Semantic Security in the Presence of Active Adversaries

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Joint work with Paul Cuff and Haim Permuter

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ECE Department Seminar, NJIT

March 8th, 2016

Motivation

Information Theoretic Security over Noisy Channels

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Our Goal: Stronger metric and remove "known channel" assumption.

Some Background

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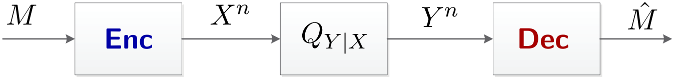
$$\star \quad I(X;Y) = D(P_{X,Y}||P_XP_Y) \quad \star$$

Channel Coding Theorem

[Shannon 1948]



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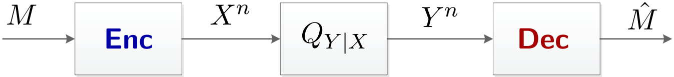

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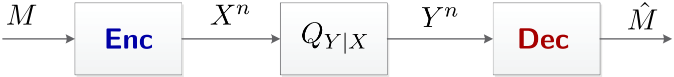

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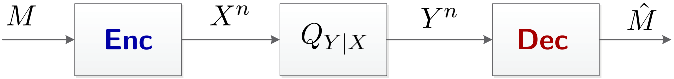

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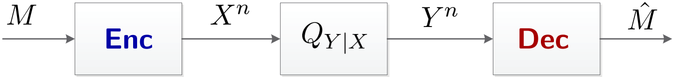

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- **Capacity:** $C \triangleq \sup\Big\{R \Big| \exists(n,R) - \text{codes s.t. } \mathbb{P}\big(M \neq \hat{M}\big) \underset{n}{\to} 0\Big\}$.

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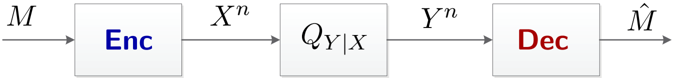

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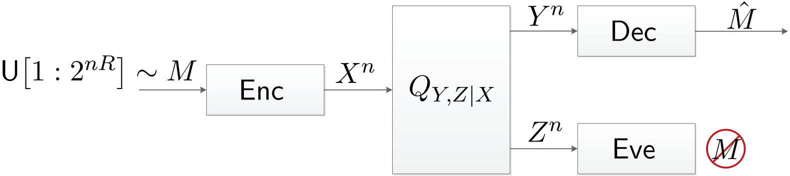
Theorem (Shannon 1948)

The capacity of a DMC $Q_{Y|X}$ *is* $C = \max_{Q_X} I(X;Y)$.

Wiretap Channels - Security Metrics

Wiretap Channels and Security Metrics

Degraded [Wyner 1975], General [Csiszár-Körner 1978]



$\mathsf{U}\left[1:2^{nR}\right] \sim M$

Enc

X^n

$Q_{Y,Z|X}$

Y^n

Dec

$\hat{M}$

Z^n

Eve

$\cancel{M}$

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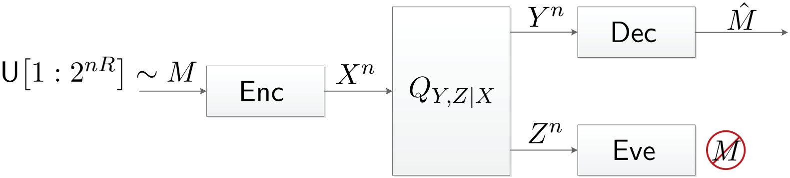

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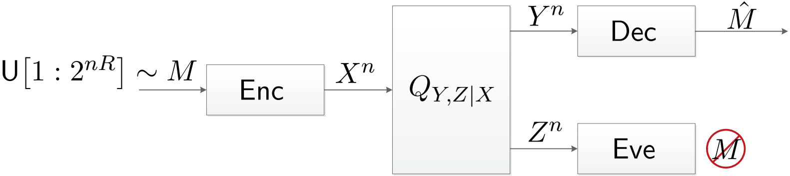

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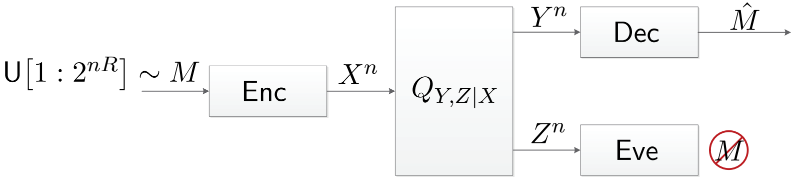

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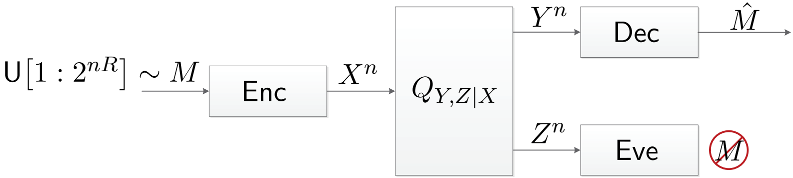

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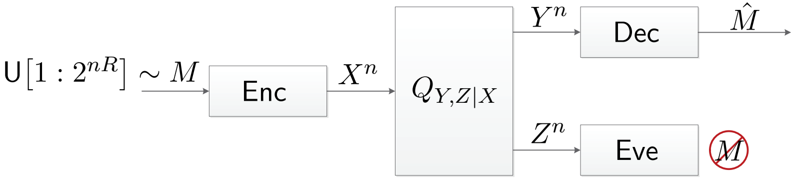

