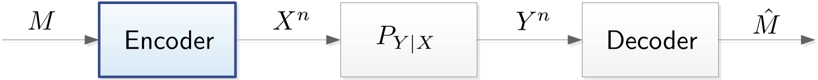
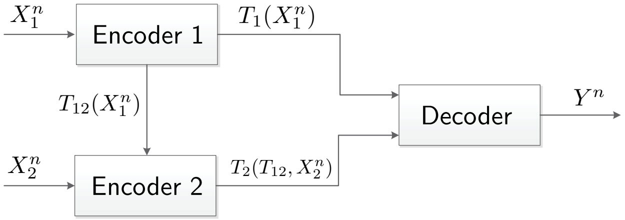


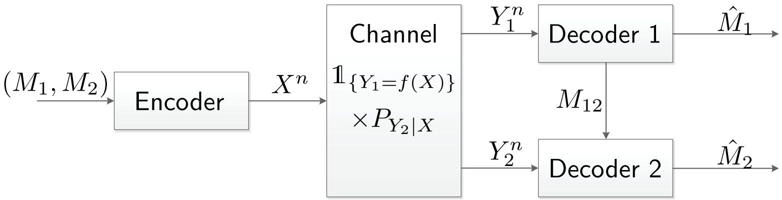


$$C = \max_{P_X} I(X;Y)$$

Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:





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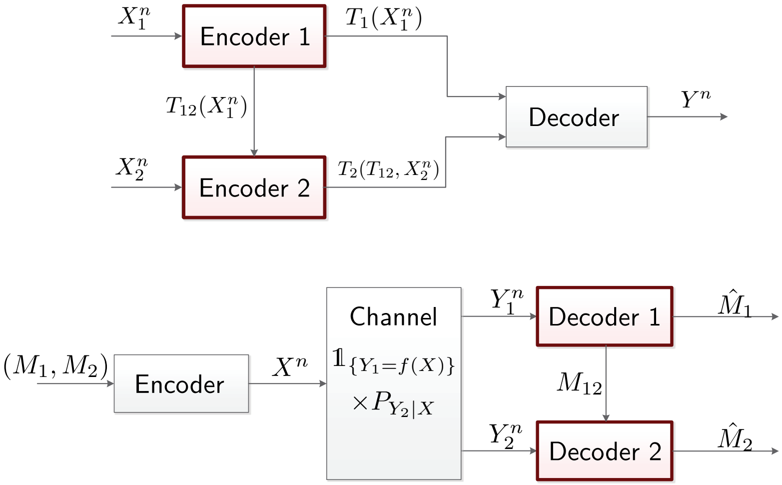

Duality - Multi-User Case

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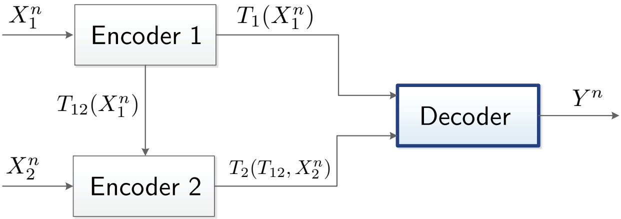

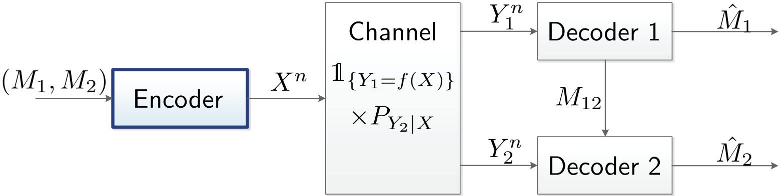

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AK Problem vs. Semi-Deterministic BC:

Probabilistic relations are preserved:

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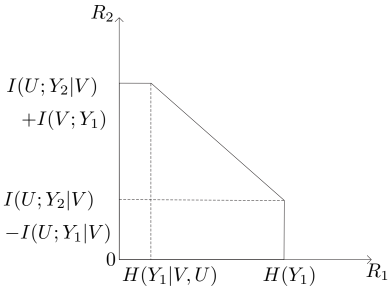
Probabilistic relations are preserved:

Semi-Deterministic BC | **AK Problem**

$$(X^n, Y_1^n, Y_2^n) \in \mathcal{T}_\epsilon^{(n)}\left(P_X^\star \mathbb{1}_{\{Y_1=f(X)\}} P_{Y_2|X}\right) \longleftrightarrow (Y^n, X_1^n, X_2^n) \in \mathcal{T}_\epsilon^{(n)}\left(P_Y \mathbb{1}_{\{X_1=f(Y)\}} P^\star_{X_2|Y}\right)$$

Duality - Corner Point Correspondence

For fixed joint distributions and R_{12}:



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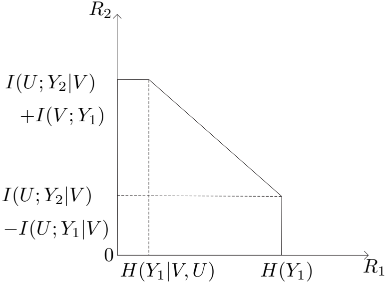

Semi-Deterministic BC with Cooperation	**Ahlswede-Körner Problem with Cooperation**
$R_{12} = I(V;Y_1) - I(V;Y_2)$	$R_{12} = I(V;X_1) - I(V;X_2)$
(R_1, R_2) at Lower Corner Point: $\Big(H(Y_1)\ ,\ I(U;Y_2\|V) - I(U;Y_1\|V) \Big)$	(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \Big)$
(R_1, R_2) at Upper Corner Point: $\Big(H(Y_1\|V,U)\ ,\ I(U;Y_2\|V) + I(V;Y_1) \Big)$	(R_1, R_2) at Upper Corner Point: $\Big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1) \Big)$

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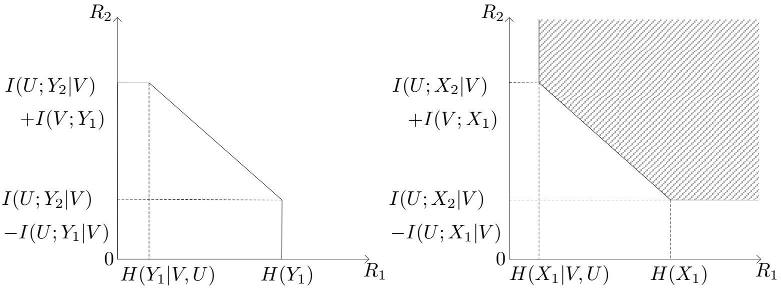

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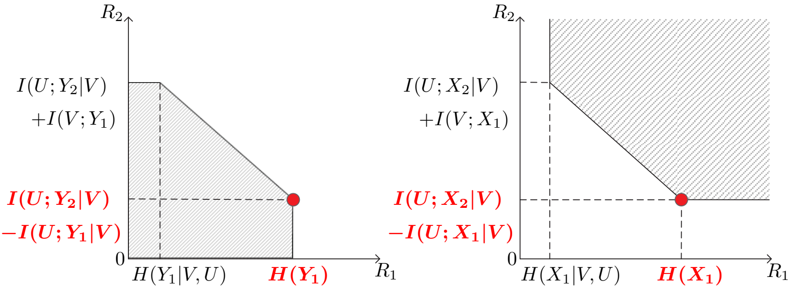

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Duality - Corner Point Correspondence

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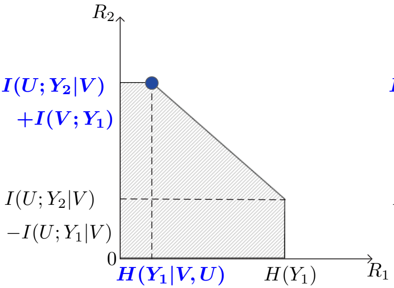

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Summary

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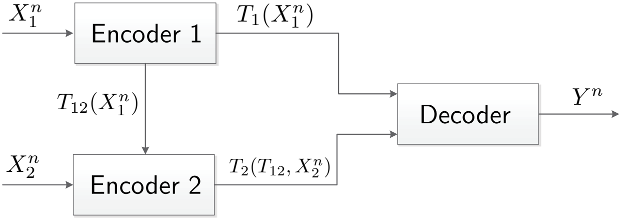
Thank you!

