The Ahlswede-Körner Coordination Problem with One-Sided Encoder Cooperation

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Outline

- Motivation and past work
- AK problem with one-sided encoder cooperation
- Semi-deterministic BC with one-sided decoder cooperation
- Duality
- Summary

Motivation and Past Work

- The two-encoder multiterminal source coding problem [Berger, 1978], [Tung, 1978].

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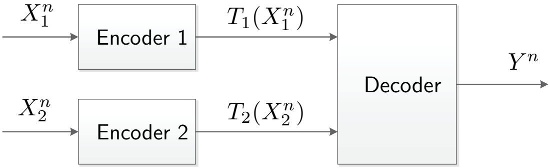

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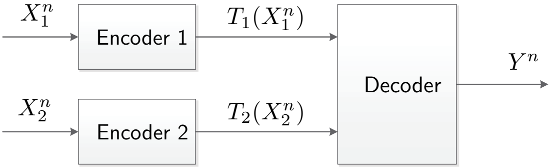

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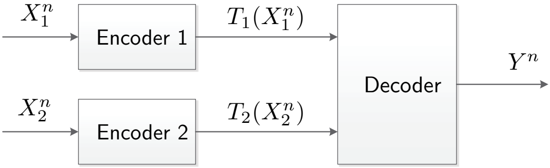

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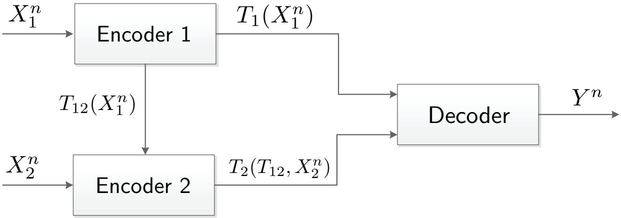
- Ahlswede-Körner problem (1975)
- Cooperation can dramatically boost performance of a network.

AK Problem with Cooperation - Definition

Without cooperation [Ahlswede-Körner, 1975]



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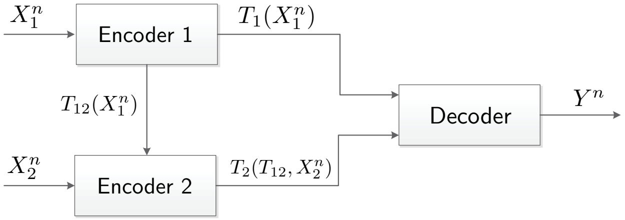

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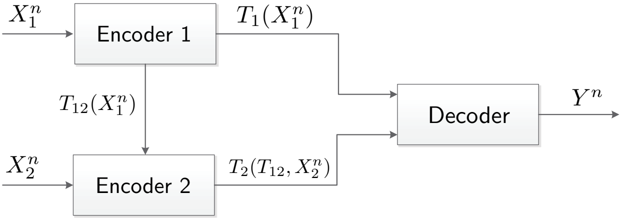

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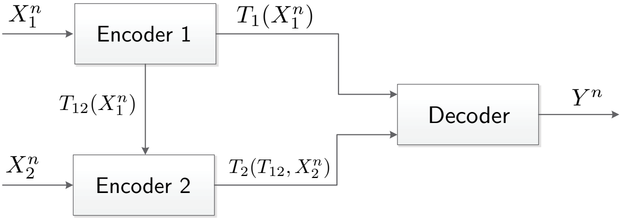

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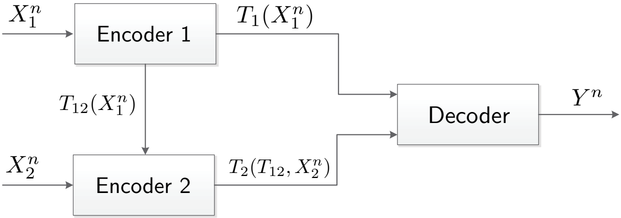

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- **Decoder output:** $(X_1^n, X_2^n, Y^n) \in \mathcal{T}_\epsilon^{(n)}\big(f(Y), X_2, Y\big)$

AK Problem with Cooperation - Solution

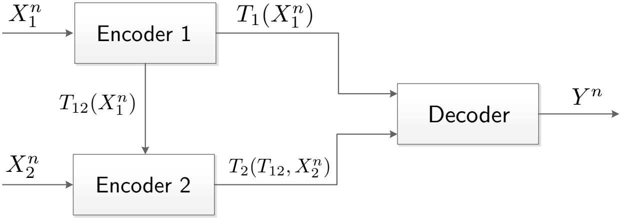
Theorem (Coordination-Capacity Region)

