Semantic-Security Capacity for Wiretap Channels of Type II

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Motivation

Information Theoretic Security over Noisy Channels

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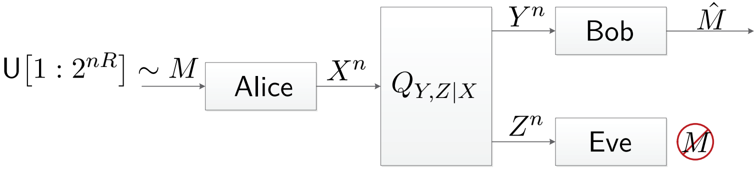
Our Goal: Stronger metric and weaken "known channel" assumption.

Wiretap Channels - Security Metrics

Wiretap Channels and Security Metrics

Degraded [Wyner 1975], General [Csiszár-Körner 1978]



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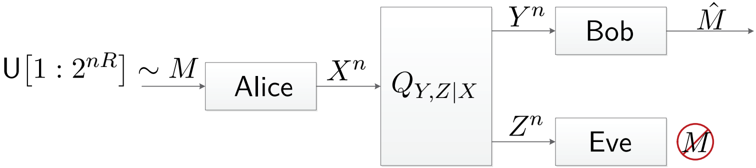

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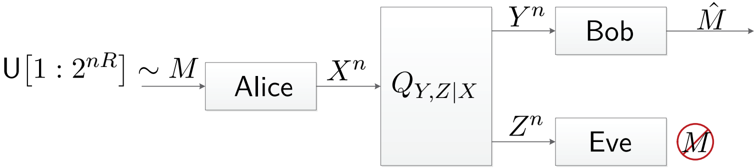

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- **Weak-Secrecy:** $\frac{1}{n} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$

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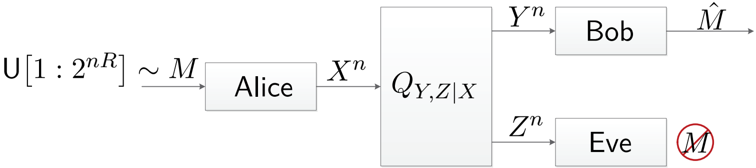

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- **Weak-Secrecy:** $\frac{1}{n}I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$ Only leakage **rate** vanishes

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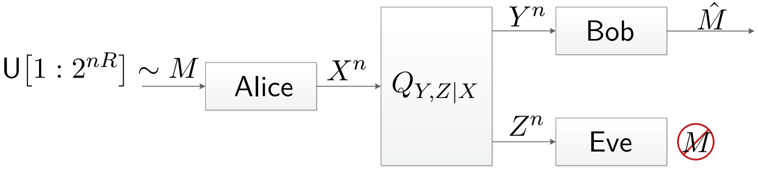

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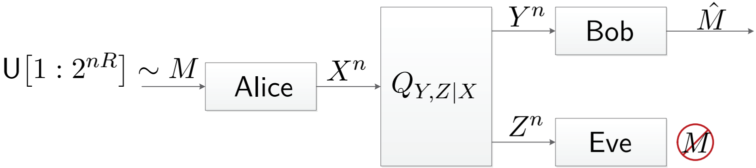

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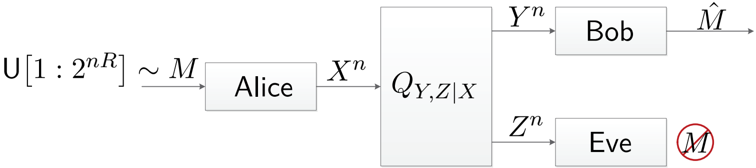

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- **Strong-Secrecy:** $I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$ Security only on **average**

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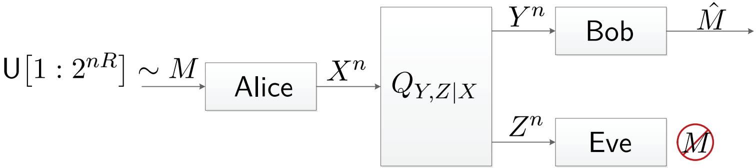

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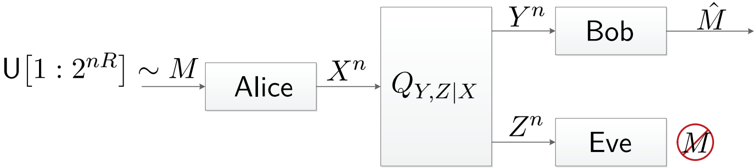

