MIMO Gaussian Broadcast Channels with Common, Private and Confidential Messages

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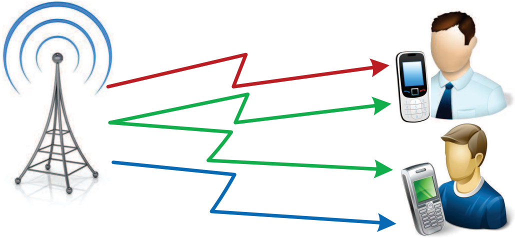
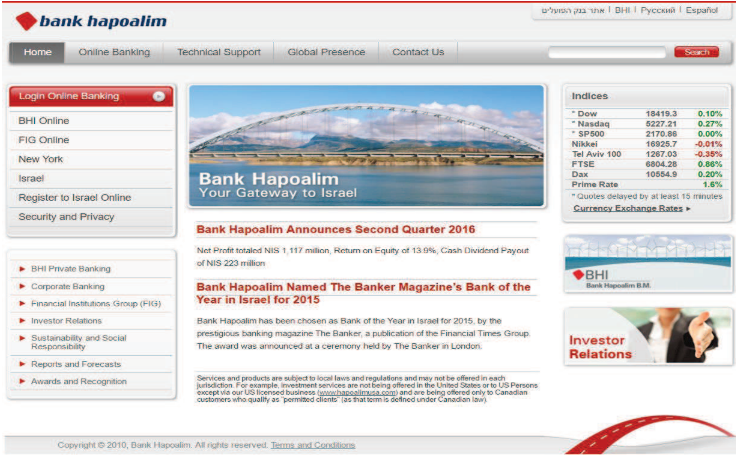


Motivation - Banking Site



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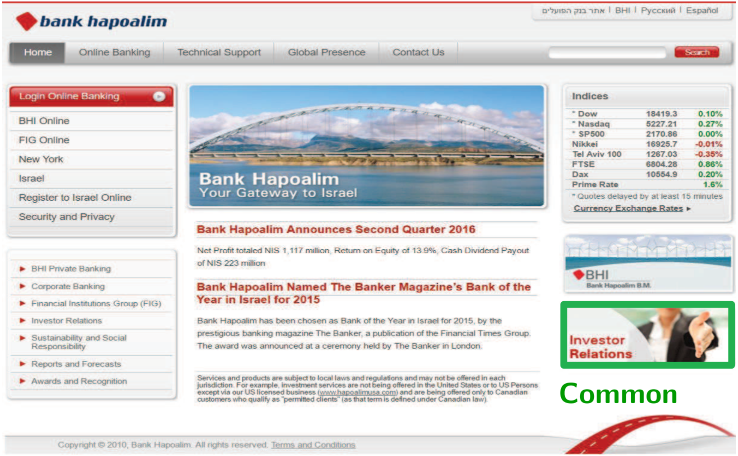

- **Common** - Advertisement.

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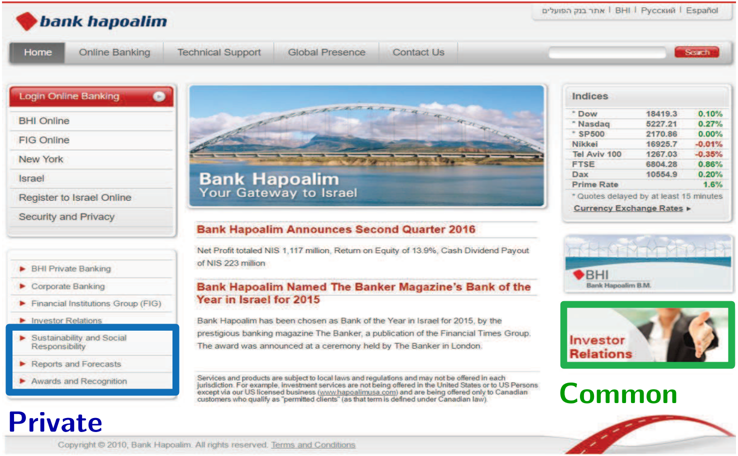