For a desired coordination distribution $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

$$\mathcal{C}_{AK} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;X_1) - I(V;X_2) \\ R_1 \geq H(X_1|V,U) \\ R_2 \geq I(U;X_2|X_1,V) \\ R_1 + R_2 \geq H(X_1|V,U) + I(V,U;X_1,X_2) \end{array} \right\}$$

where the union is over all $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

AK Problem with Cooperation - Achievability Outline



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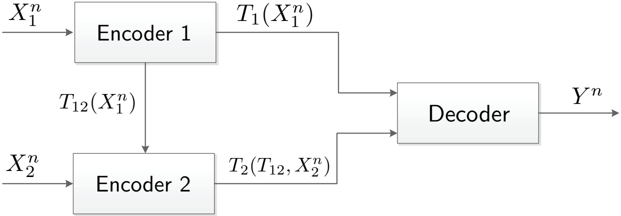

Rate	Corner Point 1	Corner Point 2
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$H(X_1\|V,U)$
R_2	$I(U;X_2\|V) - I(U;X_1\|V)$	$I(U;X_2\|V) + I(V;X_1)$

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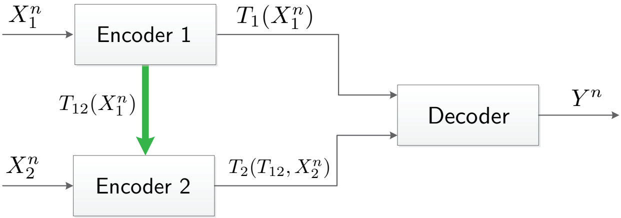

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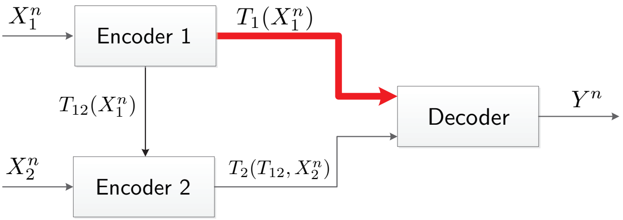

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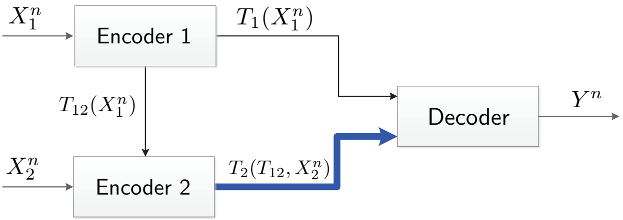

Rate	Corner Point 1	**Corner Point 2**
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$\boldsymbol{H(X_1\|V,U)}$
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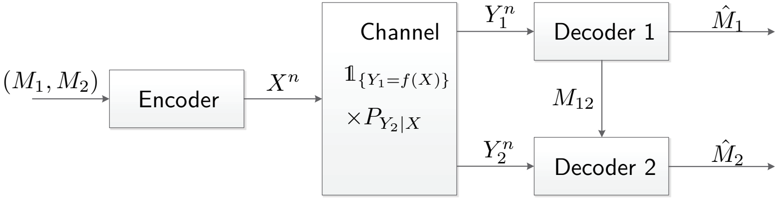
- **Cooperation:** Wyner-Ziv scheme to convey V^n via cooperation link.
- **Lower CP:** V^n is transmitted to dec. by Enc. 1 within X_1^n.
- **Corner Point 2:** V^n is explicitly transmitted to dec. by Enc. 2.

Semi-Deterministic BC with Cooperation - Definition

Without cooperation [Gelfand and Pinsker, 1980]



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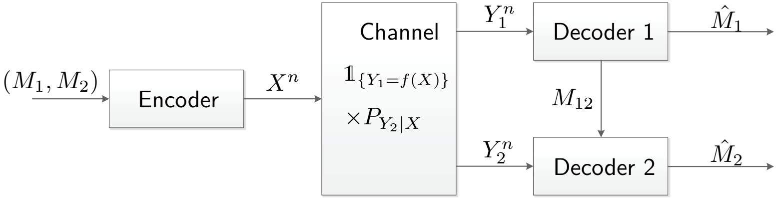

- **Messages:** $(M_1, M_2) \sim \text{Unif}\{1, \ldots, 2^{nR_1}\} \times \{1, \ldots, 2^{nR_2}\}$.