AK Problem with Cooperation - Achievability Outline



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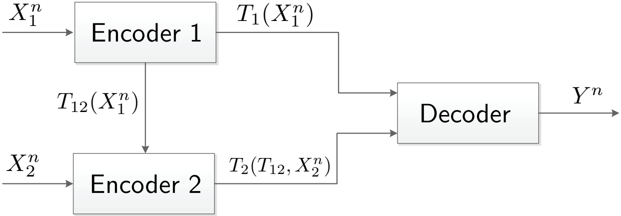

Rate	Corner Point 1	Corner Point 2
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$H(X_1\|V,U)$
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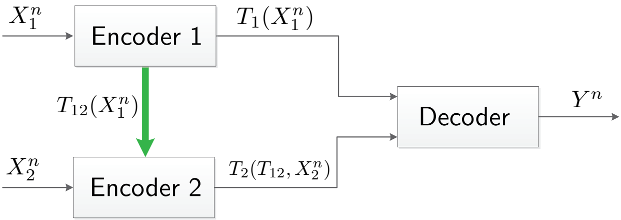
AK Problem with Cooperation - Achievability Outline



Rate	Corner Point 1	Corner Point 2
R_{12}	$\boldsymbol{I(V;X_1) - I(V;X_2)}$	$\boldsymbol{I(V;X_1) - I(V;X_2)}$
R_1	$H(X_1)$	$H(X_1\|V,U)$
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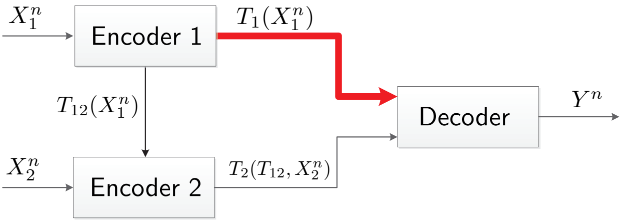
- **Cooperation:** Wyner-Ziv scheme to convey V^n via cooperation link.

AK Problem with Cooperation - Achievability Outline



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R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
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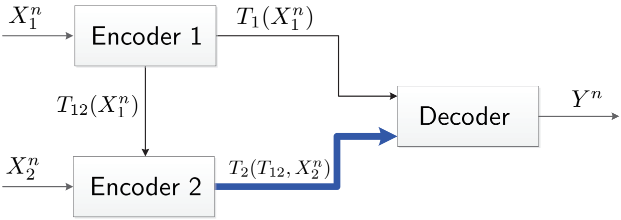
AK Problem with Cooperation - Achievability Outline



Rate	Corner Point 1	**Corner Point 2**
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
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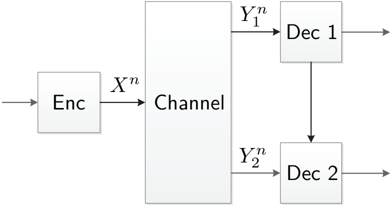
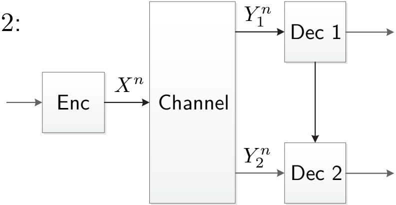
Semi-Deterministic BC with Cooperation - Achievability Outline



Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:



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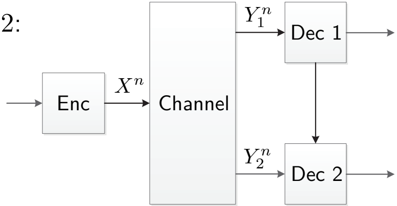
  - (M_{10}, M_{20}) - Public message;



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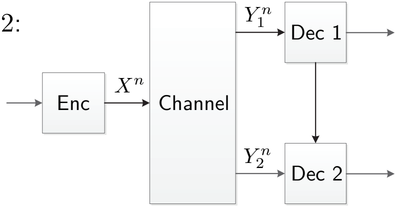

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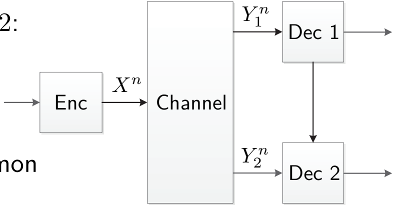

- **Codebook Structure:** Marton (with common message).

