Cooperative Broadcast Channels with a Secret Message

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Joint work with Paul Cuff

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Outline

- Motivation
- Channel resolvability for strong-secrecy in Marton codes
- Cooperative BCs with a confidential message
- Strong-secrecy-capacity results
- Summary

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- Help while concealing.

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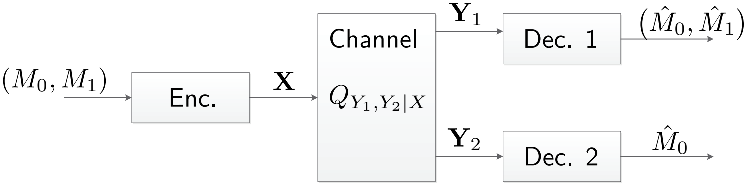

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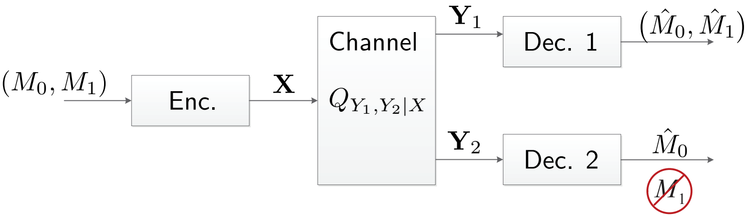

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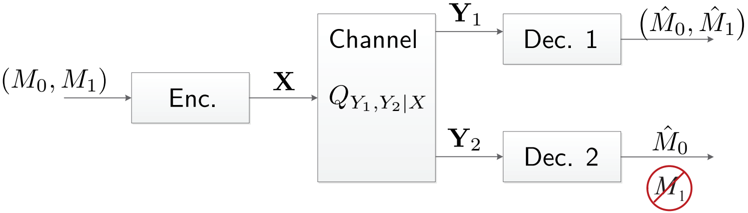

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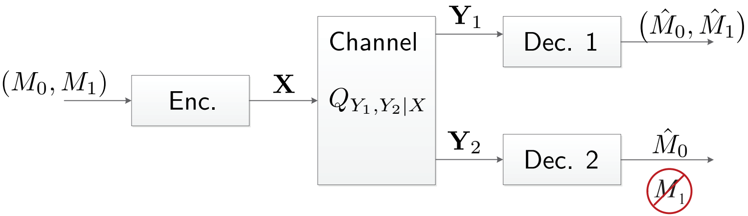

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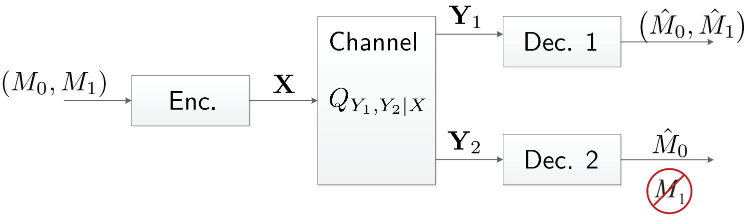

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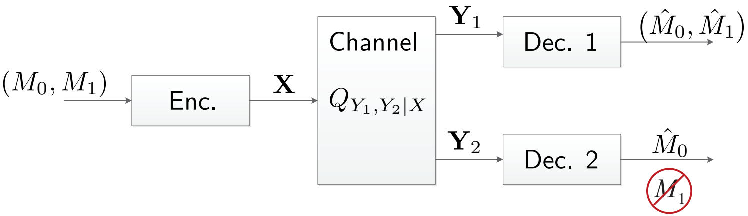

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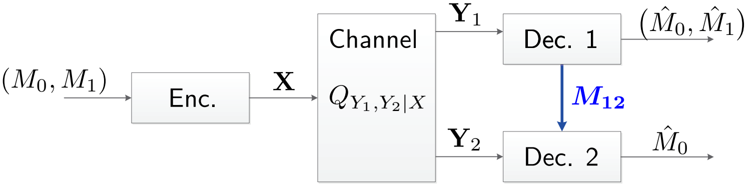

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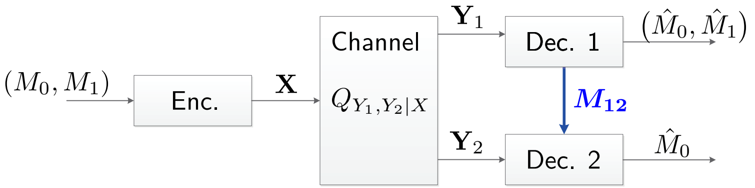

