Broadcast Channels with Cooperation: Capacity and Duality for the Semi-Deterministic Case

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Outline

- Channel-source duality for BCs
- Semi-deterministic BC with decoder cooperation
- Source coding dual
- Capacity results
- Summary

Duality - Preface

"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
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- The solutions are dual - Information measures coincide.
- A formal proof of duality is still absent.
- Solving one problem $\implies$ Valuable insight into solving dual.

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Point-to-Point Case:

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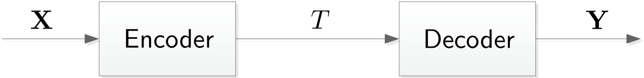

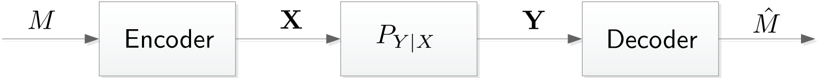

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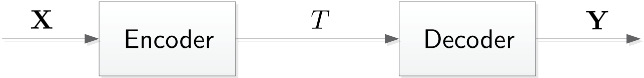

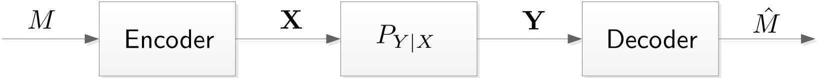

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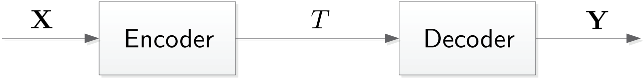

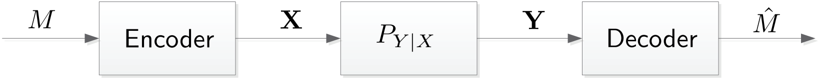

Fixed-Type Code: $(\mathbf{X},\mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}(P^\star_X P_{Y|X})$

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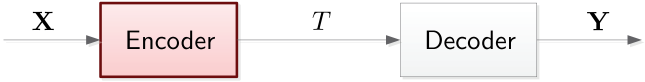

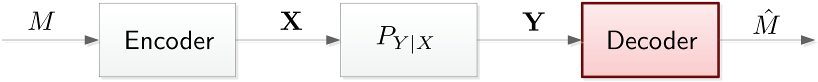

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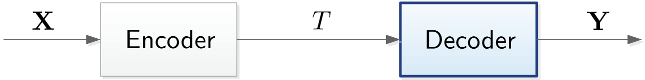

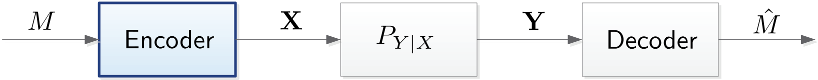

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Point-to-Point Case:

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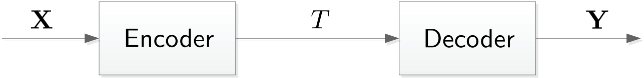

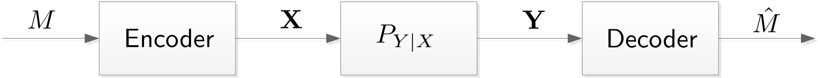

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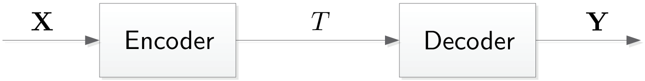

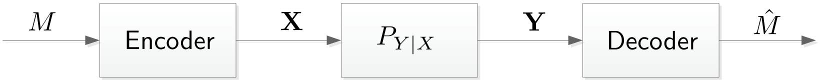

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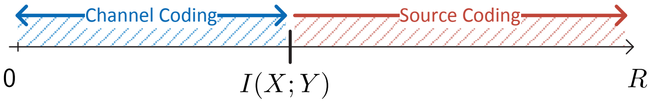
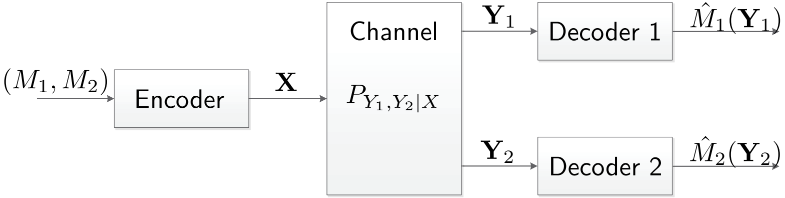
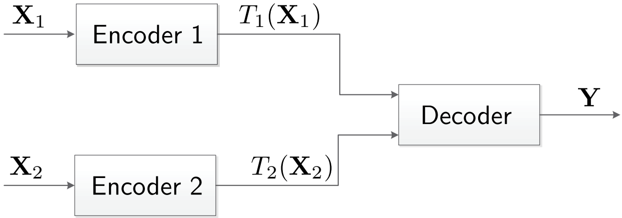


Multi-User Duality - Broadcast Channels





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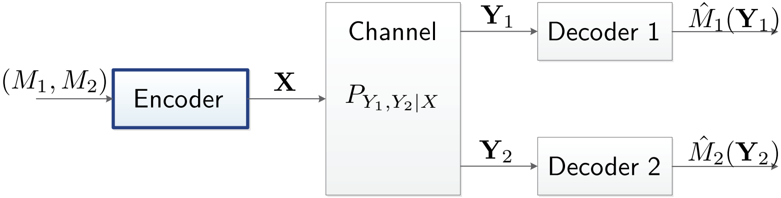