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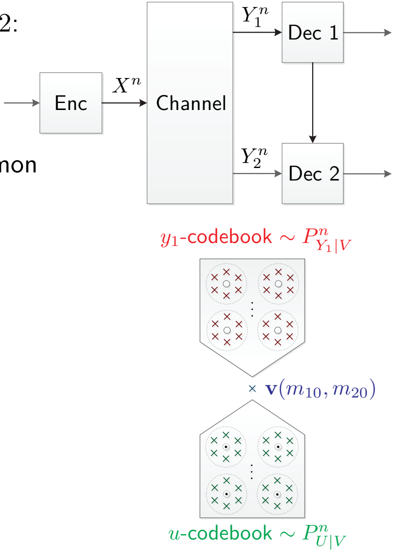

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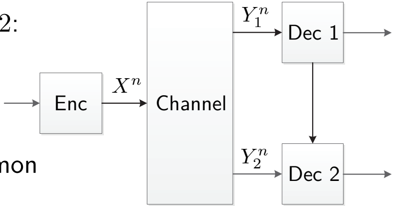

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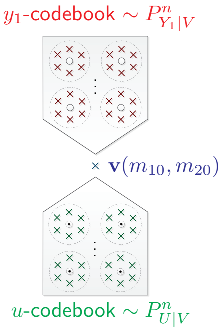

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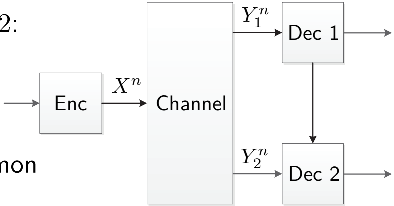

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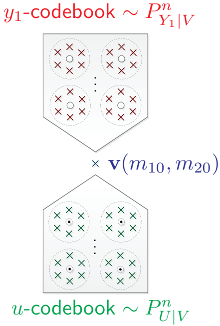
  1. Partition common message c.b into $2^{nR_{12}}$ bins.



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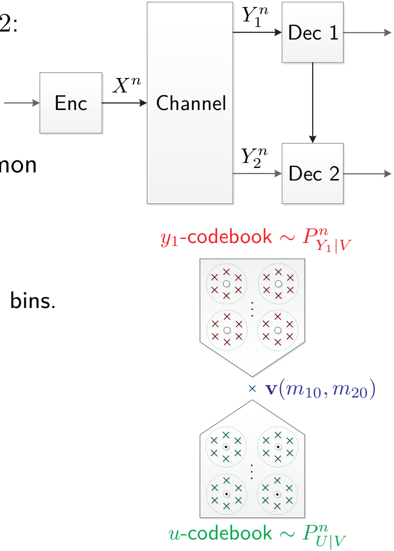

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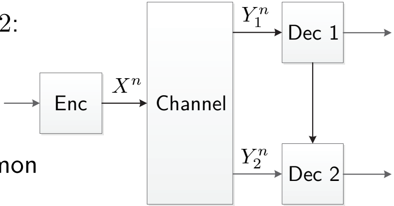

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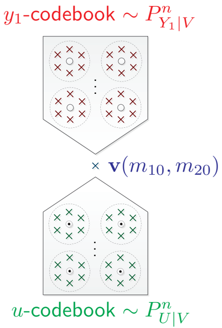

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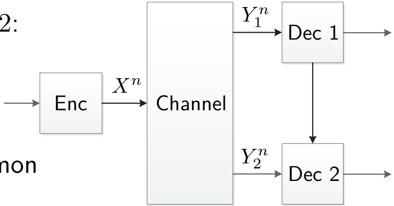

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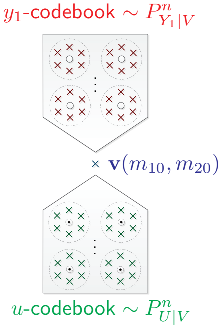

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

Semi-Deterministic BC with Cooperation - Converse Outline

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1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

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$$= \sum_{i=1}^{n} \left[\boldsymbol{I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i)} \right] + I(M_2; M_{12})$$

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$$= \boldsymbol{\sum_{i=1}^{n} \Big[I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i) \Big]} + I(M_2; M_{12})$$

$$= \boldsymbol{\sum_{i=1}^{n} \Big[I(M_2; Y_{2,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_{1,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \Big]} + I(M_2; M_{12})$$

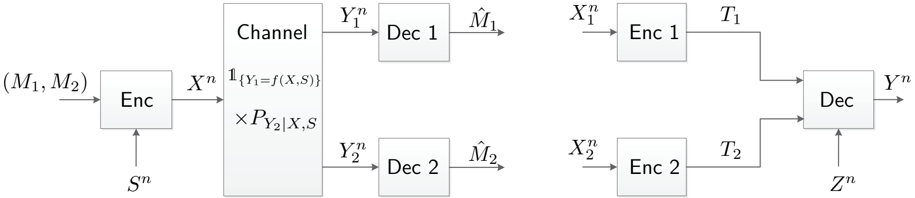
Replaces 4 uses of Csiszár Sum Identity!

Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

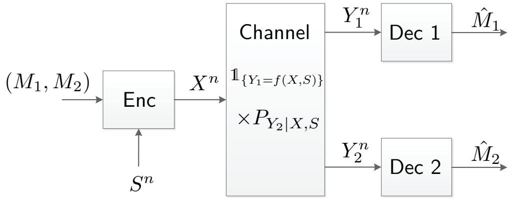
Multi-User Duality - Additional Examples

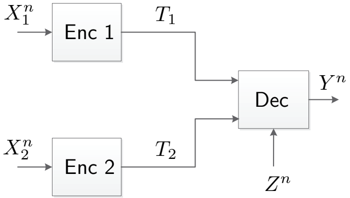
State-Dependant Semi-Deterministic BC vs. Dual:

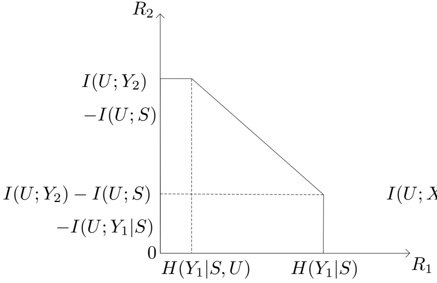


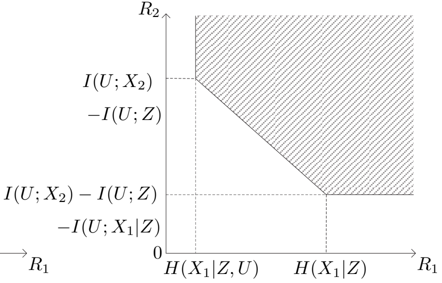
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:









Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:

Multi-User Duality - Additional Examples

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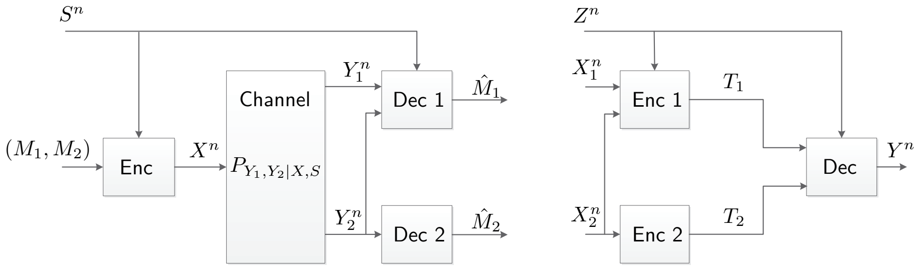

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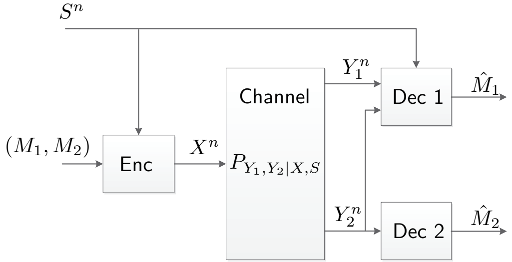

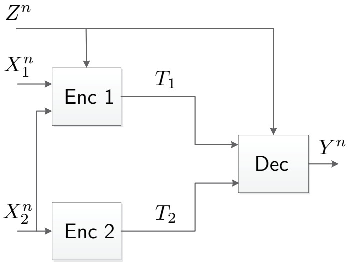

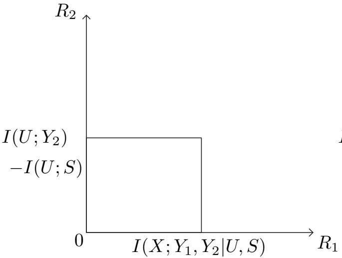

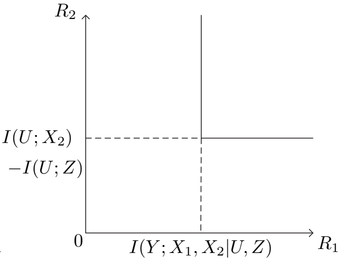

Multi-User Duality - Additional Examples

Action-Dependant Output-Degraded BC vs. Dual:

Multi-User Duality - Additional Examples

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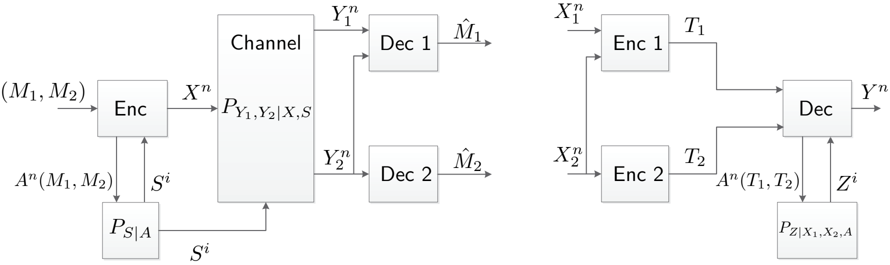

Multi-User Duality - Additional Examples

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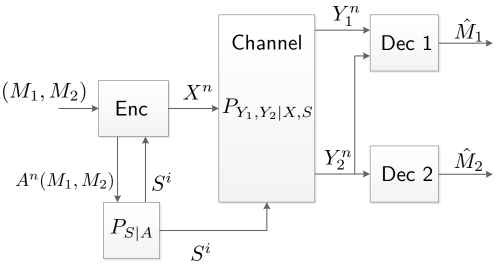

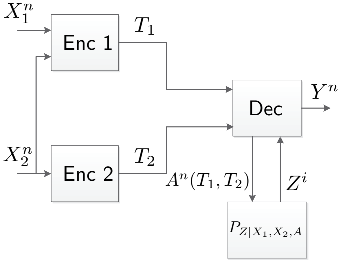

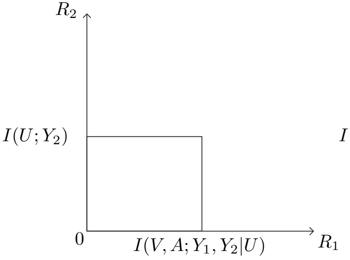

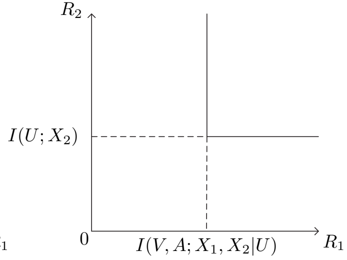

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 1:

$$\Big(\, I(V;X_1|X_2)\ ,\ H(X_1)\ ,\ I(U;X_2|X_1,V)\,\Big).$$

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- **Encoder 1 to Decoder:** Conveys X_1^n to the decoder in a lossless manner.
- **Encoder 2 to Decoder:** The decoder knows X_1^n and therefore V^n. Wyner-Ziv coding to convey U^n.

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

$$\Big(\, I(V;X_1|X_2)\ ,\ H(X_1|V,U)\ ,\ I(U;X_2|V)+I(V;X_1)\,\Big).$$

AK Problem with Cooperation - Achievability Outline

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- **Cooperation:** Same.

AK Problem with Cooperation - Achievability Outline

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AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

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- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows V^n. Conveys the index of V^n and uses superposition coding to convey U^n.
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AK Problem with Cooperation - Proof Outline

Converse:

AK Problem with Cooperation - Proof Outline

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- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\setminus i}, X_{2,i+1}^n),$$
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for every $1 \le i \le n$.