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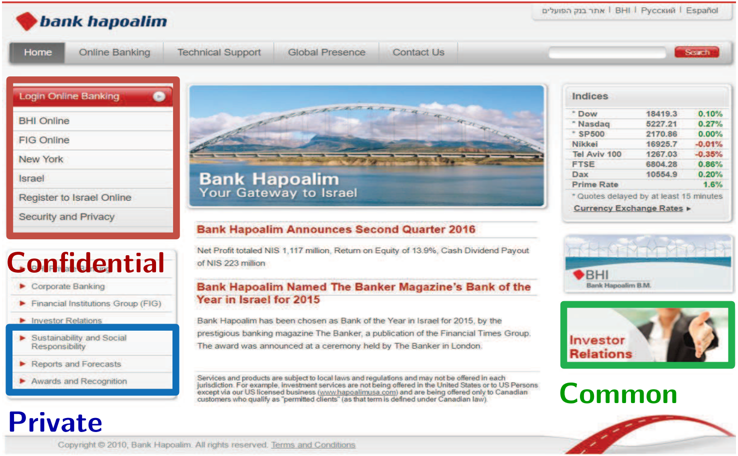
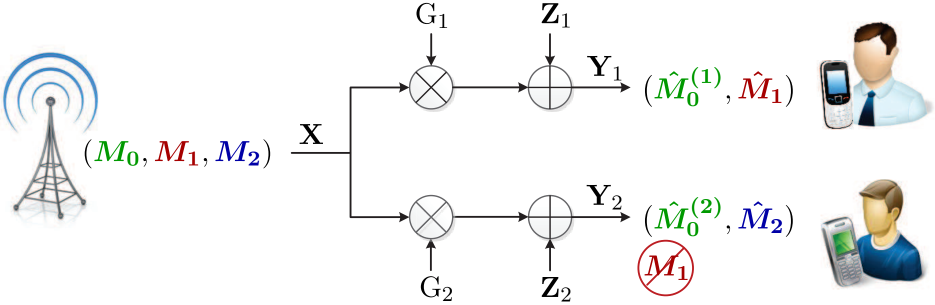


- **Common** - Advertisement.
- **Private** - On-demand Public info (programs, reports, forecasts).
- **Confidential** - Online banking (access account, transactions).

MIMO Gaussian BC - Problem Setup



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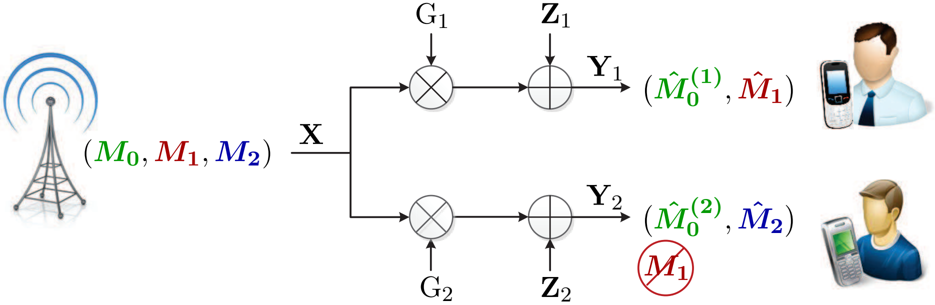

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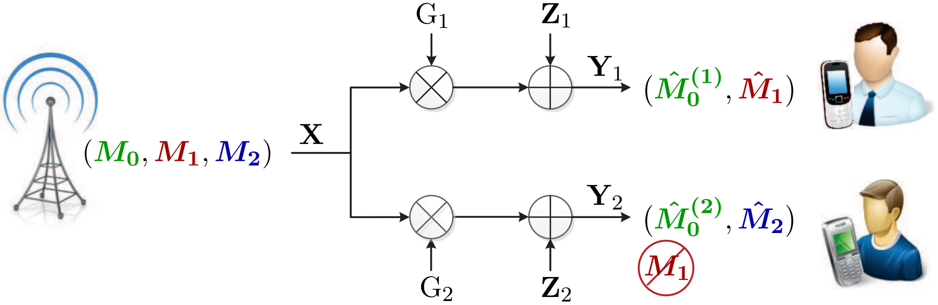

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- **Dimensions:** $\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Z}_1, \mathbf{Z}_2 \in \mathbb{R}^t \;\; ; \; G_1, G_2 \in \mathbb{R}^{t \times t}$.