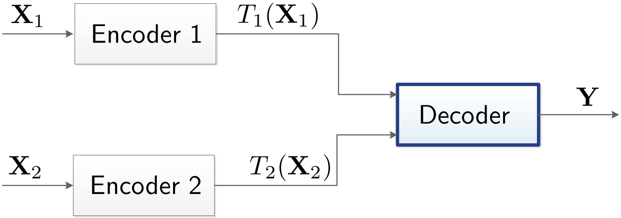

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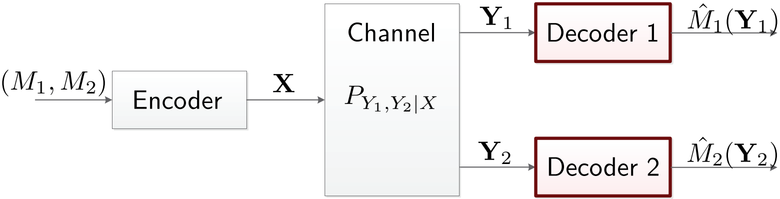

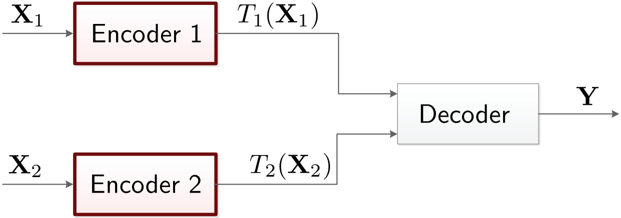

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Broadcast Channel		**Dual Source Coding Setting**
$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}\left(P_X^\star P_{Y_1,Y_2\|X}\right)$	$\longleftrightarrow$	$(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}\left(P_{X_1,X_2} P^\star_{Y\|X_1,X_2}\right)$

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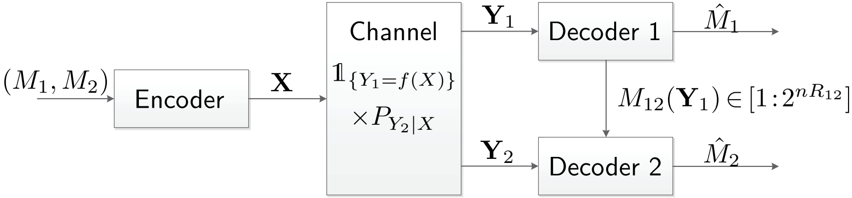
⋆ **Result Duality:** Information measures at the corner points coincide! ⋆

Cooperative SD-BC vs. Cooperative WAK Problem

Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]



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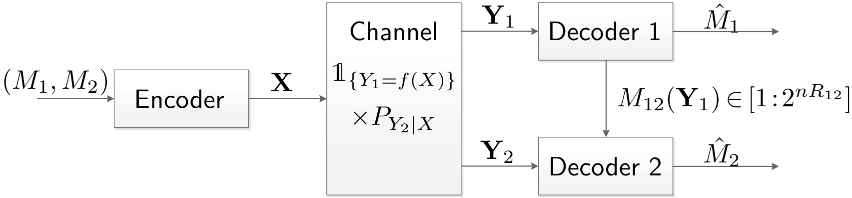

BCs with Cooperation:

- Physicaly degraded (PD) [Dabora and Servetto, 2006].
- Relay-BC [Liang and Kramer, 2007].
- State-dependent PD [Dikstein, Permuter and Steinberg, 2014].
- Degraded message sets / PD with parallel conf. [Steinberg, 2015].

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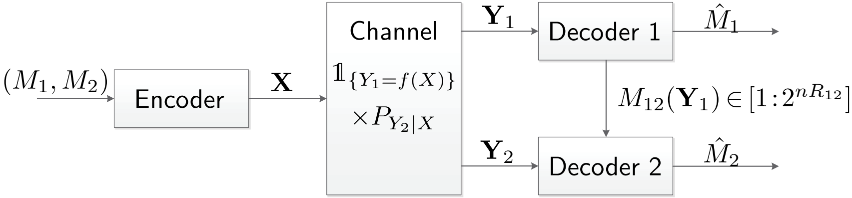

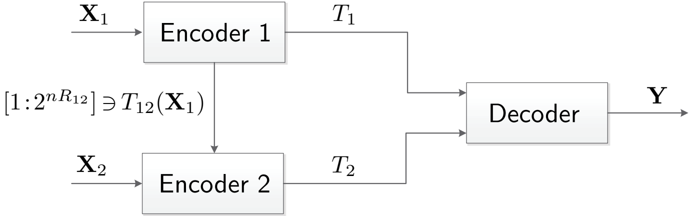

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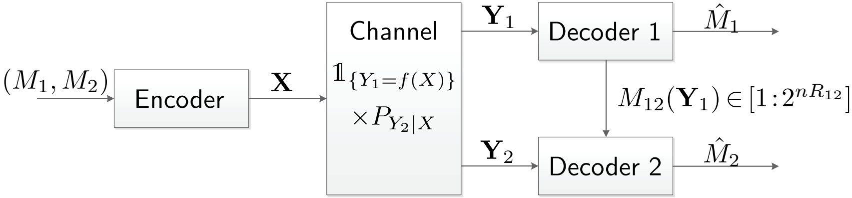

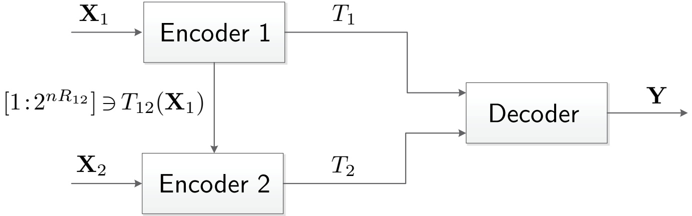

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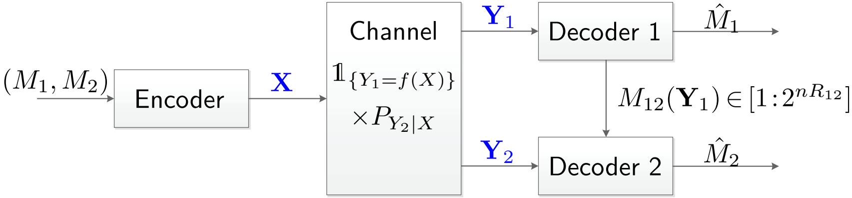