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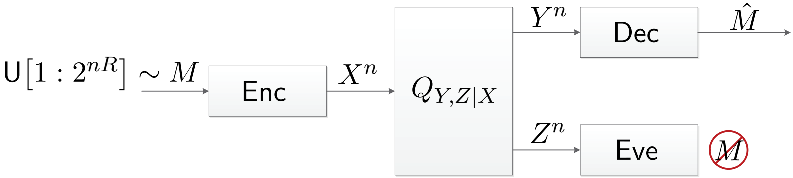

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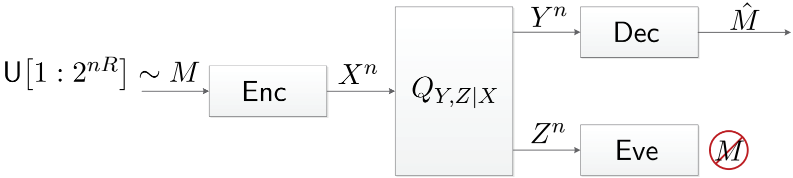

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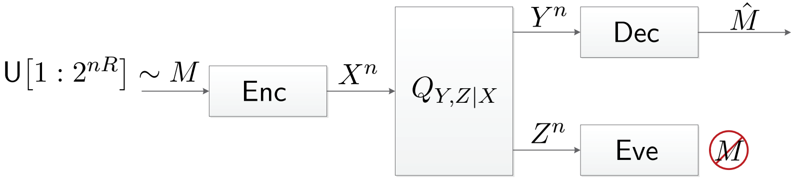

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★ A stronger secrecy metric is required for applications ★

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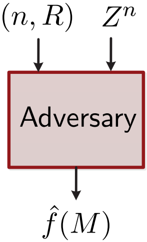

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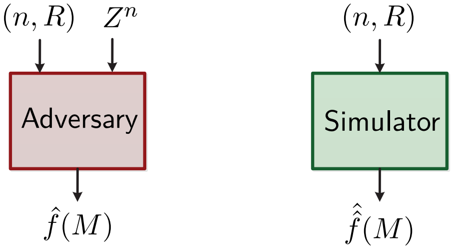

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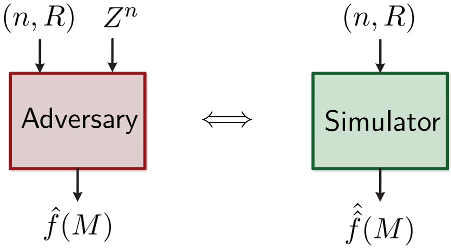

(n, R) Z^n

Adversary

$\Longleftrightarrow$

(n, R)

Simulator

$\hat{f}(M)$

$\hat{\hat{f}}(M)$

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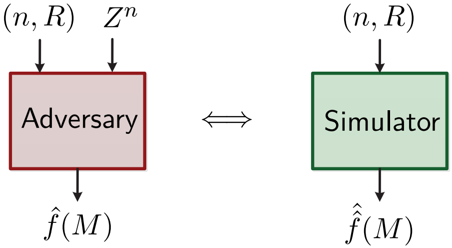

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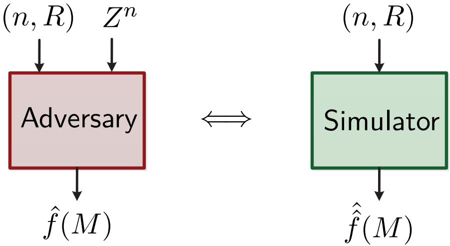

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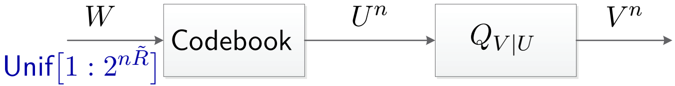
★ A single code must work well for all message PMFs ★

A Stronger Soft-Covering Lemma

Soft-Covering - Setup



W

$\mathsf{Unif}[1:2^{n\tilde{R}}]$

Codebook

U^n

$Q_{V|U}$

V^n

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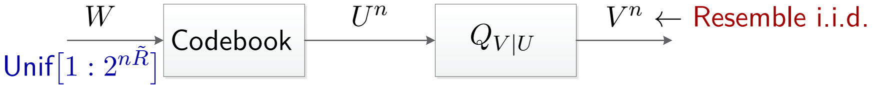

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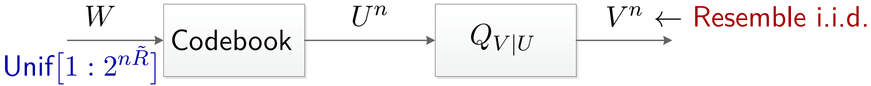

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$V^n \leftarrow$ Resemble i.i.d.

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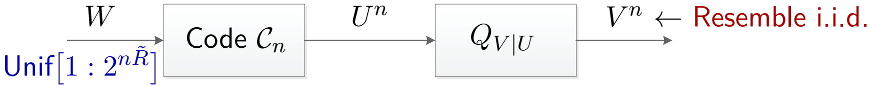

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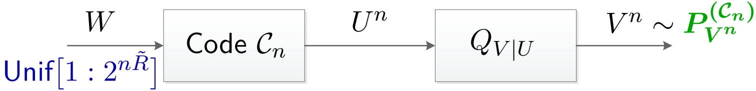

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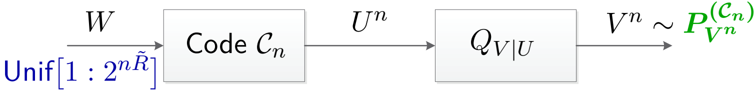