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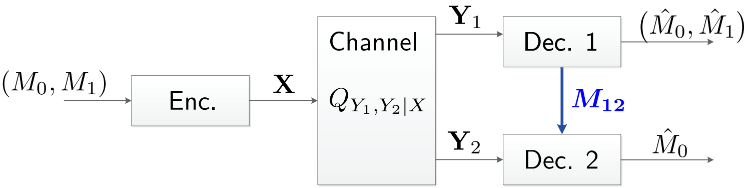

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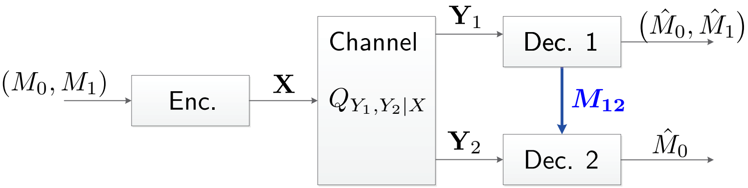

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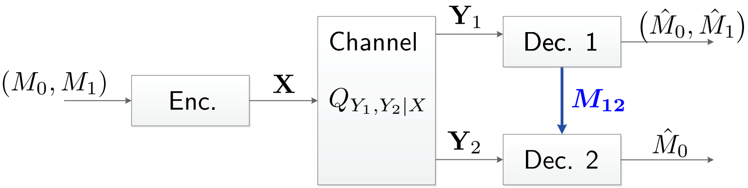

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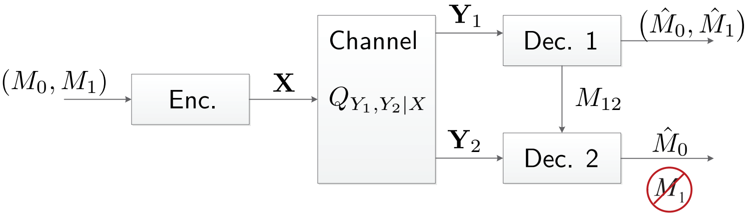

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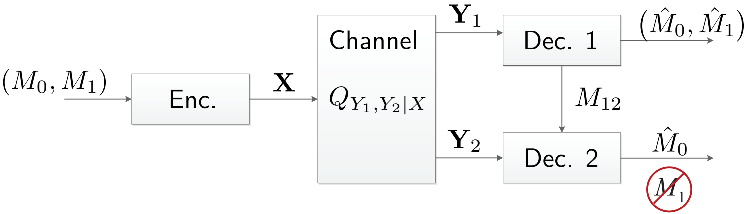

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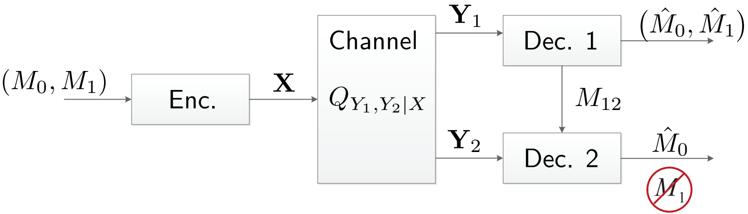

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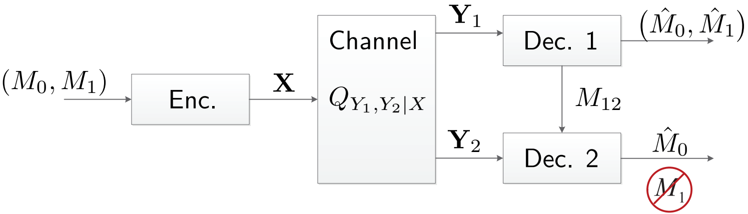

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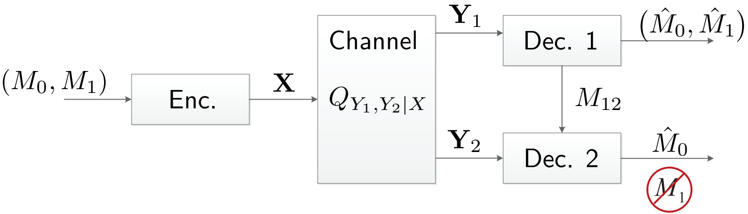