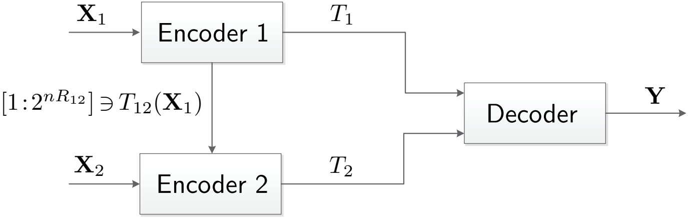

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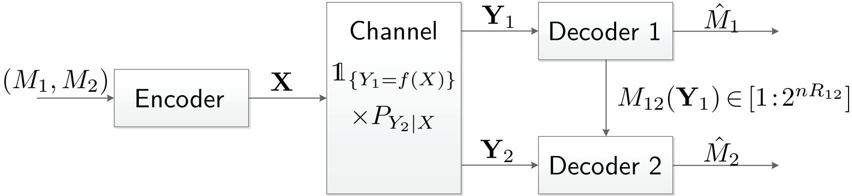

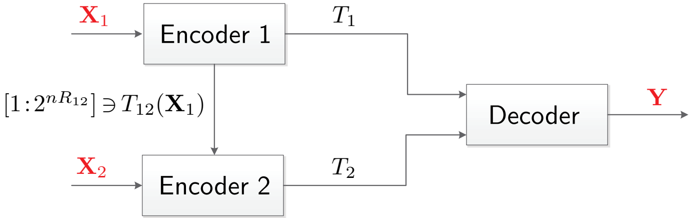

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Cooperative WAK Problem - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination PMF $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

$$\mathcal{C}_{\mathsf{WAK}} = \bigcup \left\{ \begin{array}{r} R_{12} \geq I(V;X_1) - I(V;X_2) \\ R_1 \geq H(X_1|V,U) \\ R_2 \geq I(U;X_2|V) - I(U;X_1|V) \\ R_1 + R_2 \geq H(X_1|V,U) + I(V,U;X_1,X_2) \end{array} \right\}$$

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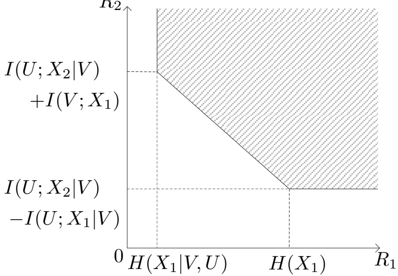
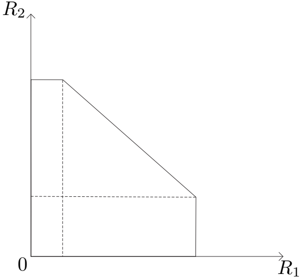
Corner Point Correspondence

For fixed joint PMFs and R_{12}:





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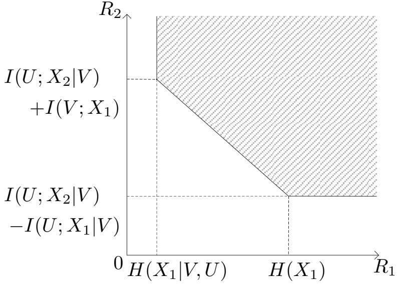

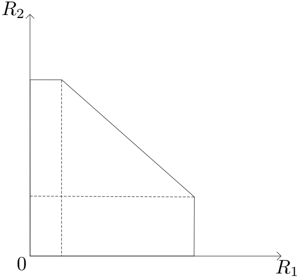

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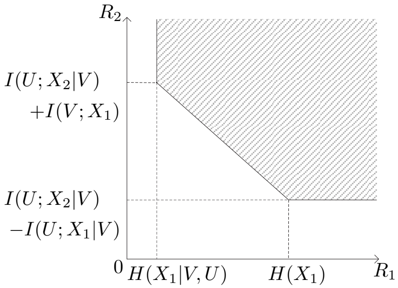
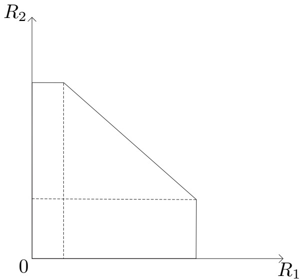
$R_{12} = I(V;X_1) - I(V;X_2)$	
(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2|V) - I(U;X_1|V) \Big)$	(R_1, R_2) at Lower Corner Point:
(R_1, R_2) at Upper Corner Point: $\Big(H(X_1|V,U)\ ,\ I(U;X_2|V) + I(V;X_1) \Big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

For fixed joint PMFs and R_{12}:





Cooperative WAK Problem	**Cooperative Semi-Deterministic BC**
$\boldsymbol{R_{12} = I(V;X_1) - I(V;X_2)}$	$\boldsymbol{R_{12} = I(V;Y_1) - I(V;Y_2)}$
(R_1,R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2|V) - I(U;X_1|V) \Big)$	(R_1,R_2) at Lower Corner Point:
(R_1,R_2) at Upper Corner Point: $\Big(H(X_1|V,U)\ ,\ I(U;X_2|V) + I(V;X_1) \Big)$	(R_1,R_2) at Upper Corner Point:

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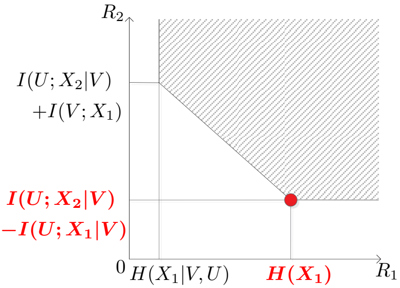

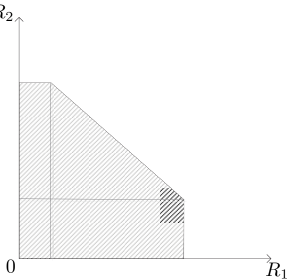

Cooperative WAK Problem	Cooperative Semi-Deterministic BC
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \Big)$	(R_1, R_2) at Lower Corner Point:
(R_1, R_2) at Upper Corner Point: $\Big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1) \Big)$	(R_1, R_2) at Upper Corner Point:

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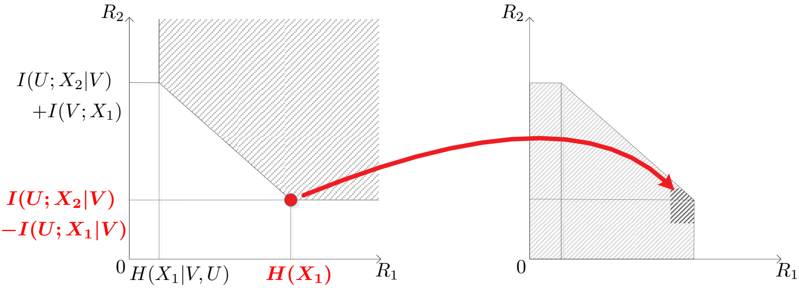

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$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\Big(\boldsymbol{H(X_1)}\ ,\ \boldsymbol{I(U;X_2|V) - I(U;X_1|V)}\Big)$	(R_1, R_2) at Lower Corner Point:
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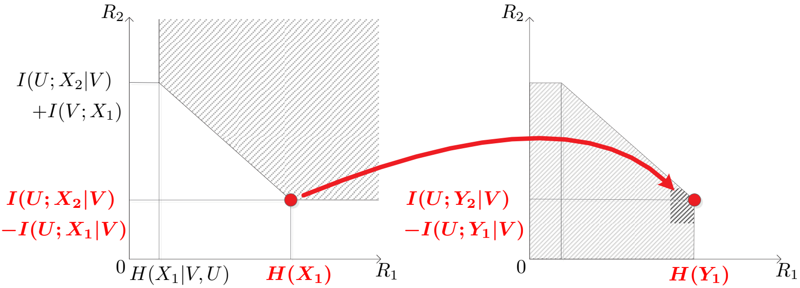

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Corner Point Correspondence

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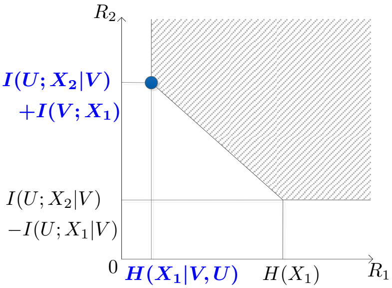

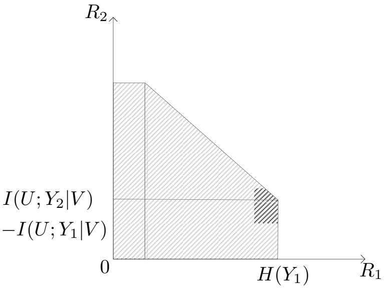

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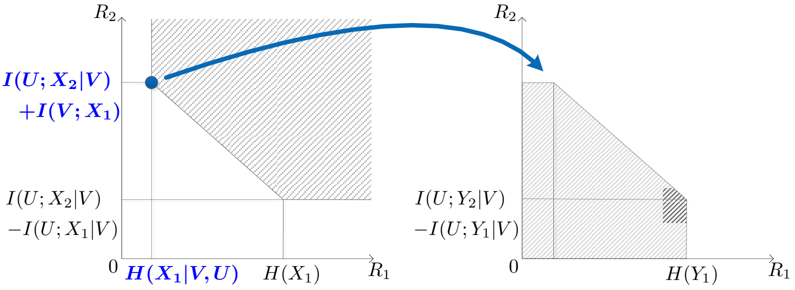

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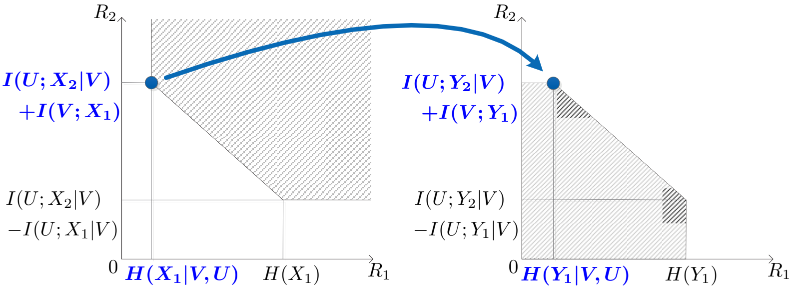

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(R_1, R_2) at Upper Corner Point: $\boldsymbol{\Big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1)\Big)}$	(R_1, R_2) at Upper Corner Point: $\boldsymbol{\Big(H(Y_1\|V,U)\ ,\ I(U;Y_2\|V) + I(V;Y_1)\Big)}$

Semi-Deterministic BC with Cooperation - Solution

Theorem (Capacity Region)

The capacity region is:

$$\mathcal{C}_{BC} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;Y_1) - I(V;Y_2) \\ R_1 \leq H(Y_1) \\ R_2 \leq I(V,U;Y_2) + R_{12} \\ R_1 + R_2 \leq H(Y_1|V,U) + I(U;Y_2|V) + I(V;Y_1) \end{array} \right\}$$

where the union is over all $P_{V,U,Y_1,X} P_{Y_2|X} \mathbb{1}_{\{Y_1=f(X)\}}$.

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- Later: Achievability and converse proofs for an alternative region.

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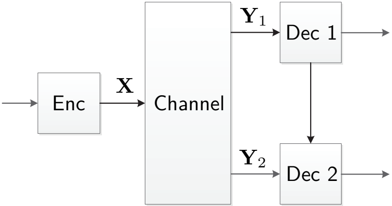
- Later: Achievability and converse proofs for an alternative region.
- $\mathcal{C}_{\text{BC}}$ emphasizes duality.