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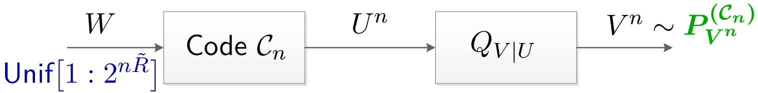

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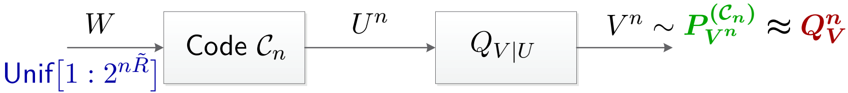

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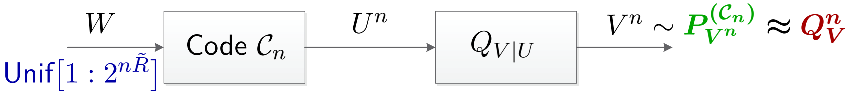

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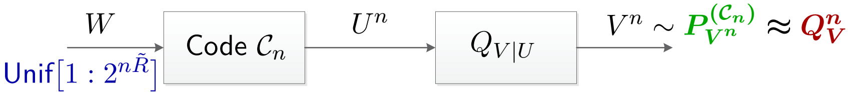
★ **Goal:** Choose $\boldsymbol{\tilde{R}}$ (codebook size) s.t. $\boldsymbol{P_{V^n}^{(\mathcal{C}_n)}} \approx \boldsymbol{Q_V^n}$ ★

Soft-Covering - Setup



W

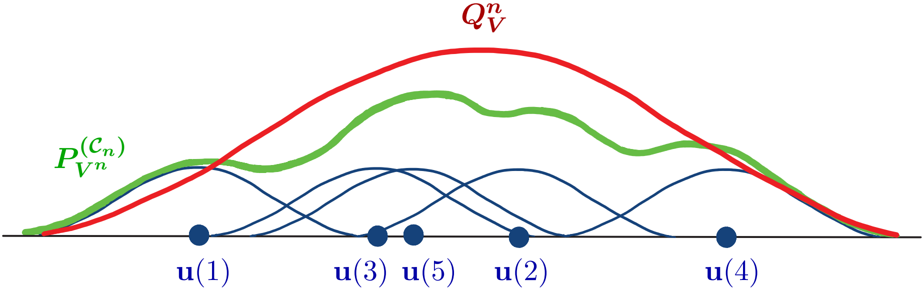
Unif$[1:2^{n\tilde{R}}]$

Code $\mathcal{C}_n$

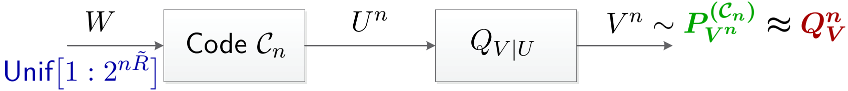
U^n

$Q_{V|U}$

$V^n \sim P_{V^n}^{(\mathcal{C}_n)} \approx Q_V^n$

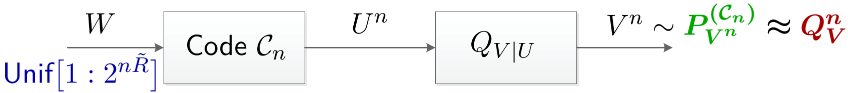


Soft-Covering - Results



$$\tilde{R} > I_Q(U;V) \implies P_{V^n}^{(\mathcal{C}_n)} \approx Q_V^n$$

Soft-Covering - Results



$$\boldsymbol{\tilde{R} > I_Q(U;V)} \implies \boldsymbol{P^{(\mathcal{C}_n)}_{V^n}} \approx \boldsymbol{Q^n_V}$$

- **Wyner 1975**: $\mathbb{E}_{\mathbb{C}_n}\frac{1}{n}D\left(P^{(\mathbb{C}_n)}_{V^n}\middle|\middle|Q^n_V\right) \xrightarrow[n\to\infty]{} 0.$

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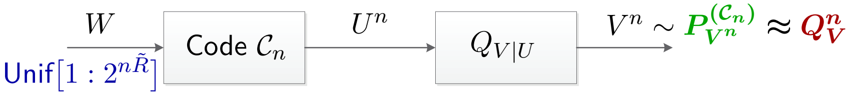

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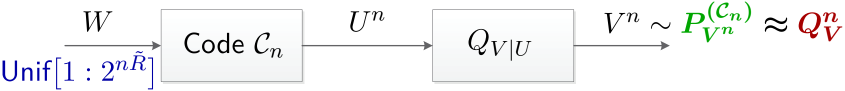

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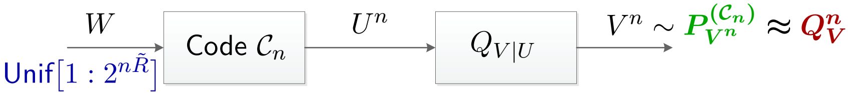

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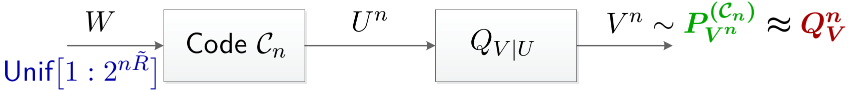
A Stronger Soft-Covering Lemma



Lemma (Cuff 2015)

If $\tilde{R} > I_Q(U;V)$ *and* $|\mathcal{V}| < \infty$*, then there exists* $\gamma_1, \gamma_2 > 0$ *s.t.*