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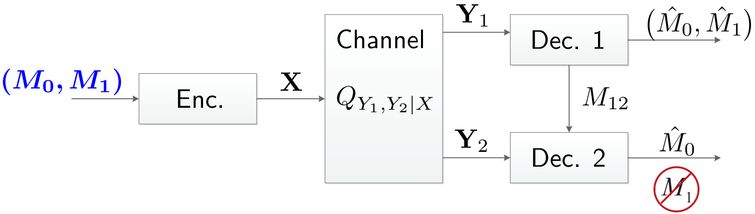

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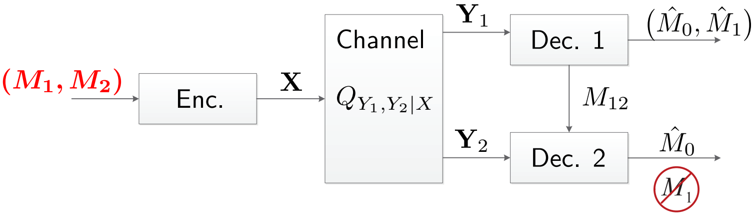

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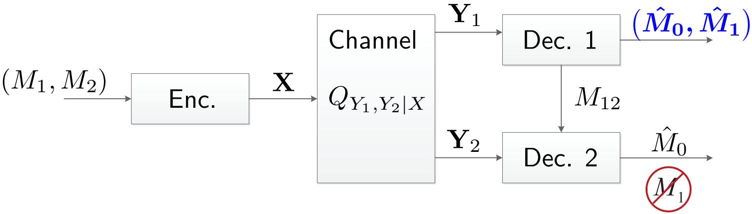

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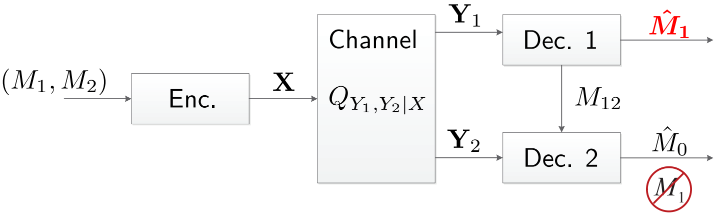

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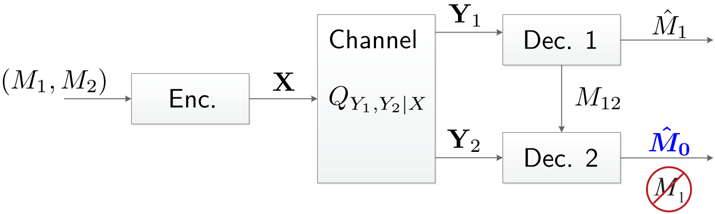

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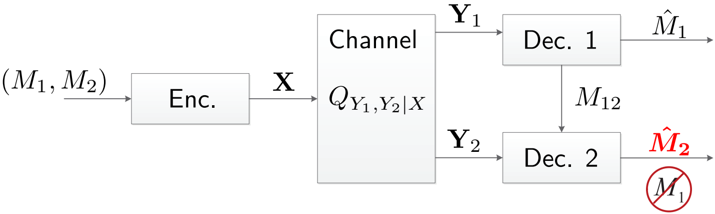

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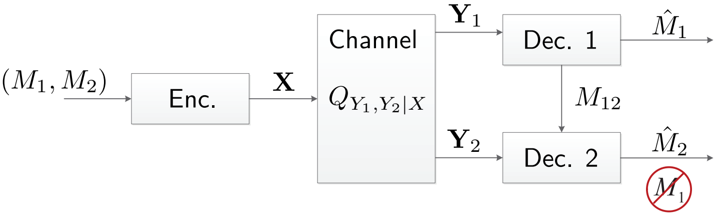

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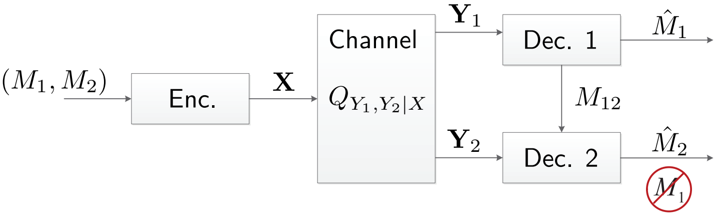