Cooperative Semi-Deterministic BC - Achievability Outline



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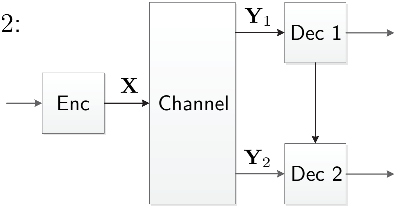

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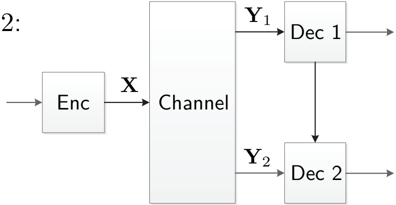

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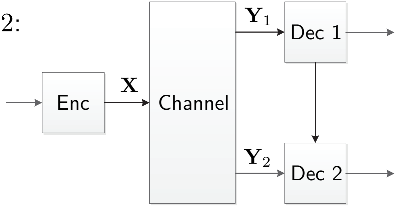

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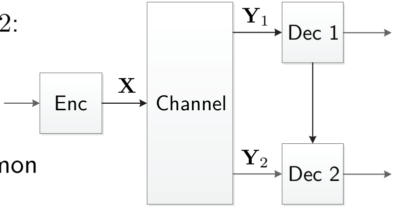

- **Codebook Structure:** Marton (with common message).

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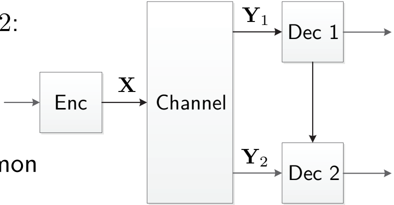

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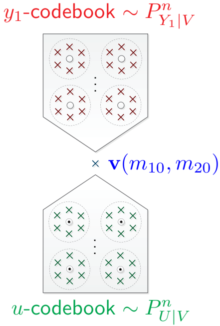

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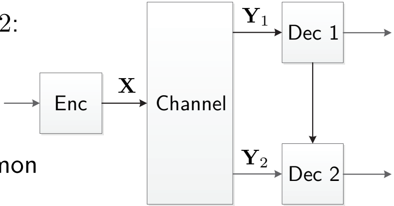

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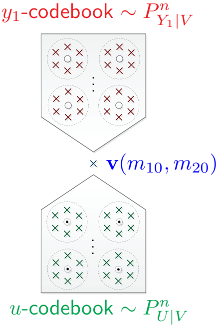

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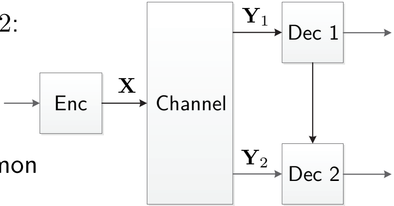

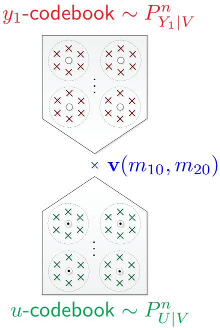

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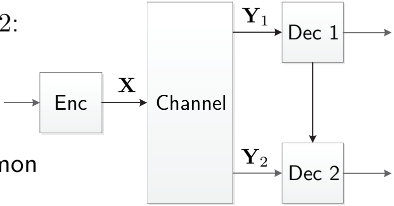

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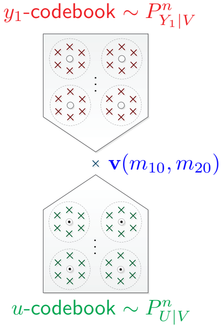

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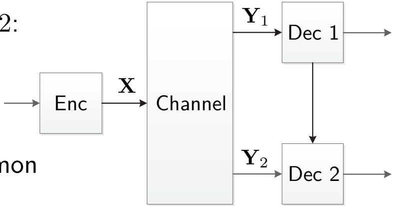

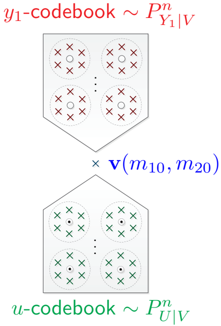

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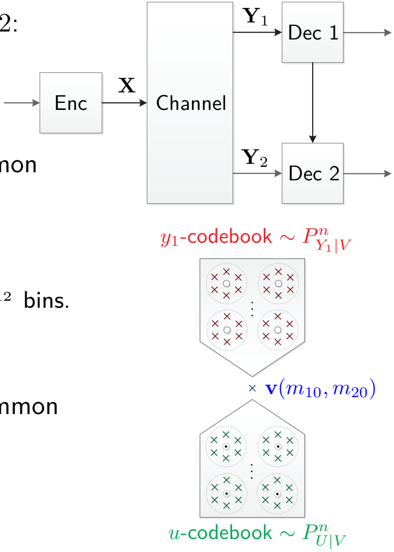

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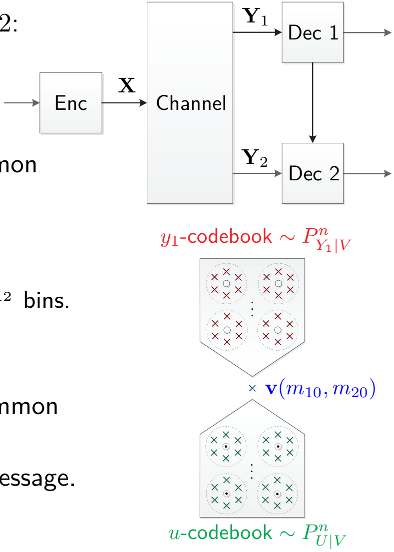

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  $\Longrightarrow$ More channel resources for private message.