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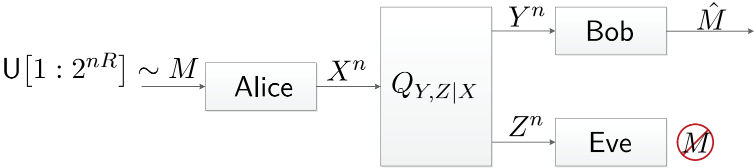

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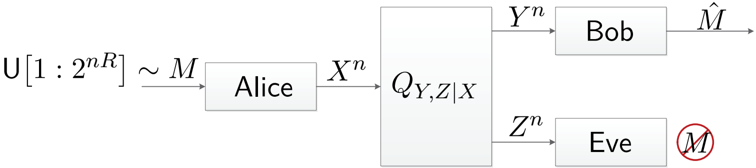

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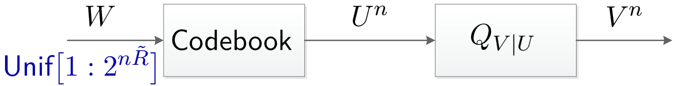
⋆ A single code that satisfied exponentially many secrecy constraints ⋆

A Stronger Soft-Covering Lemma

Soft-Covering - Setup



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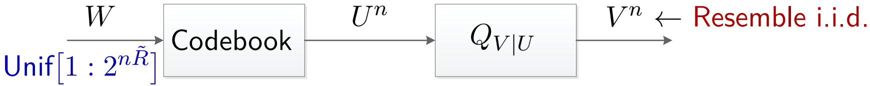

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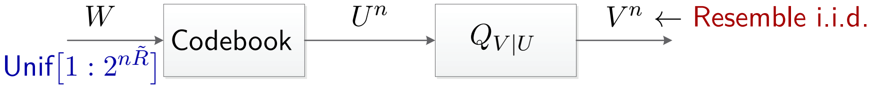

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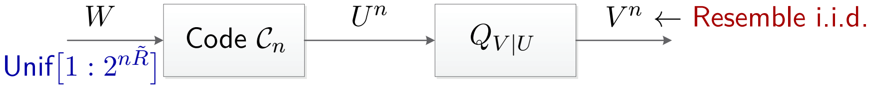

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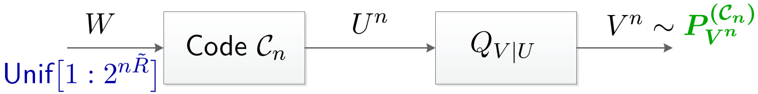

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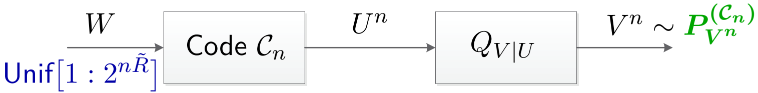

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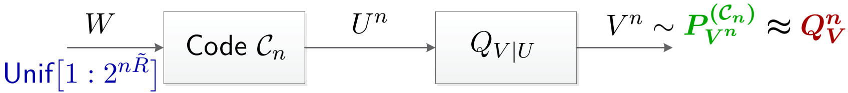

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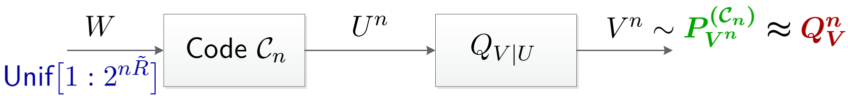

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⋆ **Goal:** Choose $\tilde{R}$ (codebook size) s.t. $P_{V^n}^{(\mathcal{C}_n)} \approx Q_V^n$ ⋆

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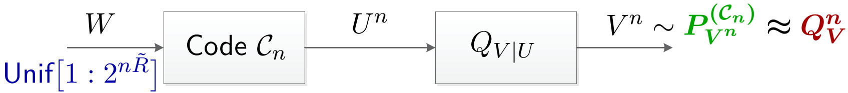

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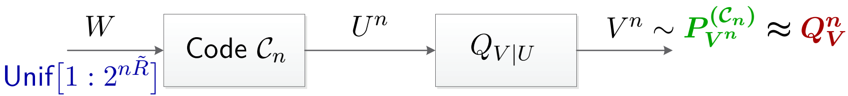