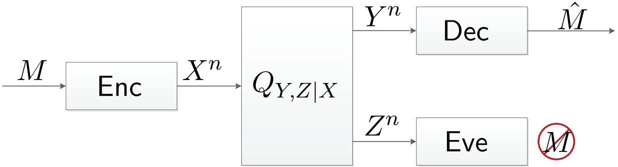
$$\mathbb{P}_{\mathbb{C}_n}\left(D\left(P_{V^n}^{(\mathbb{C}_n)} \middle\| Q_V^n \right) > e^{-n\gamma_1} \right) \leq e^{-e^{n\gamma_2}}$$

for n *sufficiently large.*

Revisit Wiretap Channels - Semantic Security

Semantic Security for Wiretap Channels

DM [Bellare-Tessaro-Vardy 2012], Gaussian [Tyagi-Vardy 2014]



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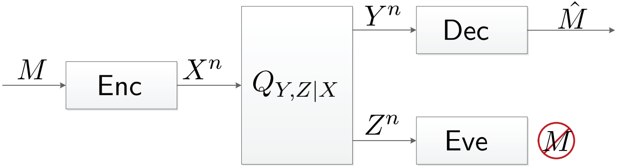

- **Security Metric:** $\max_{P_M} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$

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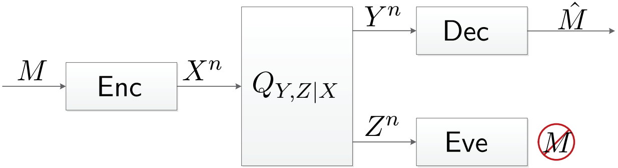

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Theorem

$$C_{\text{Semantic}} = C_{\text{Weak}} = \max_{Q_{U,X}} \Big[I(U;Y) - I(U;Z) \Big]$$

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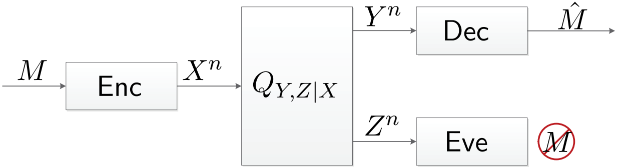

- **Security Metric:** $\max\limits_{P_M} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$

Theorem

$$C_{\text{Semantic}} = C_{\text{Weak}} = \max_{Q_{U,X}} \Big[I(U;Y) - I(U;Z) \Big]$$

- **Our Derivation:** **Union bound** & **Stronger soft-covering lemma**.

Semantic Security for Wiretap Channels - Derivation

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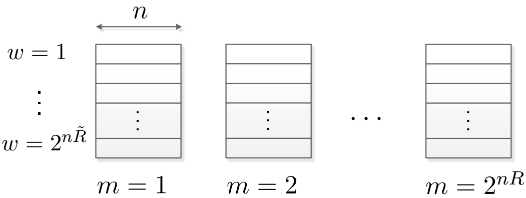

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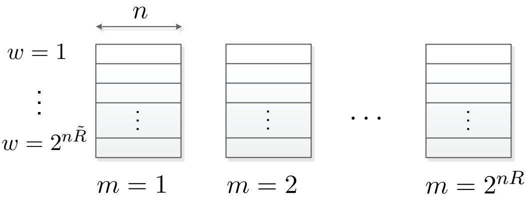

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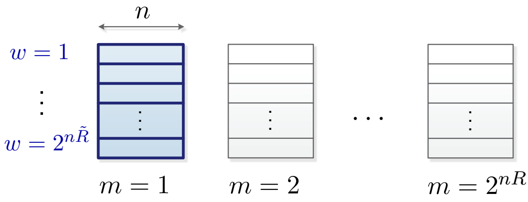

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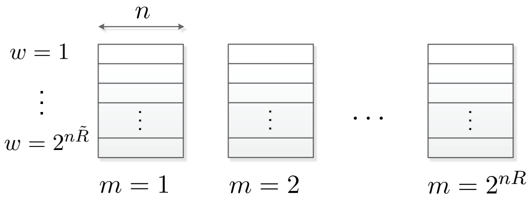

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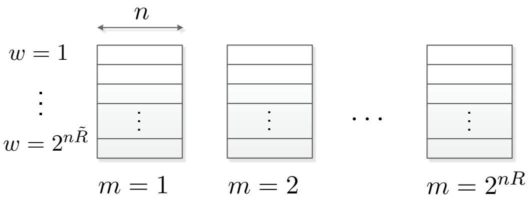

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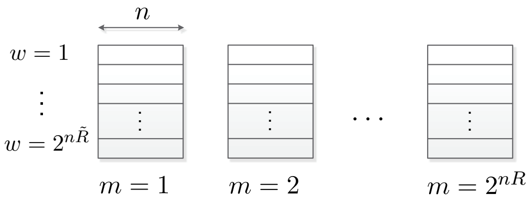

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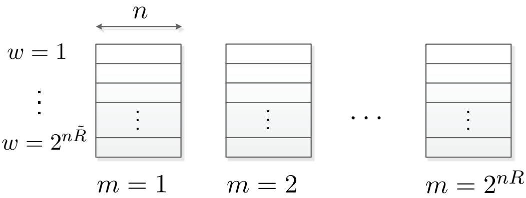

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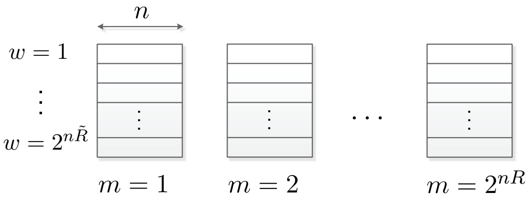