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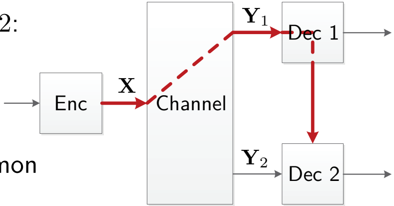

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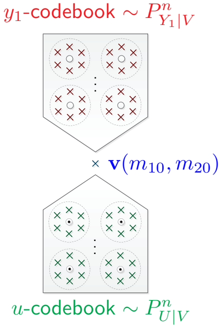

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Summary

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Thank you!

Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

Multi-User Duality - Additional Examples

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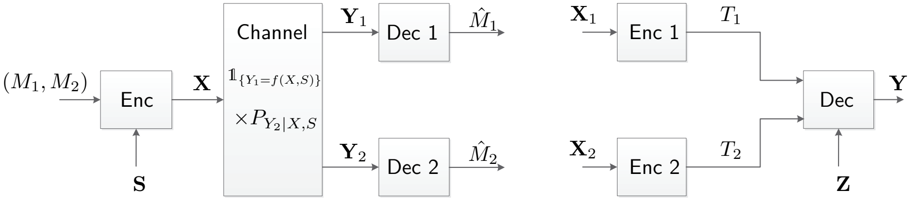

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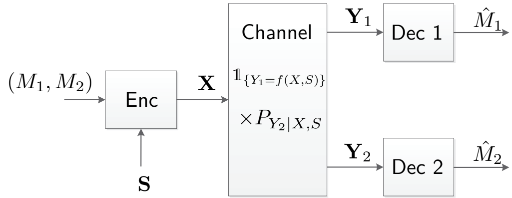
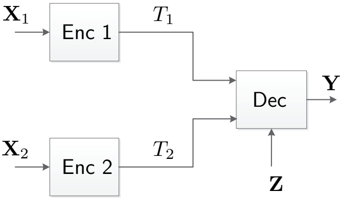
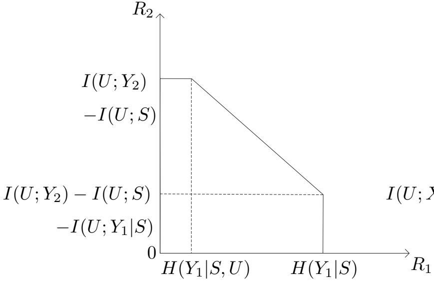
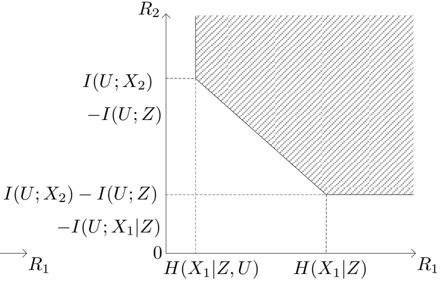






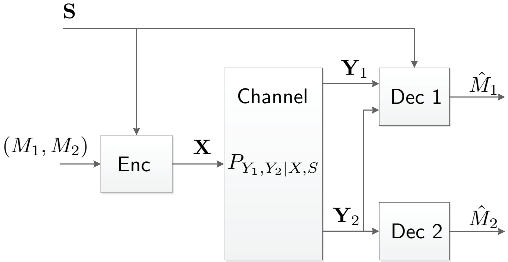


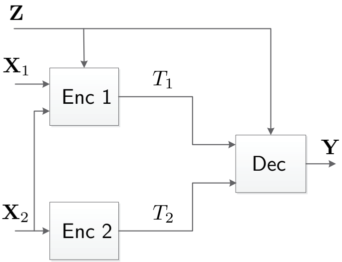
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:

Multi-User Duality - Additional Examples

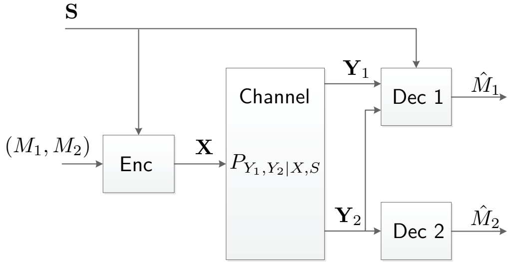
State-Dependant Output-Degraded BC vs. Dual:

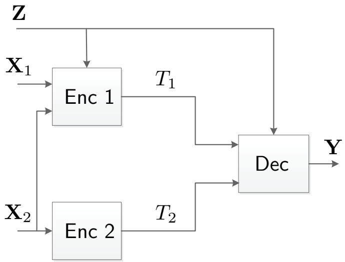


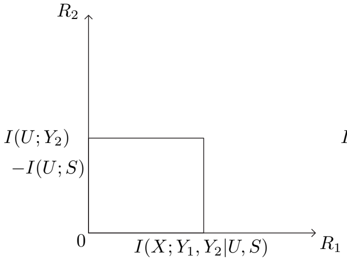


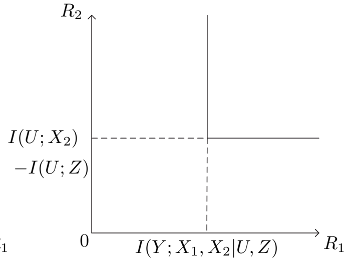
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:





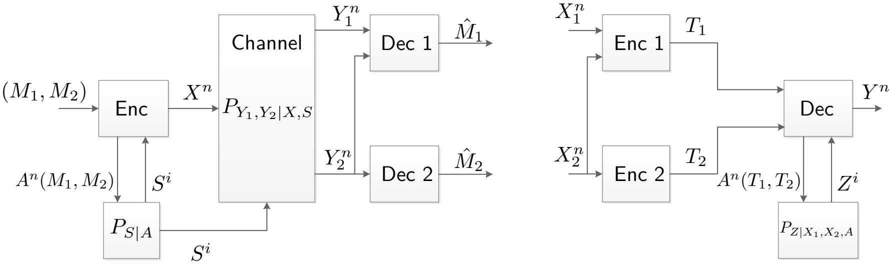




Action-Dependant Output-Degraded BC vs. Dual:

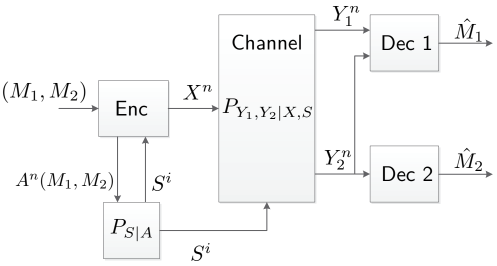
Multi-User Duality - Additional Examples

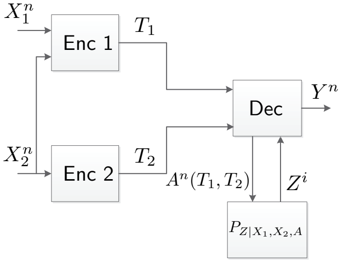
Action-Dependant Output-Degraded BC vs. Dual:

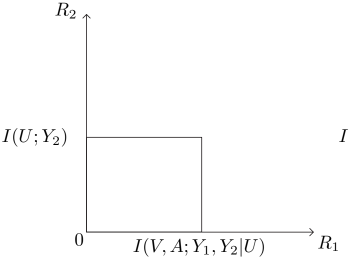


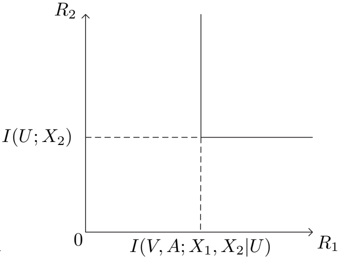
Multi-User Duality - Additional Examples

Action-Dependant Output-Degraded BC vs. Dual:

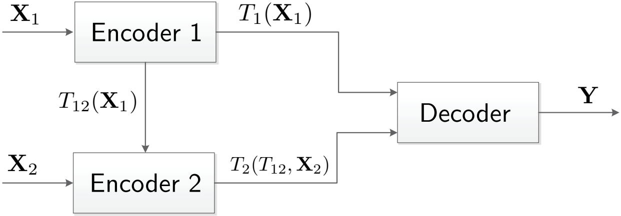








AK Problem with Cooperation - Achievability Outline



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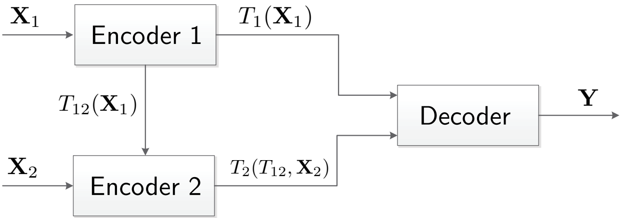

Rate	Corner Point 1	Corner Point 2
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$H(X_1 \vert V, U)$
R_2	$I(U;X_2 \vert V) - I(U;X_1 \vert V)$	$I(U;X_2 \vert V) + I(V;X_1)$

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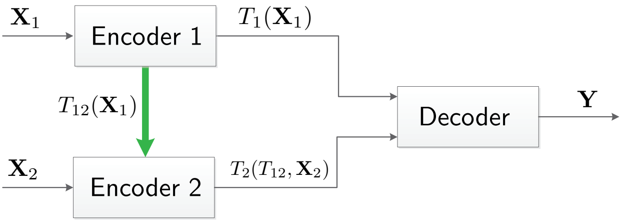

Rate	Corner Point 1	Corner Point 2
R_{12}	$\boldsymbol{I(V;X_1) - I(V;X_2)}$	$\boldsymbol{I(V;X_1) - I(V;X_2)}$
R_1	$H(X_1)$	$H(X_1 \vert V, U)$
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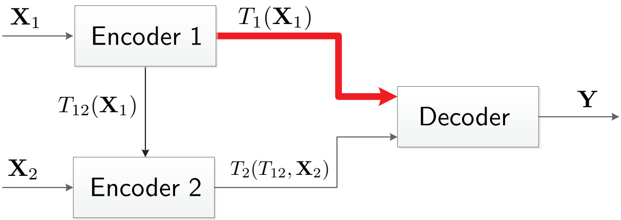
- **Cooperation:** Wyner-Ziv scheme to convey $\mathbf{V}$ via cooperation link.

AK Problem with Cooperation - Achievability Outline



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R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$\boldsymbol{H(X_1)}$	$H(X_1 \vert V, U)$
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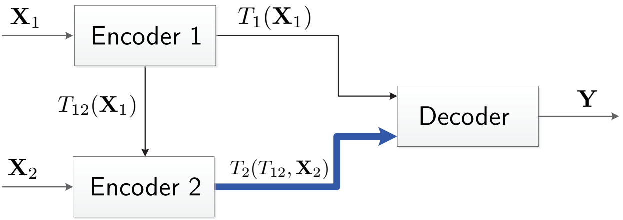

Rate	Corner Point 1	**Corner Point 2**
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$\boldsymbol{H(X_1 \vert V, U)}$
R_2	$I(U;X_2 \vert V) - I(U;X_1 \vert V)$	$\boldsymbol{I(U;X_2 \vert V) + I(V;X_1)}$

- **Cooperation:** Wyner-Ziv scheme to convey $\mathbf{V}$ via cooperation link.
- **Corner Point 1:** $\mathbf{V}$ is transmitted to dec. by Enc. 1 within $\mathbf{X}_1$.
- **Corner Point 2:** $\mathbf{V}$ is explicitly transmitted to dec. by Enc. 2.

AK Problem with Cooperation - Proof Outline

Converse:

AK Problem with Cooperation - Proof Outline

Converse:

- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\setminus i}, X_{2,i+1}^n),$$
$$U_i = T_2,$$

for every $1 \leq i \leq n$.