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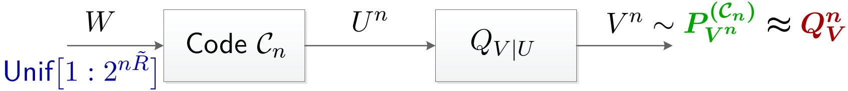

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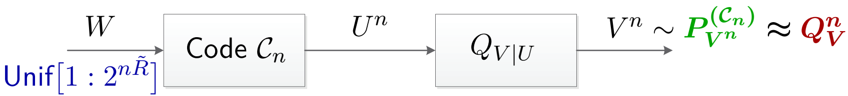

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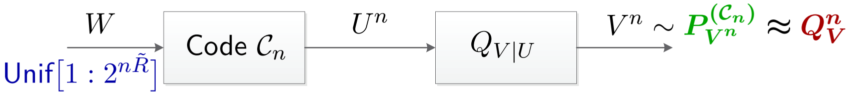

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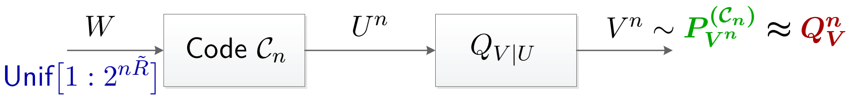
A Stronger Soft-Covering Lemma



Theorem (Cuff 2015, ZG-Cuff-Permuter 2016)

If $\tilde{R} > I_Q(U;V)$ *and* $|\mathcal{V}| < \infty$, *then there exists* $\gamma_1, \gamma_2 > 0$ *s.t.*

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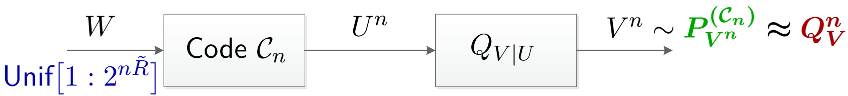

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- **More on the stronger SCL:** Paul Cuff on Fr-PM-1-7 at 15:50.

Wiretap Channels of Type II

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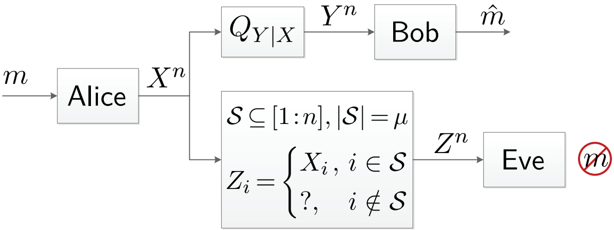
[Ozarow-Wyner 1984]



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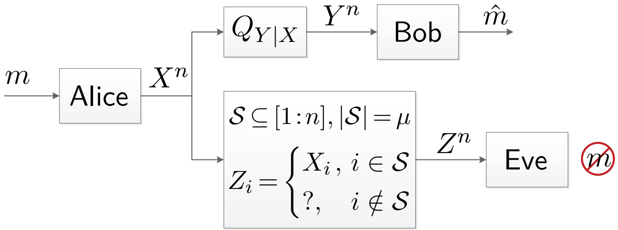

- **Eavesdropper:** Can observe a subset $\boldsymbol{\mathcal{S}} \subseteq [1:n]$ of size $\boldsymbol{\mu = \lfloor \alpha n \rfloor}$, $\boldsymbol{\alpha \in [0,1]}$, of transmitted symbols.

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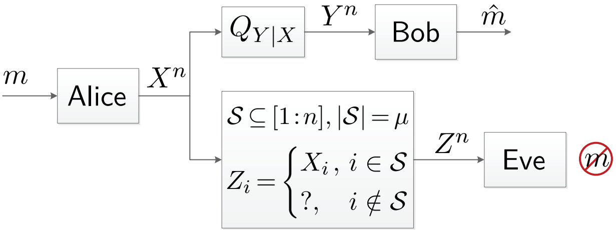

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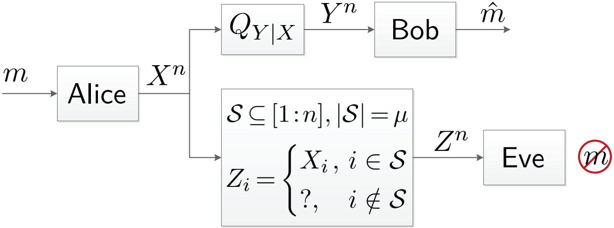