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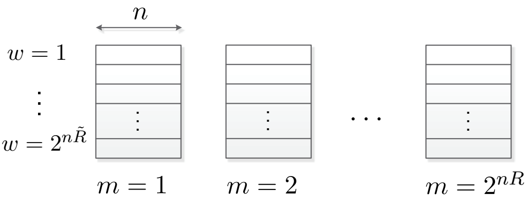

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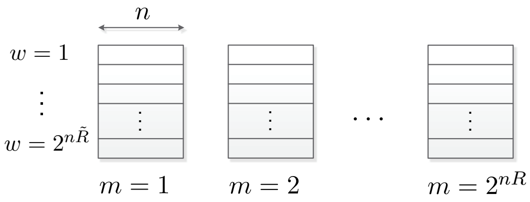

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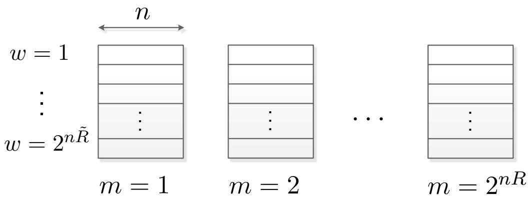

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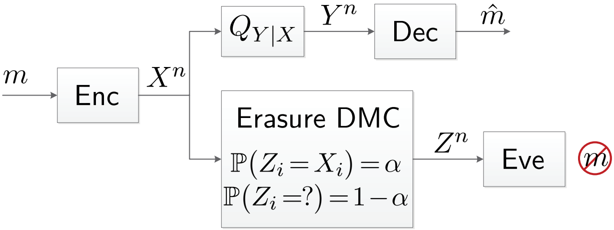
Wiretap Channels of Type II

Wiretap Channels of Type II - Preliminary

[Ozarow and Wyner 1984]



- **WTC I with Erasure DMC to Eve:**

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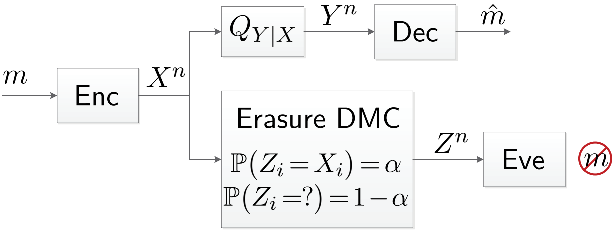
[Ozarow and Wyner 1984]



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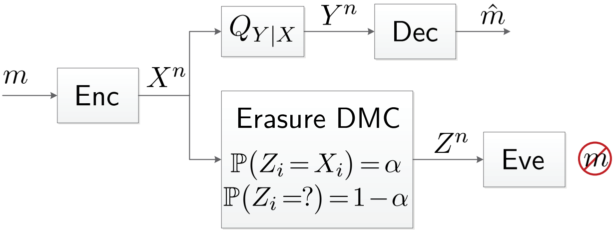

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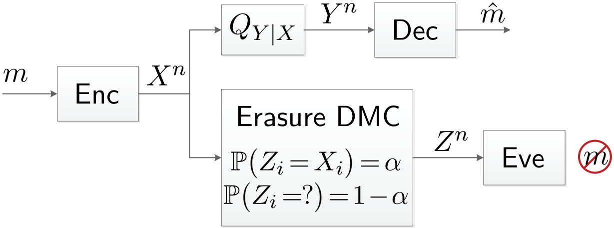

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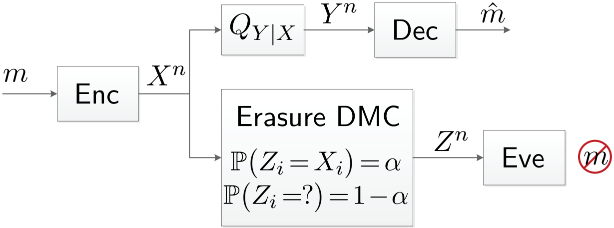

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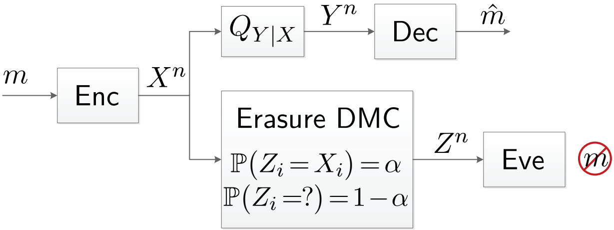
- **WTC II: Stronger Eve.**
  - Eve chooses which αn symbols of X^n to observe.

Wiretap Channels of Type II - Preliminary

[Ozarow and Wyner 1984]



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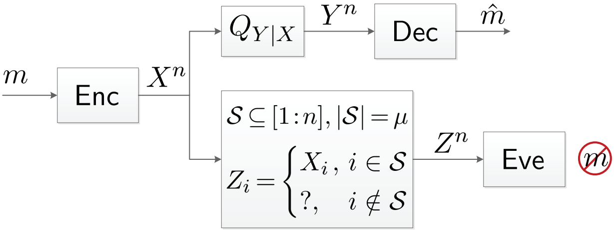
- **WTC II: Stronger Eve.**
  - Eve chooses which αn symbols of X^n to observe.
  - Ensure security versus all possible choices of observations.