AK Problem with Cooperation - Proof Outline

Converse:

- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\setminus i}, X_{2,i+1}^n),$$
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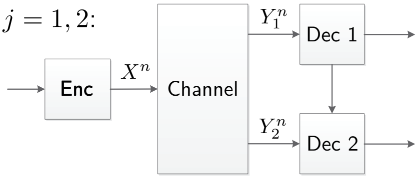
  for every $1 \leq i \leq n$.
- Time mixing properties.

Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:



Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
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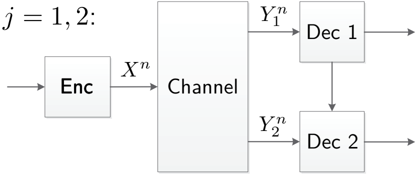

Semi-Deterministic BC with Cooperation - Achievability Outline

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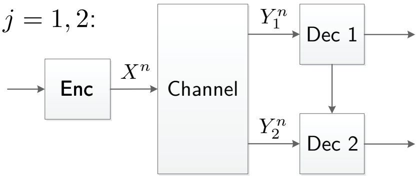

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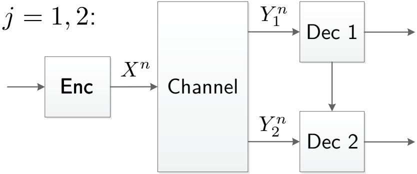

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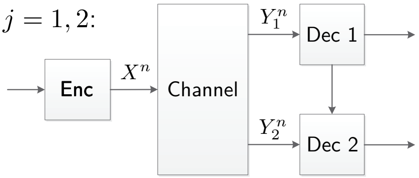

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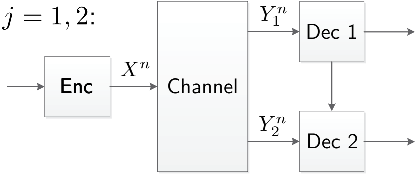

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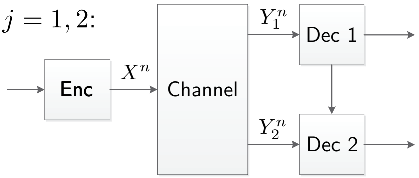

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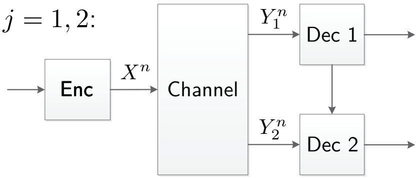

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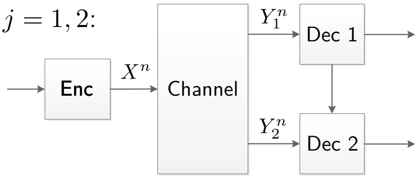

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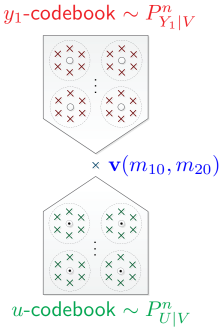

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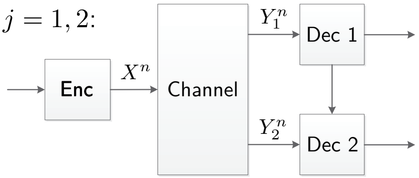

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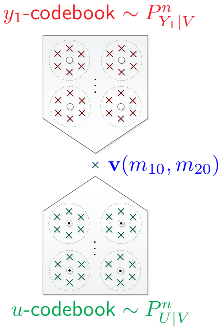

- **Decoding:** Joint typicality decoding.

Semi-Deterministic BC with Cooperation - Achievability Outline

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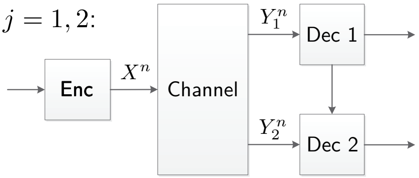

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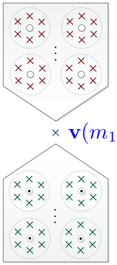

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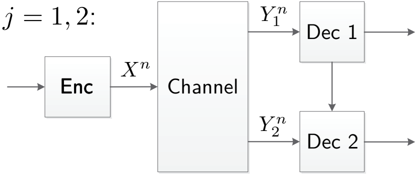

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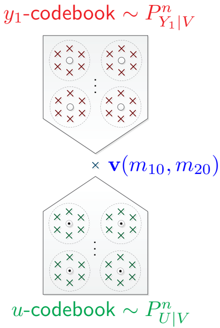

- **Decoding:** Joint typicality decoding.
- **Cooperation:** Bin number of V^n - $2^{nR_{12}}$ bins.
- **Gain:** Dec. 2 reduces search space of $\mathbf{V}$ by R_{12}.

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

Semi-Deterministic BC with Cooperation - Converse Outline

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$$= \sum_{i=1}^{n} \left[\boldsymbol{I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i)} \right] + I(M_2; M_{12})$$

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$$
\begin{aligned}
H(M_2) - n\epsilon_n &\leq \boldsymbol{I(M_2; Y_2^n | M_{12})} + I(M_2; M_{12}) \\
&= \boldsymbol{\sum_{i=1}^{n} \Big[I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^{i}) \Big]} + I(M_2; M_{12}) \\
&= \boldsymbol{\sum_{i=1}^{n} \Big[I(M_2; Y_{2,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_{1,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \Big]} \\
&\quad + I(M_2; M_{12})
\end{aligned}
$$

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$$
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&H(M_2) - n\epsilon_n \leq \boldsymbol{I(M_2; Y_2^n | M_{12})} + I(M_2; M_{12}) \\
&= \sum_{i=1}^{n} \Big[I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i) \Big] + I(M_2; M_{12}) \\
&= \sum_{i=1}^{n} \Big[I(M_2; Y_{2,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_{1,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \Big] \\
&\qquad + I(M_2; M_{12})
\end{aligned}
$$

⋆ Replaces 2 uses of Csiszár Sum Identity.