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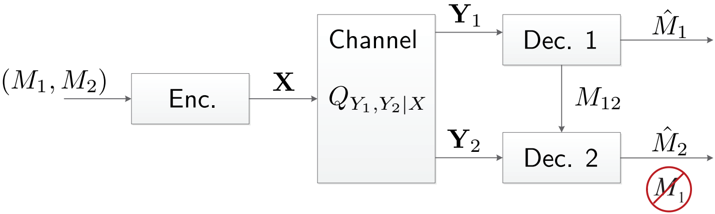

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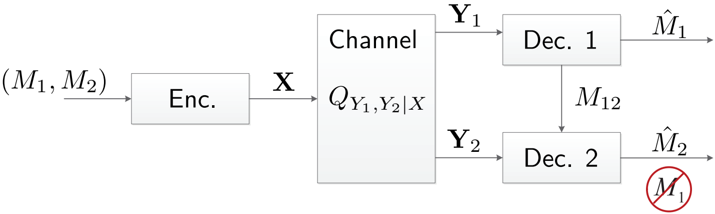

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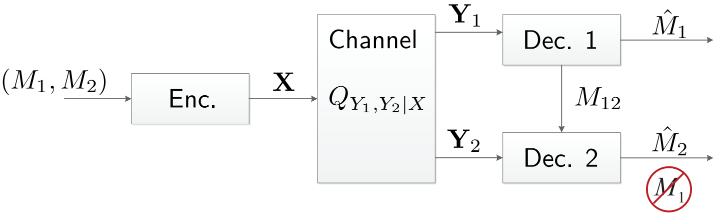

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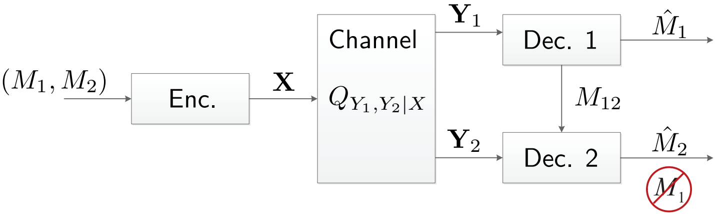

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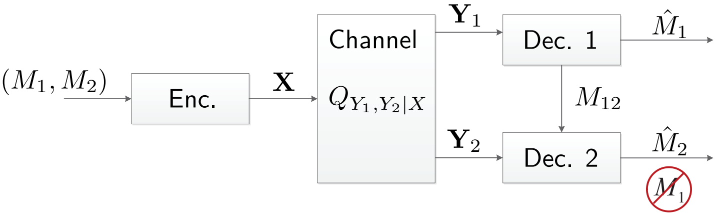

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⋆ Strong-secrecy for Marton codes (superposing & multi-coding) ⋆

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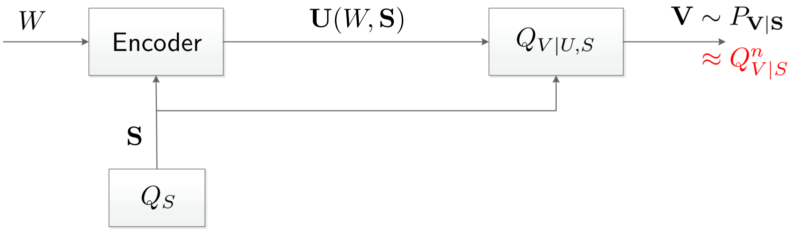
Classic Case - PTP codebook [Wyner, 1975], [Han & Verdú, 1993]



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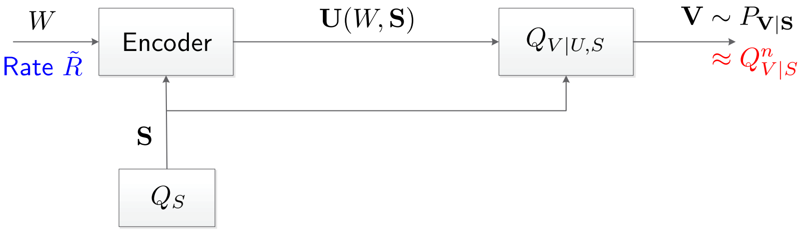