Wiretap Channels of Type II - Definition

[Ozarow-Wyner 1984]



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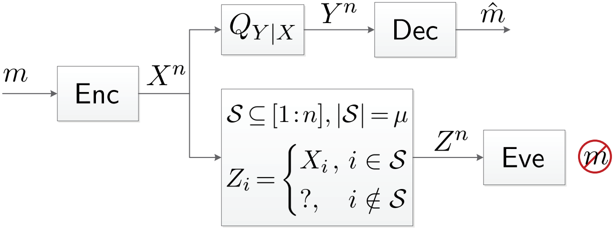

- **Eavesdropper:** Can observe a subset $\boldsymbol{\mathcal{S}} \subseteq [1:n]$ of size $\boldsymbol{\mu = \lfloor \alpha n \rfloor}$, $\boldsymbol{\alpha \in [0,1]}$, of transmitted symbols.

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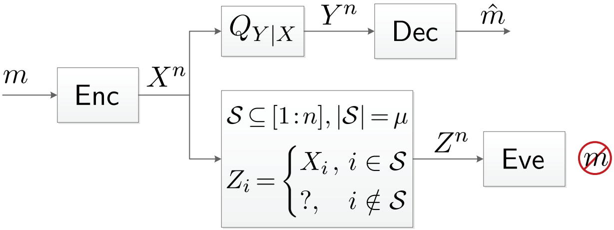

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$n = 10$ $\boldsymbol{\alpha = 0.63}$

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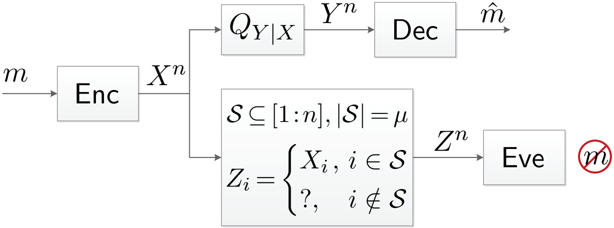

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Wiretap Channels of Type II - Past Results

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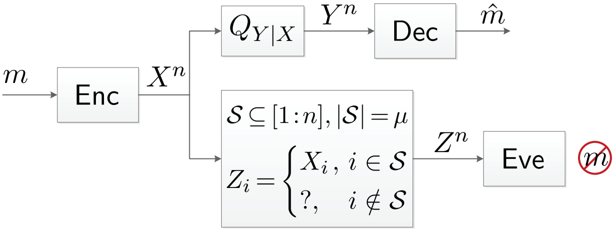

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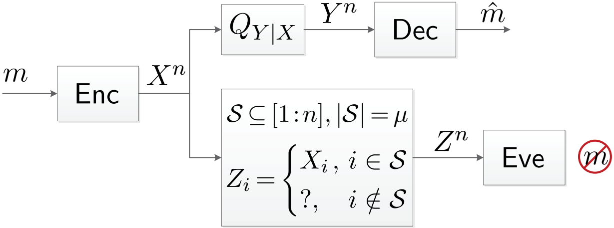

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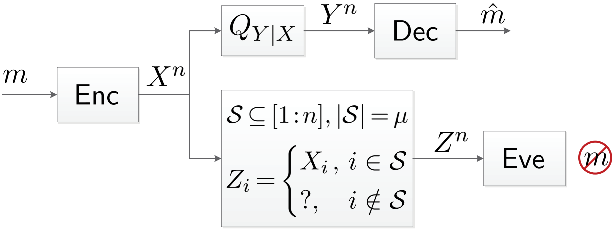

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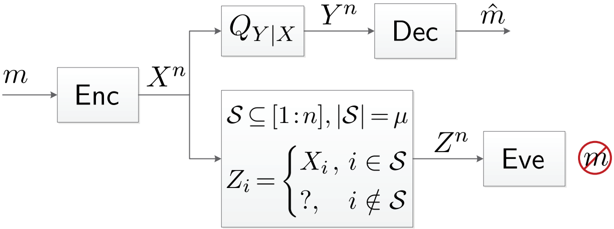

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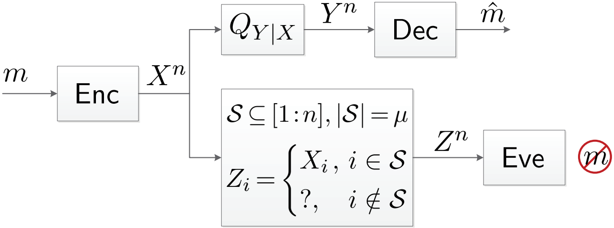

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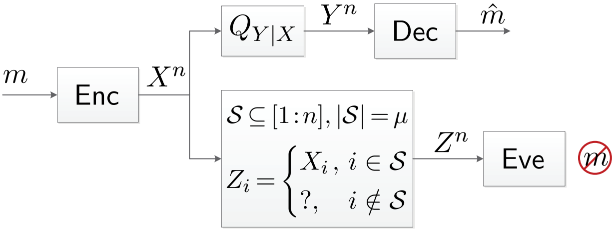

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  - Lower & upper bounds - Not match in general.