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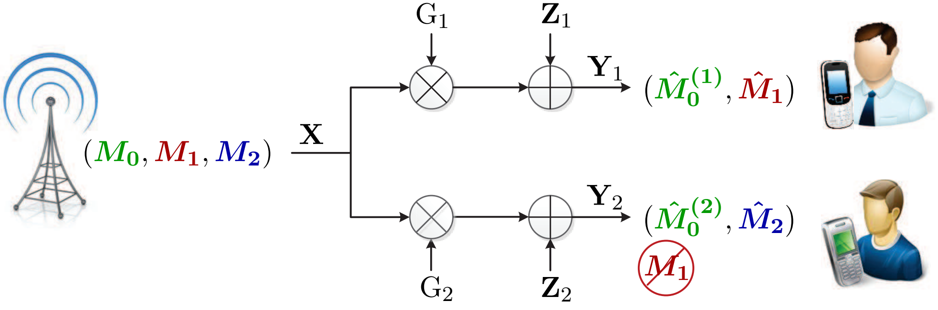

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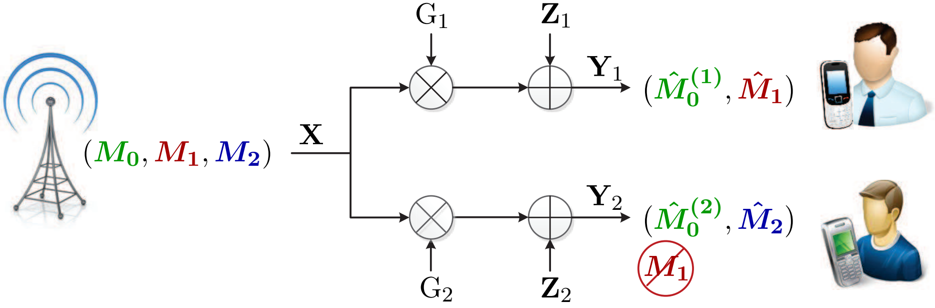

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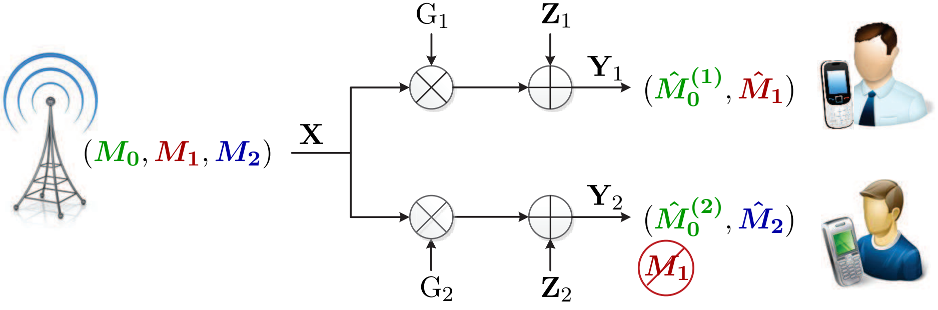

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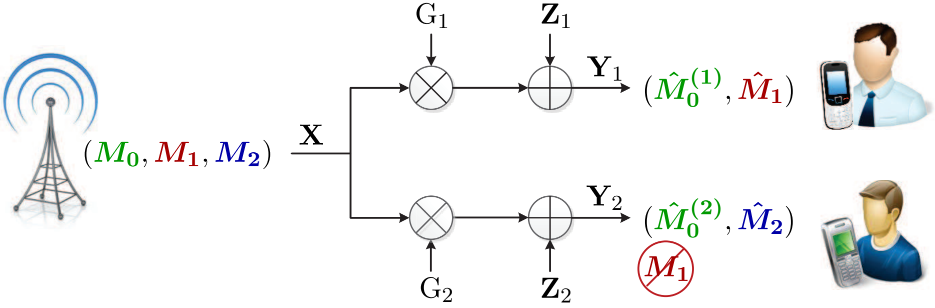
- **Input Covariance Constraint:** $\frac{1}{n}\sum_{i=1}^{n} \mathbb{E}\left[\mathbf{X}(i)\mathbf{X}^\top(i)\right] \preceq \mathrm{K}$.
- **Security Criterion:** $\frac{1}{n} I(M_1; \mathbf{Y}_2^n) \xrightarrow[n\to\infty]{} 0$.

MIMO Gaussian BC - Goals



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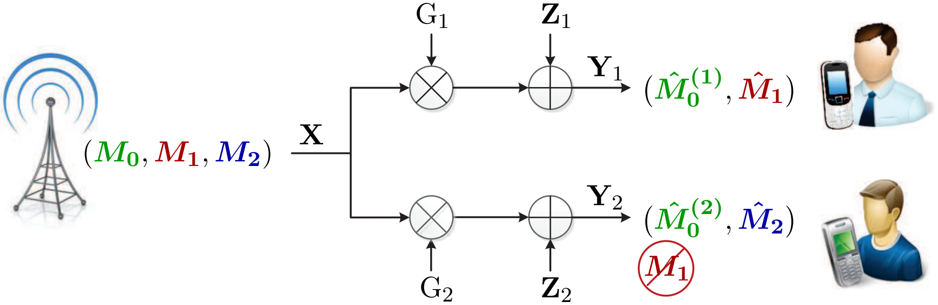