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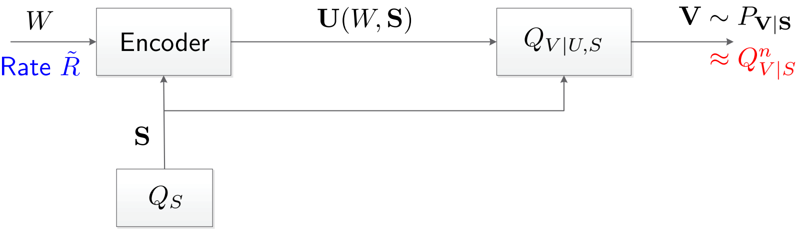

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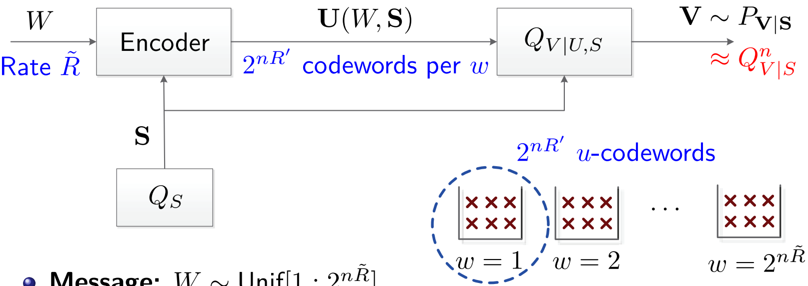

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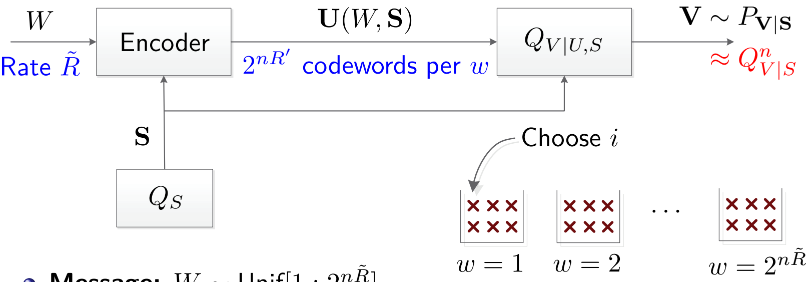

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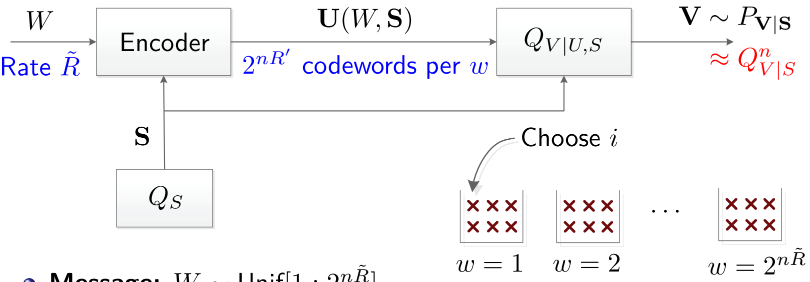

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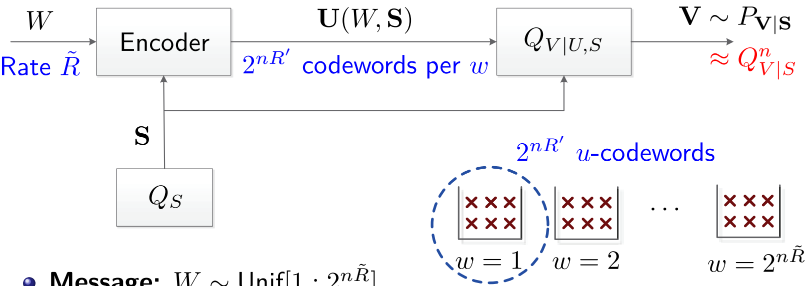

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- **Goal:** Choose $(\tilde{R}, R')$ s.t. $\mathbb{E}_{\mathbb{C}_n}\Big[D\big(P_{\mathbf{V}|\mathbf{S},\mathbb{C}_n}\big|\big|Q^n_{V|S}\big|Q^n_S\big)\Big] \to 0$.