Wiretap Channels of Type II - SS-Capacity

Semantic Security:

Wiretap Channels of Type II - SS-Capacity

Semantic Security: $$\max_{\substack{P_M,\mathcal{S}:\\|\mathcal{S}|=\mu}} I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0.$$

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Theorem (ZG-Cuff-Permuter 2015)

For any $\alpha \in [0,1]$

$$C^{(\mathrm{II})}_{\mathrm{Semantic}}(\alpha) = C^{(\mathrm{II})}_{\mathrm{Weak}}(\alpha) = \max_{Q_{U,X}} \Big[I(U;Y) - \alpha I(U;X) \Big]$$

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WTC II SS-Capacity - Achievability for U=X

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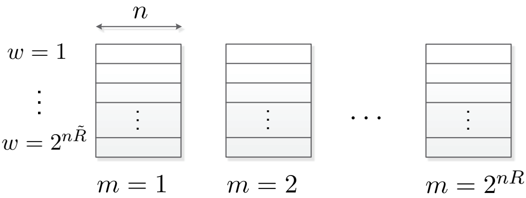

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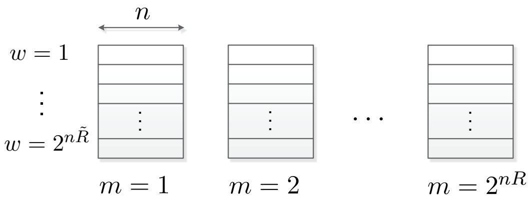

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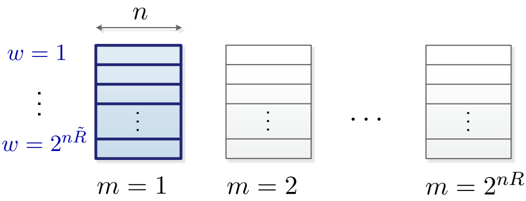

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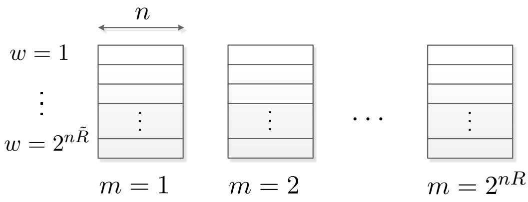

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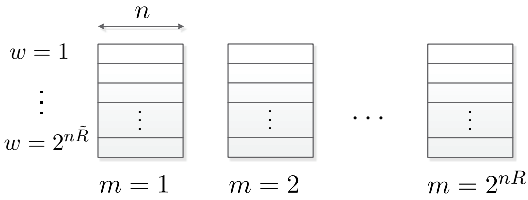

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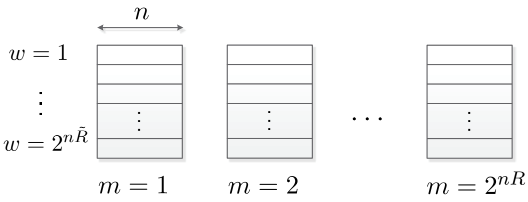

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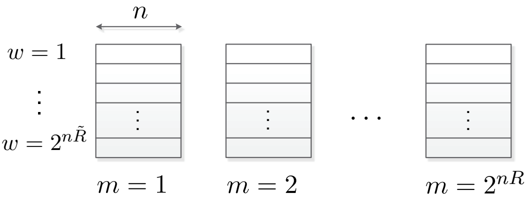

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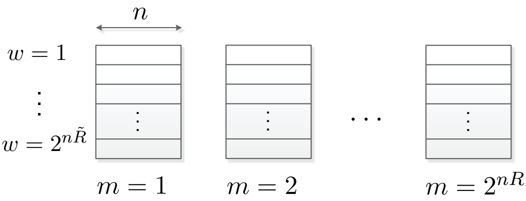

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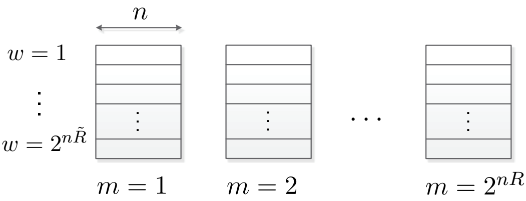

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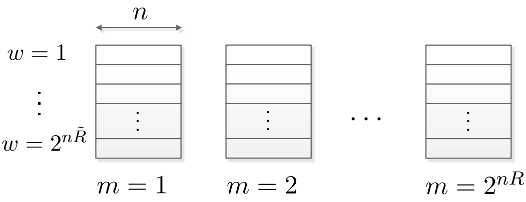

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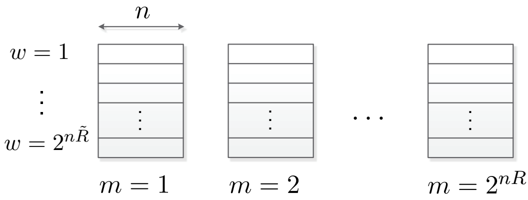

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WTC II SS-Capacity - Achievability for U=X

Finalization:

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WTC II SS-Capacity - Achievability for U=X

Finalization:

- **Semantic Security:** Satisfied if $\tilde{R} > \alpha H(X)$
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WTC II SS-Capacity - Achievability for U=X

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- **Rate Bound:** $R < I(X;Y) - \alpha H(X)$ is achievable.
- **Channel Prefixing:** Prefixing $Q_{X|U}$ achieves $I(U;Y) - \alpha I(U;X)$.

WTC II SS-Capacity - Converse

$$\boxed{\textbf{SS-capacity WTC II} \ \leq \ \textbf{Weak-secrecy-capacity WTC I}}$$

WTC II SS-Capacity - Converse

SS-capacity WTC II $\leq$ **Weak-secrecy-capacity WTC I**

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$$\boxed{\textbf{SS-capacity WTC II} \leq \textbf{Weak-secrecy-capacity WTC I}}$$

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- **Difficulty:** Eve might observe more X_i-s in **WTC I** than in **WTC II**.

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- **WTC I** with erasure DMC to Eve - Transition probability α.