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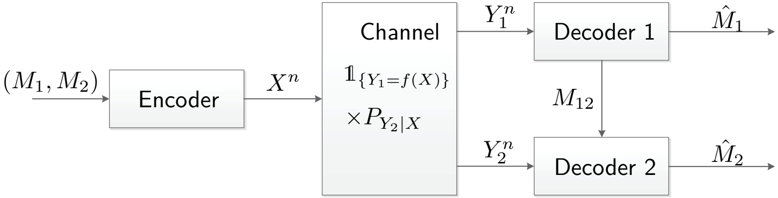

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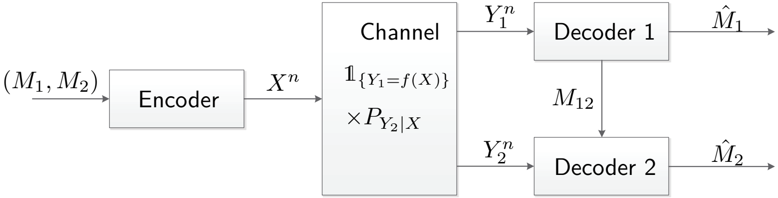

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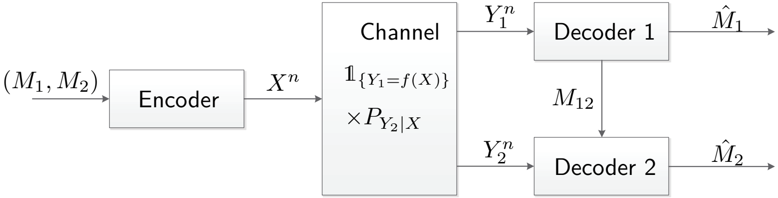

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- **Decoders' output:** $\hat{M}_1(Y_1^n)$ and $\hat{M}_2(M_{12}, Y_2^n)$;

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Theorem (Capacity Region)

The capacity region is:

$$\mathcal{C}_{BC} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;Y_1) - I(V;Y_2) \\ R_1 \leq H(Y_1) \\ R_2 \leq I(V,U;Y_2) + R_{12} \\ R_1 + R_2 \leq H(Y_1|V,U) + I(U;Y_2|V) + I(V;Y_1) \end{array} \right\}$$

where the union is over all $P_{V,U,Y_1} P_{X|V,U,Y_1} P_{Y_2|X}$ *in which* $Y_1 = f(X)$.

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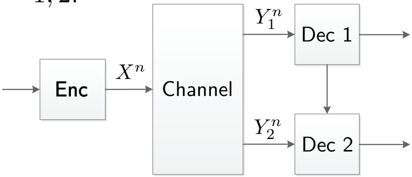

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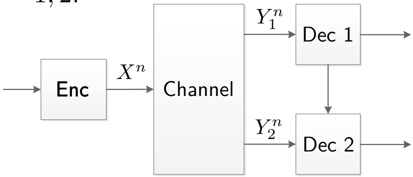

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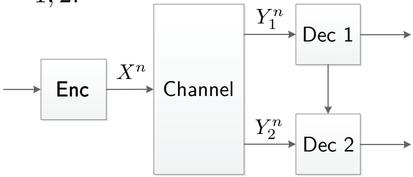

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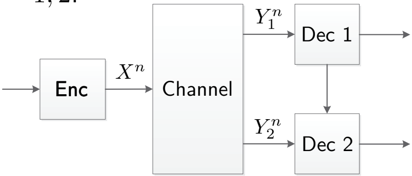

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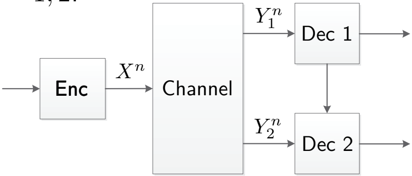

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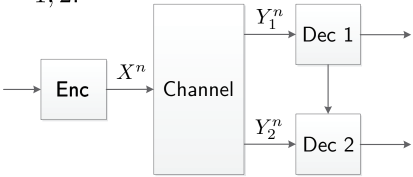