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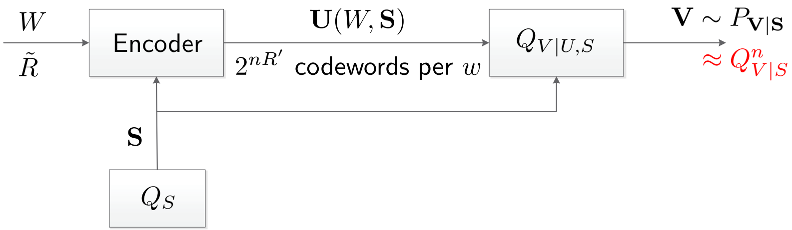
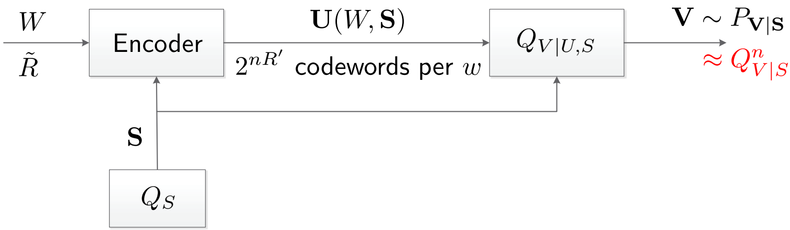


Theorem (Direct Part)

$$\begin{aligned} R' &> I(S;U) \\ R' + \tilde{R} &> I(U;S,V) \end{aligned} \quad \Longrightarrow \quad \mathbb{E}_{\mathbb{C}_n}\left[D\left(P_{\mathbf{V}|\mathbf{S},\mathbb{C}_n}\middle|\middle|Q^n_{V|S}\middle|Q^n_S\right)\right] \to 0$$

Channel Resolvability and Multi-coding (Simplified)



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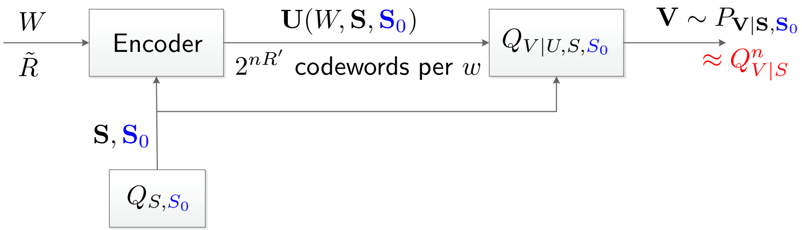

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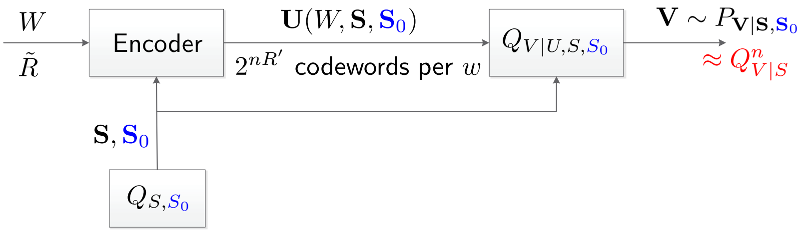

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Theorem (Direct Part with Superposing)

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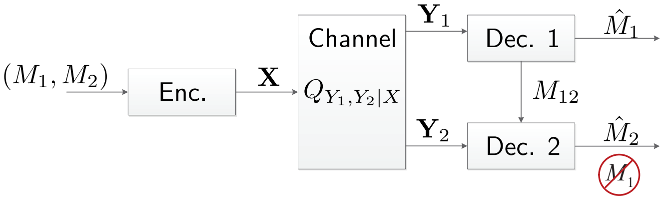
Cooperative BCs with a Confidential Message



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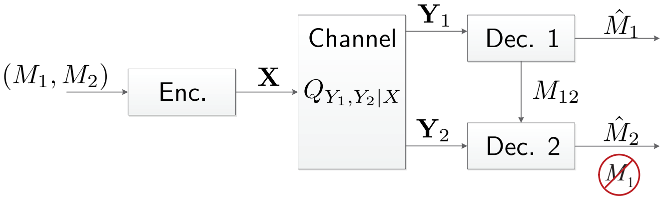