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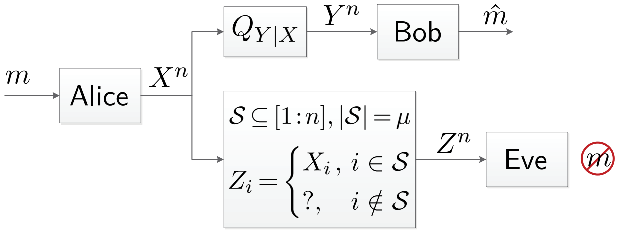

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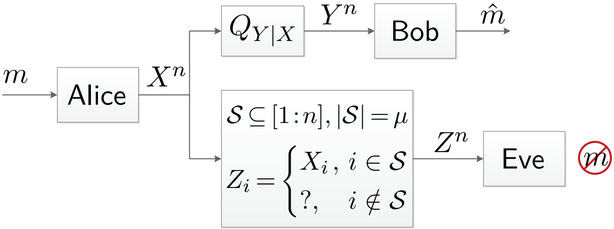
⋆ Ensure security versus all possible choices of $\boldsymbol{\mathcal{S}}$ ⋆

Wiretap Channels of Type II - Past Results

[Ozarow-Wyner 1984]



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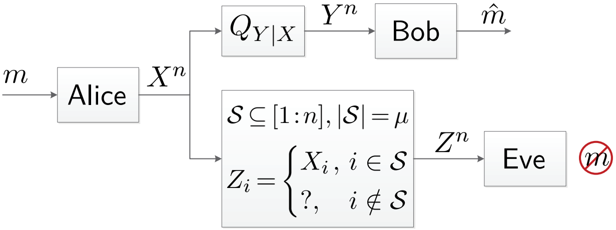

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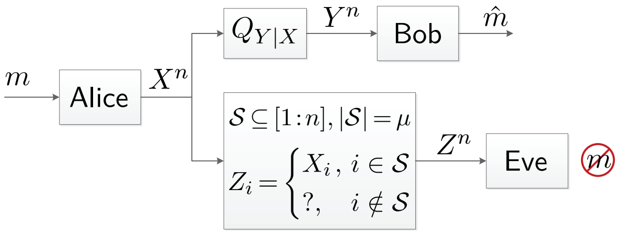

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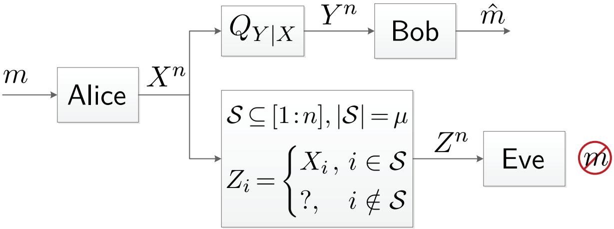

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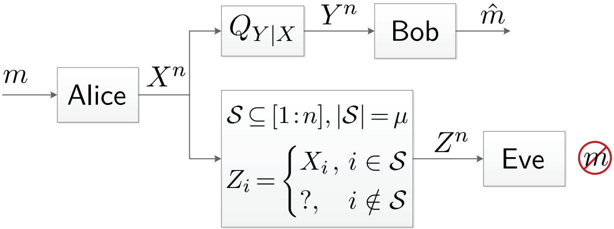

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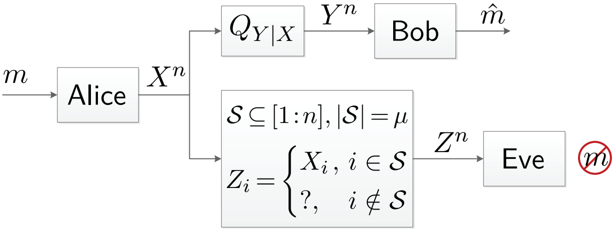
Wiretap Channels of Type II - Past Results

[Ozarow-Wyner 1984]



- **Ozarow-Wyner 1984:** Noiseless main channel
  - Rate equivocation region.
  - Coset coding.

- **Nafea-Yener 2015:** Noisy main channel
  - Built on coset code construction.
  - Lower & upper bounds - Not match in general.

Semantic Security:

Wiretap Channels of Type II - SS-Capacity

Semantic Security: $$\max_{\substack{P_M,\mathcal{S}:\\|\mathcal{S}|=\mu}} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$$

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For any $\alpha \in [0,1]$

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- RHS is the secrecy-capacity of WTC I with erasure DMC to Eve.
- Standard (erasure) wiretap code & Stronger tools for analysis.