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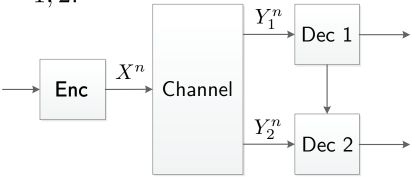

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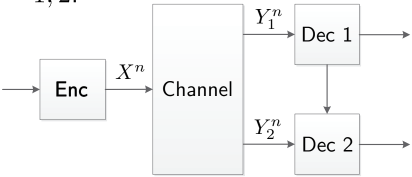

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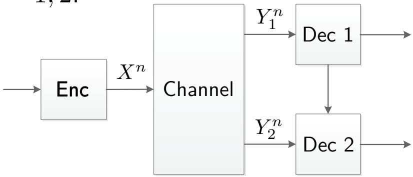

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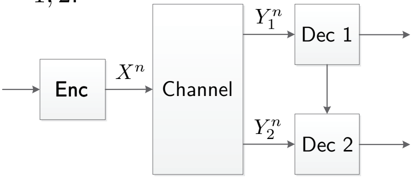

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- **Private Messages:** Marton coding:
  1. $M_{11} \longrightarrow Y_1^n$; decoded by Dec. 1.
  2. $M_{22} \longrightarrow U^n$ (superposition on V^n); decoder by Dec. 2 using V^n.

Semi-Deterministic BC with Cooperation - Converse Outline

Via novel approach - Probabilistic construction of auxiliaries:

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Generalization of [Lapidoth and Wang, 2013].

Duality - Preface

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  - Optimization domain may vary.
- A formal proof of duality is still absent.

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"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
(C. E. Shannon, 1958)

PTP Duality: [Shannon, 1958], [Cover and Chiang, 2002], [Pradhan *et al.*, 2003], [Gupta and Verdú, 2011].

- The solutions of the problems are dual.
  - Information measures admit dual forms.
  - Optimization domain may vary.
- A formal proof of duality is still absent.
- Solving one problem provides valuable insight towards the solution of the other.

Duality - Preface

Point-to-Point Case:

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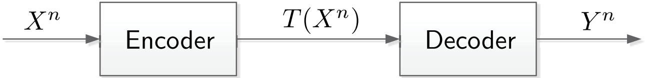

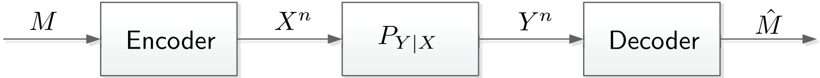

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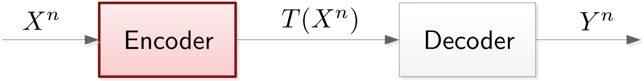

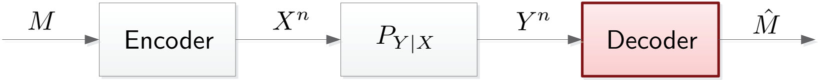

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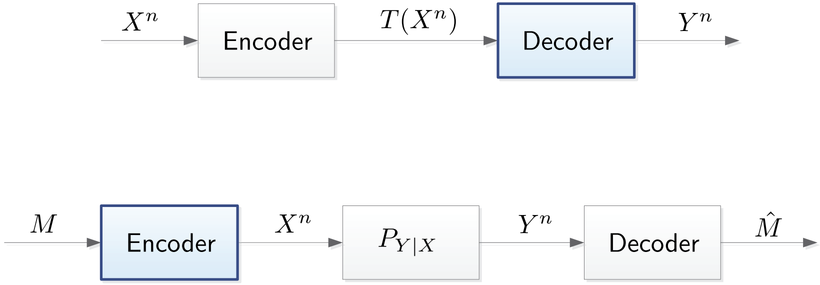
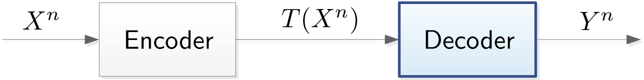
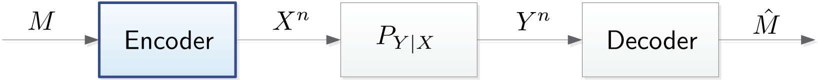