Strong-Secrecy: $I(M_1; M_{12}, \mathbf{Y}_2) \to 0.$

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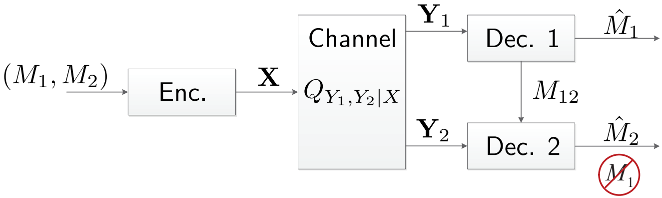

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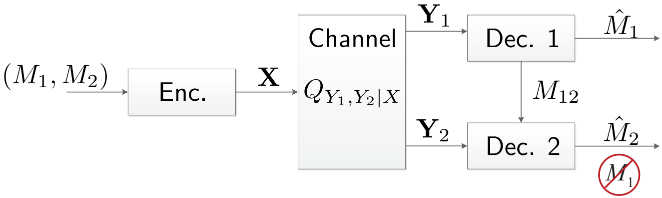
Theorem (Inner Bound)

An inner bound on the strong-secrecy-capacity region is:

$$\mathcal{R}_I = \bigcup \left\{ \begin{array}{r} R_1 \leq I(U_1;Y_1|U_0) - I(U_1;U_2|U_0) - I(U_1;Y_2|U_0,U_2) \\ R_2 \leq I(U_0,U_2;Y_2) + R_{12} \\ R_1 + R_2 \leq I(U_0,U_1;Y_1) - I(U_1;U_2|U_0) - I(U_1;Y_2|U_0,U_2) \\ +I(U_2;Y_2|U_0) \end{array} \right\}$$

where the union is over all $Q_{U_0,U_1,U_2,X}Q_{Y_1,Y_2|X}$.

Cooperative BCs with a Confidential Message



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⋆ Extension to common+private messages ⋆

Inner Bound - Proof Outline

- **Messages:** $M_2 = (M_{20}, M_{22})$.

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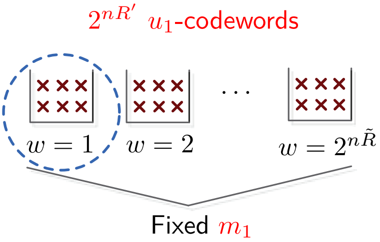

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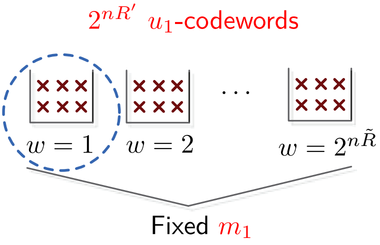
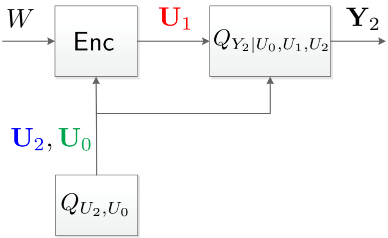


Fixed (m_{20}, m_{22}, m_1)



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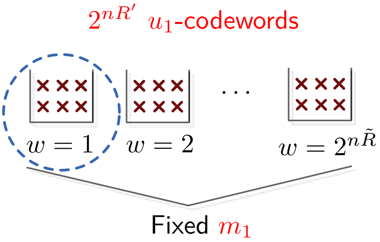

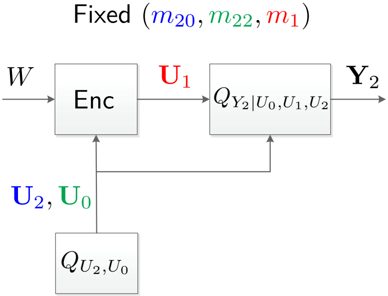

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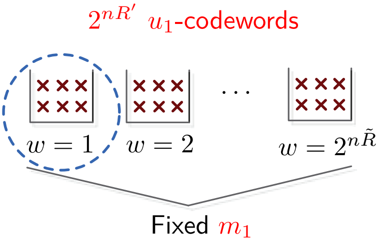

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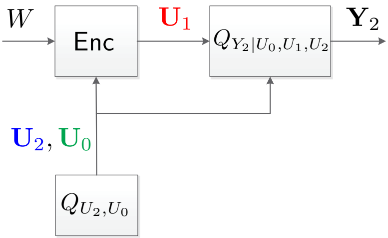

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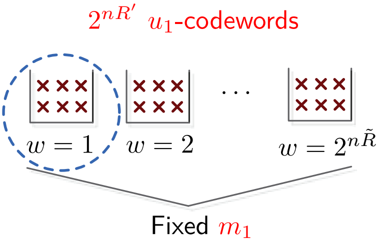

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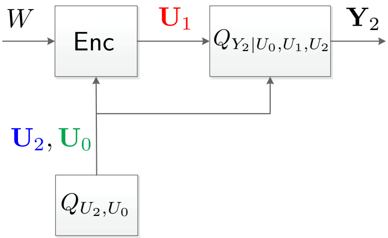

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  1. Bin M_{20} codebook into $2^{nR_{12}}$ bins.