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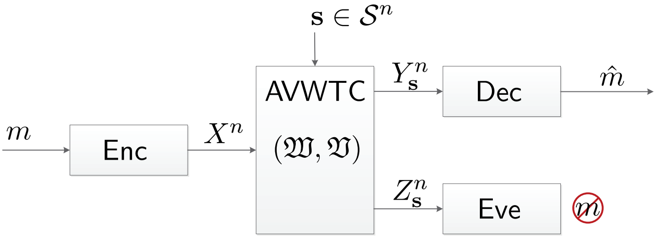
- **Solution:** Sanov's theorem & Continuity of mutual information.

Arbitrarily Varying Wiretap Channels

Arbitrarily Varying Wiretap Channels - Definition



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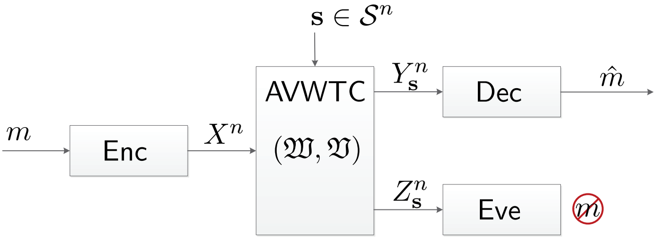

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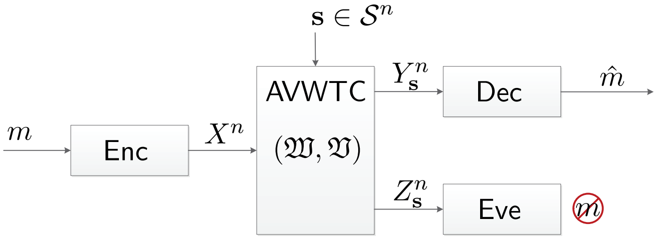

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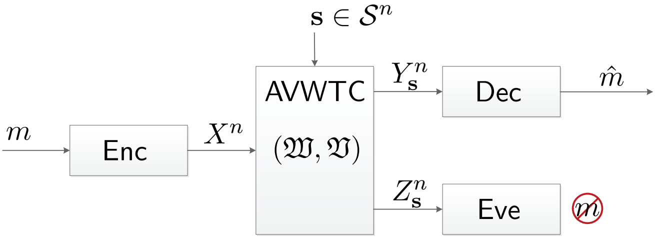

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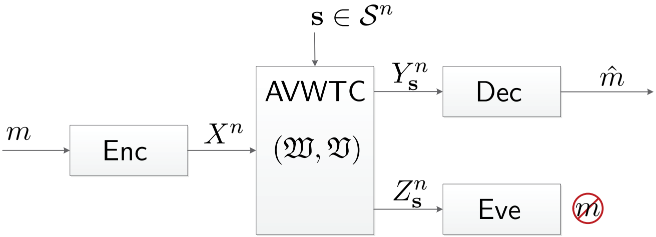

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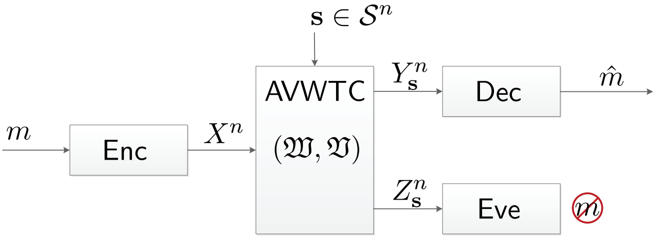

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  - CR should **not** be viewed as cryptographic key for secrecy.

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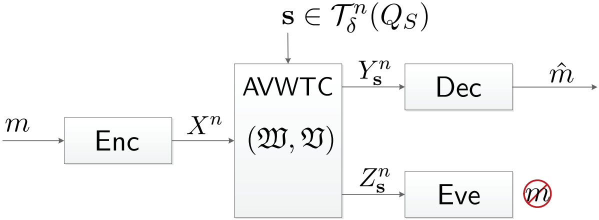

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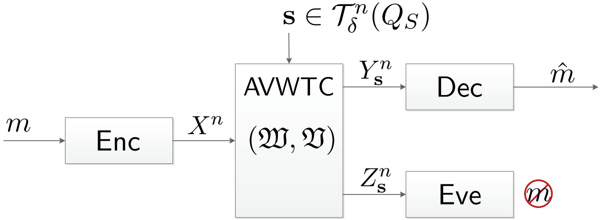

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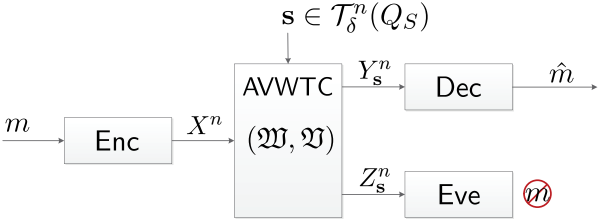

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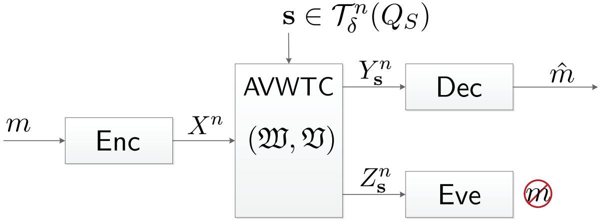

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   - $|\Gamma_n| = n^3$ - Polynomial in n.

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$\mathcal{Q}$ is convex and closed

$$C_{\mathrm{R}}(\mathfrak{W}, \mathfrak{V}, \mathcal{Q}) \geq \max_{Q_{U,X}} \left[\min_{Q_S^{(1)} \in \mathcal{Q}} I(U;Y) - \max_{Q_S^{(2)} \in \mathcal{Q}} I(U;Z|S) \right]$$

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$\mathcal{Q}$ contains only rational PMFs

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