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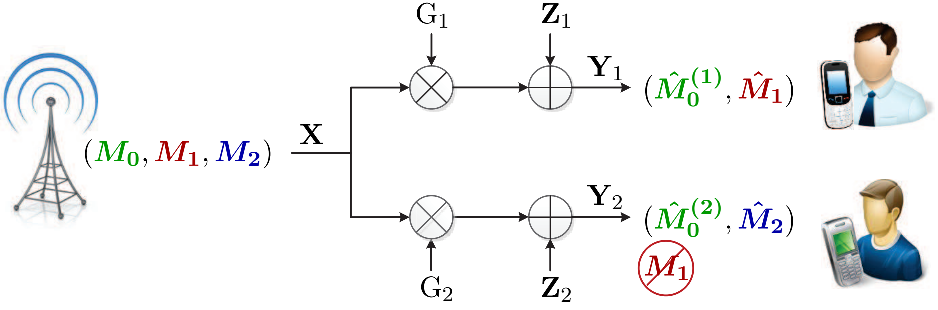

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  Q: Do Gaussian inputs achieve boundary points?

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MIMO Gaussian BCs with Eavesdropping Receivers:

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⋆ Solution for two last unsolved cases via **Upper Concave Envelopes** ⋆

MIMO Gaussian BC - Secrecy-Capacity Results

Without a Common Message: M_1 - Confidential ; M_2 - Private

Theorem (ZG 2016)

The secrecy-capacity region for a covariance constraint $\mathrm{K} \succeq 0$ *is*

$$\hat{\mathcal{C}}_{\mathrm{K}} = \bigcup_{0 \preceq \mathrm{K}^\star \preceq \mathrm{K}} \left\{ (R_1, R_2) \in \mathbb{R}_+^2 \;\middle|\; \begin{array}{l} R_1 \leq \frac{1}{2} \log \left| \dfrac{\mathrm{I} + \mathrm{G}_1 \mathrm{K}^\star \mathrm{G}_1^\top}{\mathrm{I} + \mathrm{G}_2 \mathrm{K}^\star \mathrm{G}_2^\top} \right| \\ R_2 \leq \frac{1}{2} \log \left| \dfrac{\mathrm{I} + \mathrm{G}_2 \mathrm{K} \mathrm{G}_2^\top}{\mathrm{I} + \mathrm{G}_2 \mathrm{K}^\star \mathrm{G}_2^\top} \right| \end{array} \right\}.$$

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- $\boldsymbol{R_2}$ **Bound - Capacity of MIMO Gaussian PTP to User 2:**
  Input covariance $\mathrm{K} - \mathrm{K}^\star$; Noise covariance $\mathrm{I} + \mathrm{G}_2 \mathrm{K}^\star \mathrm{G}_2^\top$.

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- R_1 **Bound: MIMO Gaussian WTC with input** K_1

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- **R_0 Bound: MIMO Gaussian PTP with remaining covariance $\mathbf{K} - (\mathbf{K}_1 + \mathbf{K}_2)$ ($\mathbf{K}_1, \mathbf{K}_2$ are noises).**

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$\implies$ $\boldsymbol{\mathcal{O}_\mathrm{K}} \subseteq$ **Region from Theorem**

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Achievevability:

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Achievability: Substituting Gaussian inputs into $\mathcal{I}_{\mathrm{K}}$.

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- **Dirty Paper Coding to cancel M_2 signal at Receiver 1.**

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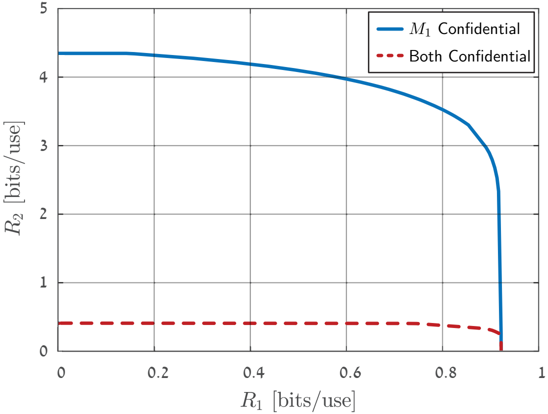

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Thank You!