AK Problem with Cooperation - Proof Outline

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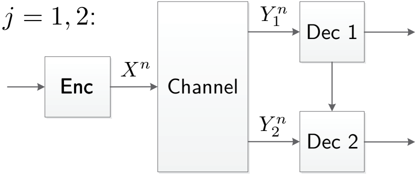
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:



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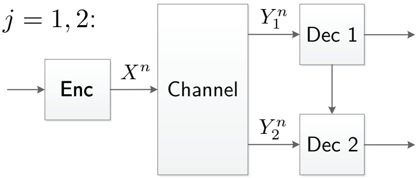

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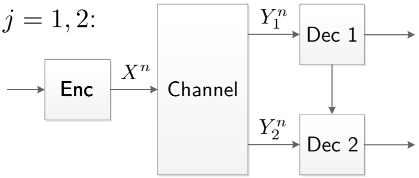

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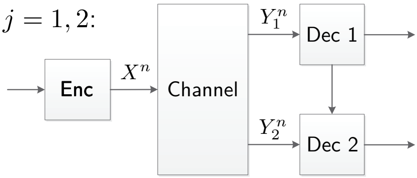

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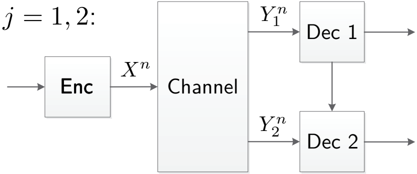

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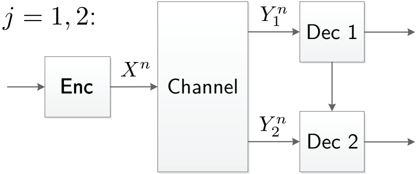

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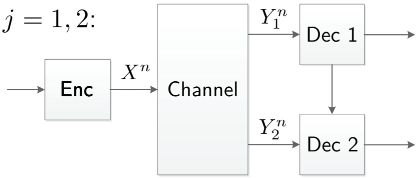

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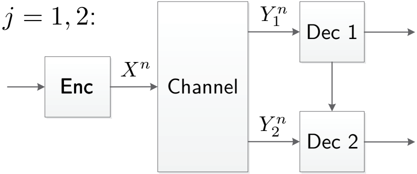

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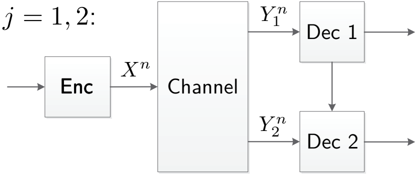

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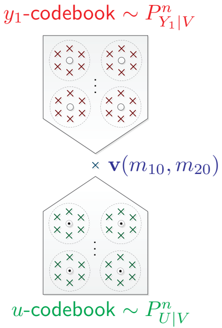

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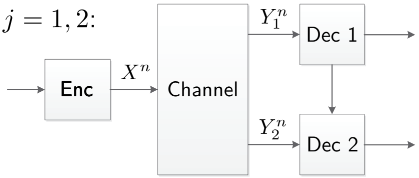

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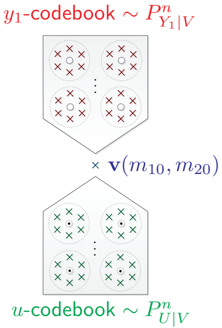

- **Decoding:** Joint typicality decoding.

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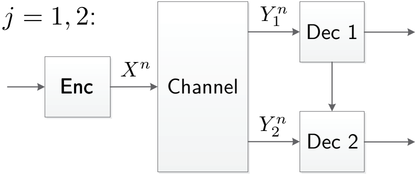

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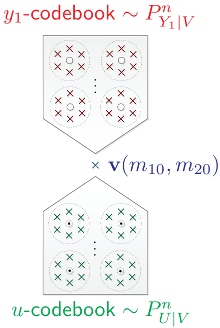

- **Decoding:** Joint typicality decoding.

- **Cooperation:** Bin number of V^n - $2^{nR_{12}}$ bins.

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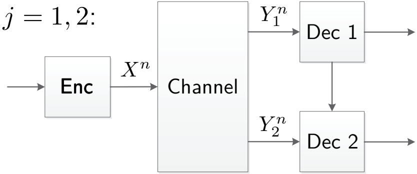

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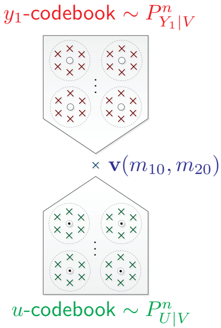

- **Decoding:** Joint typicality decoding.
- **Cooperation:** Bin number of V^n - $2^{nR_{12}}$ bins.
- **Gain:** Dec. 2 reduces search space of V^n by R_{12}.

Semi-Deterministic BC with Cooperation - Proof Outline

Achievability: Split $M_i = (M_{i0}, M_{ii})$, $i = 1, 2$. Code construction:

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