Point-to-Point Case:

$$R^\star = \min_{P_{Y|X}} I(X;Y)$$

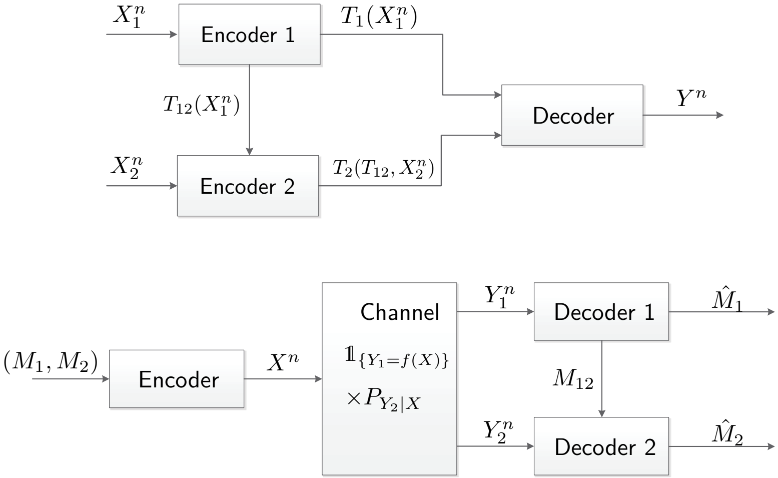




$$C = \max_{P_X} I(X;Y)$$

Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:



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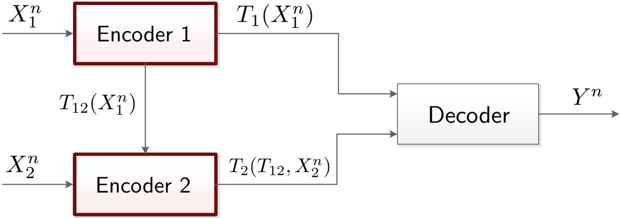

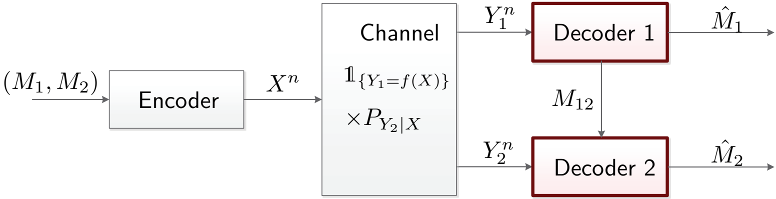

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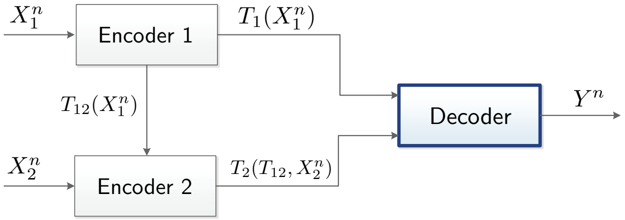
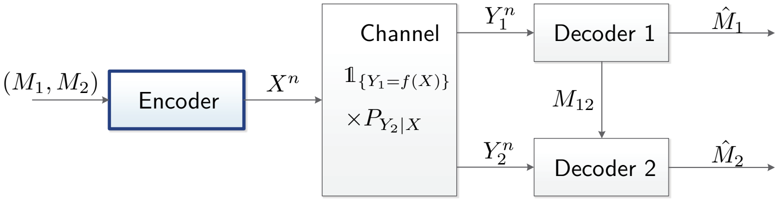




Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:

Probabilistic relations are preserved:

Duality - Multi-User Case

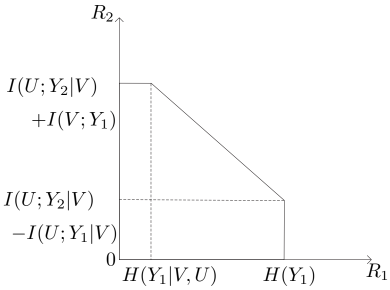
AK Problem vs. Semi-Deterministic BC:

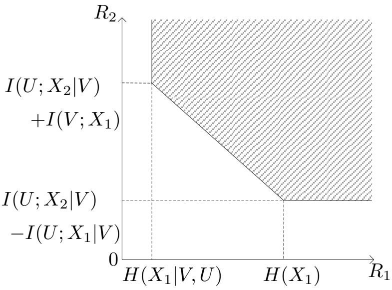
Probabilistic relations are preserved:

Semi-Deterministic BC		**AK Problem**
$(X^n, Y_1^n, Y_2^n) \in \mathcal{T}_\epsilon^{(n)}(X, f(X), Y_2)$	$\longleftrightarrow$	$(Y^n, X_1^n, X_2^n) \in \mathcal{T}_\epsilon^{(n)}(Y, f(Y), X_2)$
$(P_X^\star \mathbb{1}_{\{Y_1=f(X)\}} P_{Y_2\|X})$		$(P_Y \mathbb{1}_{\{X_1=f(Y)\}} P_{X_2\|Y}^\star)$

Duality - Corner Point Correspondence

For fixed joint distributions:





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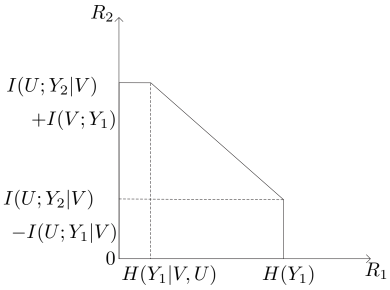

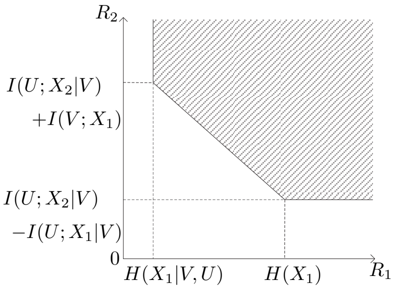

Semi-Deterministic BC with Cooperation	**Ahlswede-Körner Problem with Cooperation**
$R_{12} = I(V;Y_1) - I(V;Y_2)$	$R_{12} = I(V;X_1) - I(V;X_2)$
(R_1, R_2) at Lower Corner Point: $\Big(H(Y_1)\ ,\ I(U;Y_2\|V) - I(U;Y_1\|V) \Big)$	(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \Big)$
(R_1, R_2) at Upper Corner Point: $\Big(H(Y_1\|V,U)\ ,\ I(U;Y_2\|V) + I(V;Y_1) \Big)$	(R_1, R_2) at Upper Corner Point: $\Big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1) \Big)$

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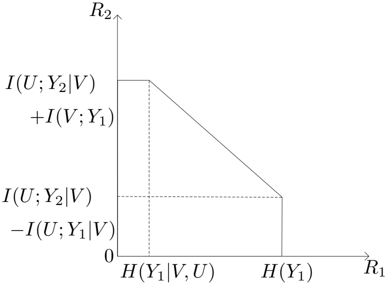

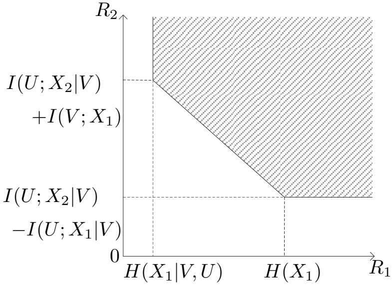

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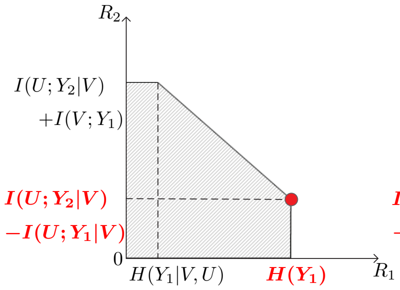

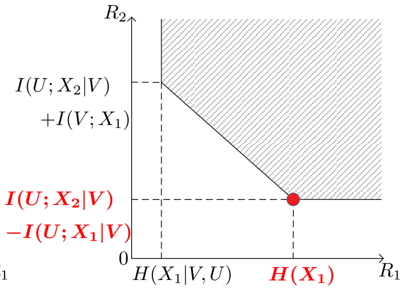

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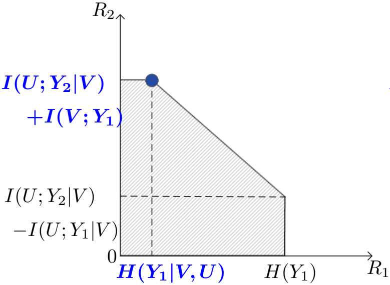

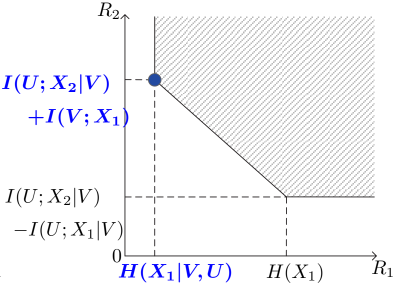

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Summary

- AK problem with cooperation.