WTC II SS-Capacity - Security Analysis

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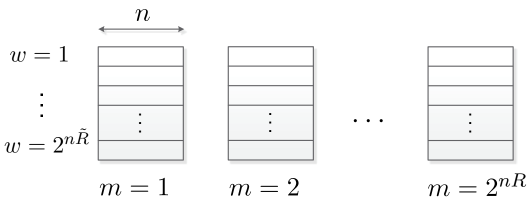

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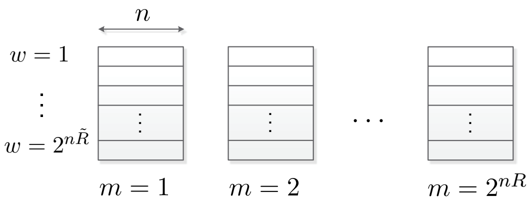

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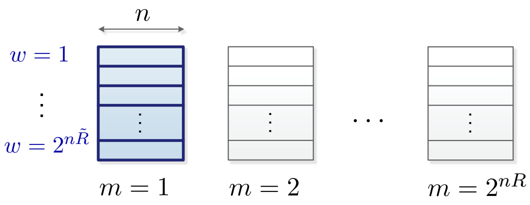

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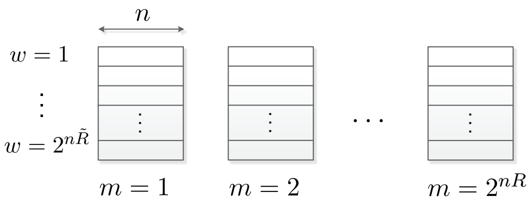

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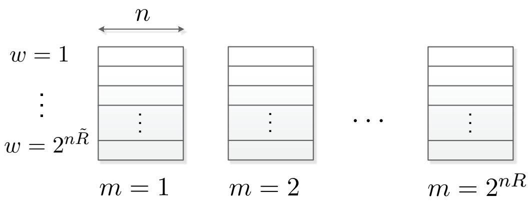

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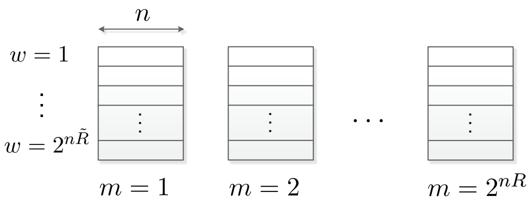

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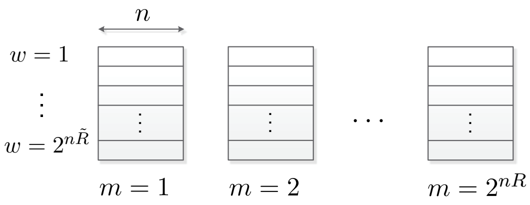

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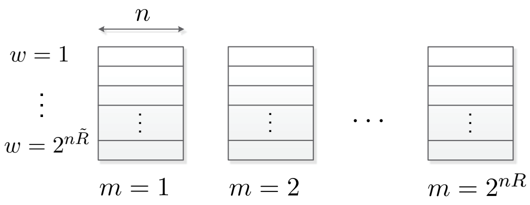

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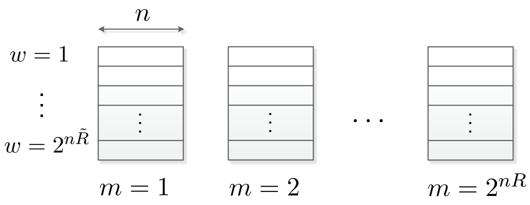

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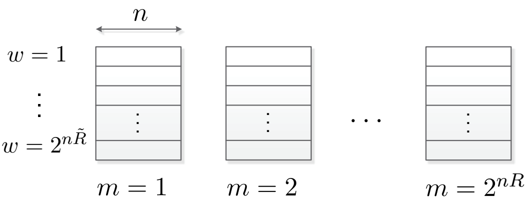

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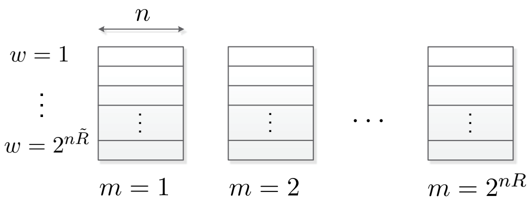

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Thank You!

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$\square$

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- **Solution:** Sanov's theorem & Continuity of mutual information.