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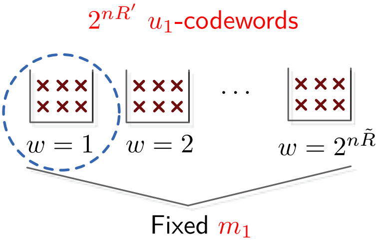

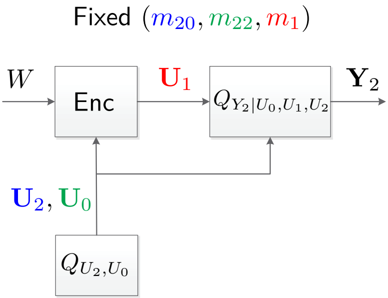

Inner Bound - Proof Outline

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Thank you!

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     - Used to conceal M_1 from Decoder 2.
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- Code construction similar to case without secrecy.
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SD-BC without Cooperation - Effect of Secrecy

Criterion	SD-BC Without Secrecy	SD-BC With M_1 Secret
Capacity	$R_1 \leq H(Y_1)$ $R_2 \leq I(U;Y_2)$ $R_1 + R_2 \leq H(Y_1\|U) + I(U;Y_2)$	$R_1 \leq H(Y_1\|U,Y_2)$ $R_2 \leq I(U;Y_2)$
CP(s)	$\big(H(Y_1\|U), I(U;Y_2)\big)$ $\big(H(Y_1), I(U;Y_2) - I(U;Y_1)\big)$	$\big(H(Y_1\|U,Y_2), I(U;Y_2)\big)$

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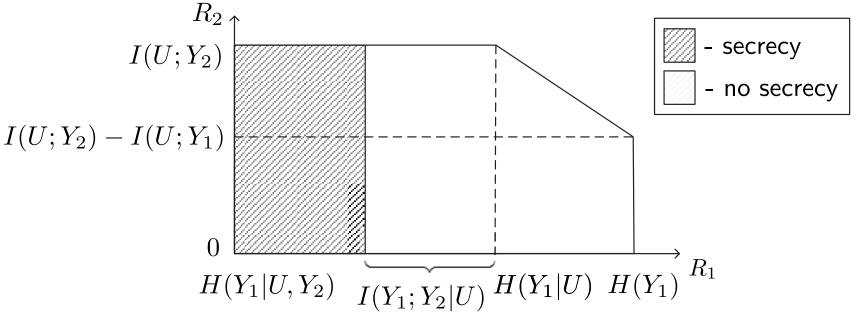
CP(s)	$\big(H(Y_1\|U), I(U;Y_2)\big)$ $\big(H(Y_1), I(U;Y_2) - I(U;Y_1)\big)$	$\big(H(Y_1\|U,Y_2), I(U;Y_2)\big)$



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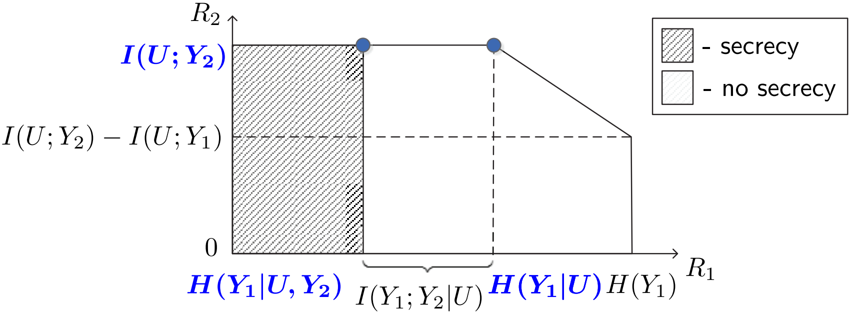

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