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Thank you!

Multi-User Duality - Additional Examples

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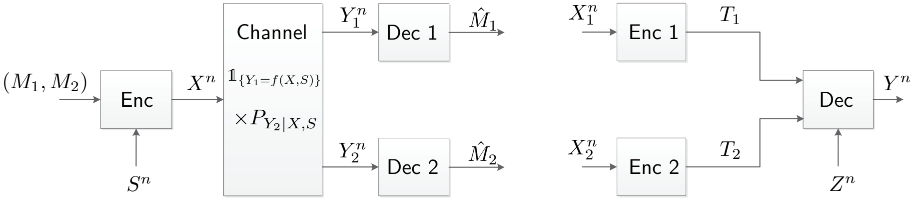

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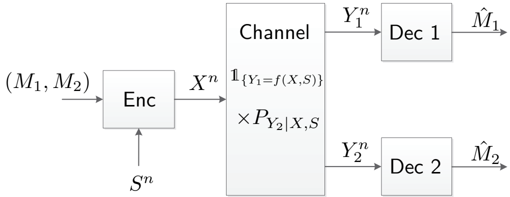

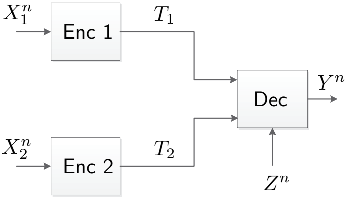

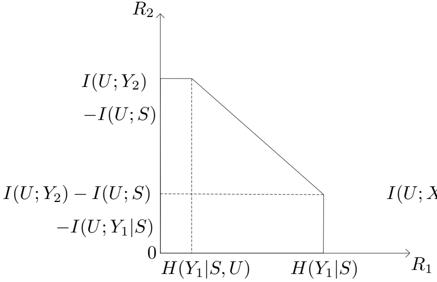

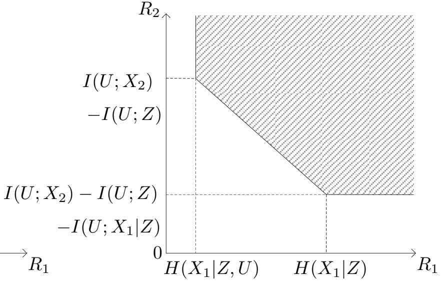

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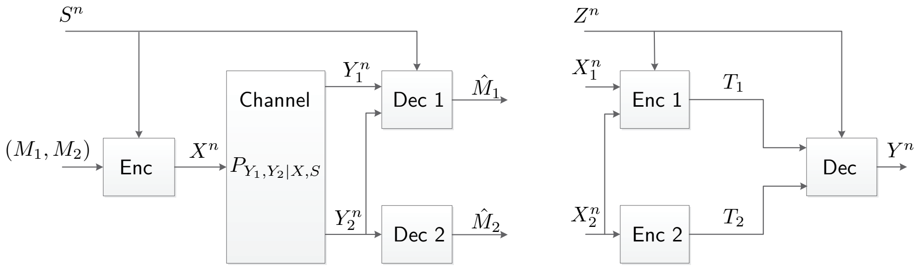

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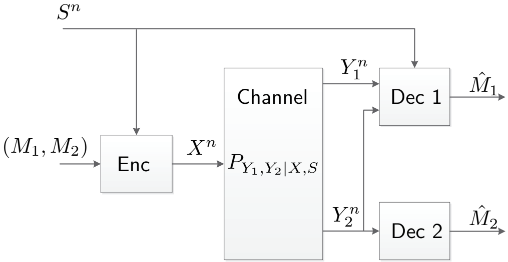

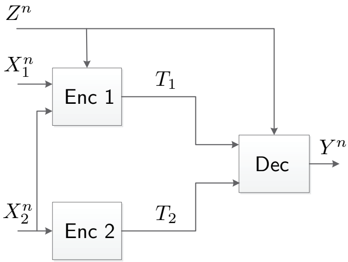

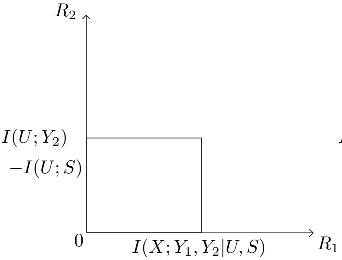

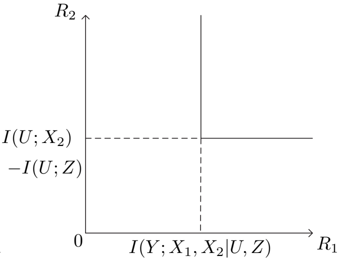

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Action-Dependant Output-Degraded BC vs. Dual:

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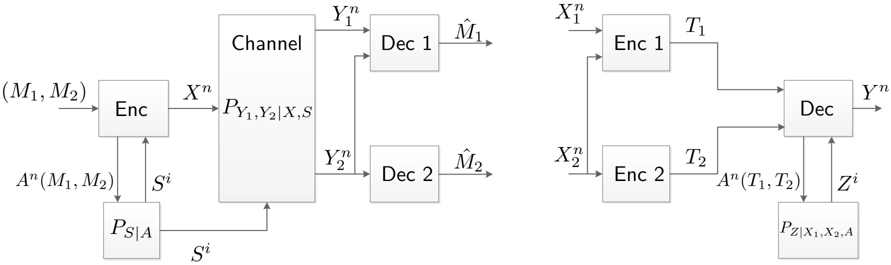

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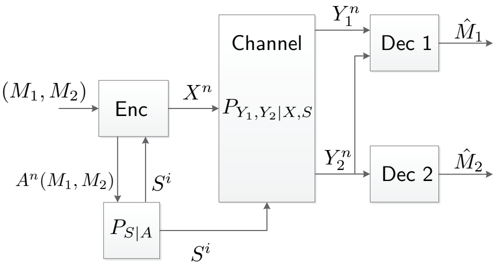

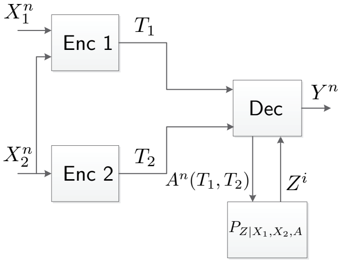

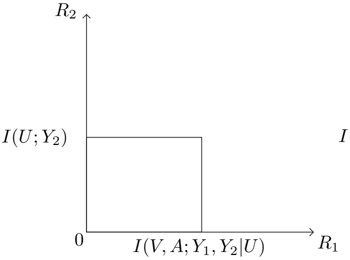

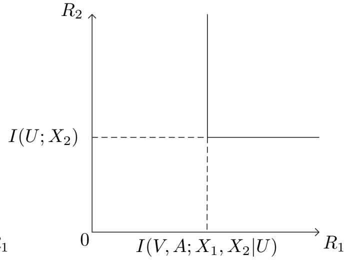

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 1:

$$\Big(\, I(V;X_1|X_2)\ ,\ H(X_1)\ ,\ I(U;X_2|X_1,V)\,\Big).$$

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- **Encoder 1 to Decoder:** Conveys X_1^n to the decoder in a lossless manner.
- **Encoder 2 to Decoder:** The decoder knows X_1^n and therefore V^n. Wyner-Ziv coding to convey U^n.

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

$$\Big(\, I(V;X_1|X_2)\ ,\ H(X_1|V,U)\ ,\ I(U;X_2|V)+I(V;X_1)\,\Big).$$

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- **Cooperation:** Same.

AK Problem with Cooperation - Achievability Outline

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- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows V^n. Conveys the index of V^n and uses superposition coding to convey U^n.

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Achieving Corner Point 2:

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- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows V^n. Conveys the index of V^n and uses superposition coding to convey U^n.
- **Encoder 1 to Decoder:** The decoder knows (V^n,U^n). Binning scheme to convey X_1^n in a lossless manner.

AK Problem with Cooperation - Proof Outline

Converse:

AK Problem with Cooperation - Proof Outline

Converse:

- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\setminus i}, X_{2,i+1}^n),$$
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for every $1 \leq i \leq n$.

AK Problem with Cooperation - Proof Outline

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- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n \setminus i}, X_{2,i+1}^n),$$
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  for every $1 \leq i \leq n$.
- Time mixing properties.

Semi-Deterministic BC with Cooperation - Proof Outline

Achievability: Split $M_i = (M_{i0}, M_{ii})$, $i = 1, 2$. Code construction:

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