The Semi-Deterministic BC with Cooperation and a Dual Source Coding Problem

Ziv Goldfeld, Haim H. Permuter and Gerhard Kramer

Ben Gurion University and Technische Universität München

June, 2014

Goal: Fundamental limits of compression and transmission:

Introduction to Information Theory

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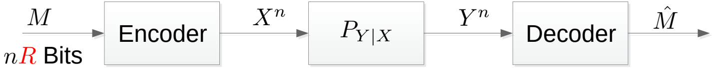

Introduction to Information Theory

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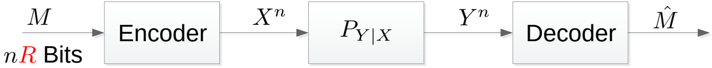

What is the highest transmission rate?

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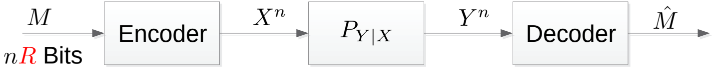

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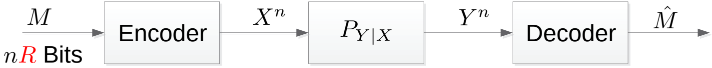

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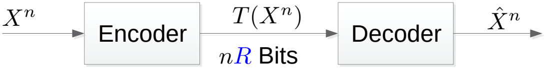

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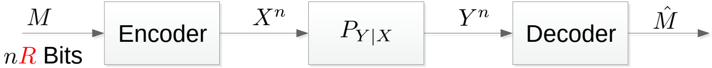

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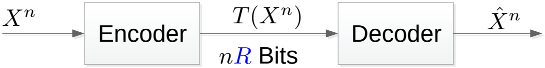

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Introduction to Information Theory - Tools

Mathematical Tools:

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   - Entropy:

$$H(X) = -\mathbb{E}\Big[\log\big(P_X(X)\big)\Big] = -\sum_{x\in\mathcal{X}} P_X(x)\log\big(P_X(x)\big)$$

$$H(X|Y) = -\sum_{x\in\mathcal{X}} P_{X,Y}(x,y)\log\big(P_{X|Y}(x|y)\big)$$

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$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X).$$

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2. Method of types:

$$\mathcal{T}_\epsilon^{(n)}(P_X) = \Big\{x^n \in \mathcal{X}^n \,\Big|\, |P_X(a) - \nu_{x^n}(a)| < \epsilon,\ \forall a \in \mathcal{X}\Big\}.$$

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3. etc.


M
nR Bits
Encoder
X^n
$P_{Y|X}$
Y^n
Decoder
$\hat{M}$

Introduction to Information Theory - Channel Coding

- **Message:** M is uniform over $\{1, \ldots, 2^{nR}\}$.

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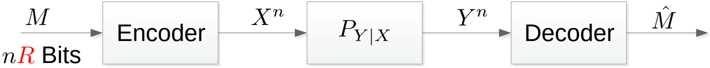

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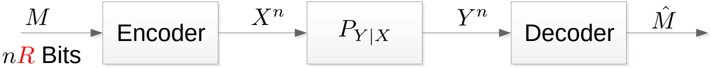

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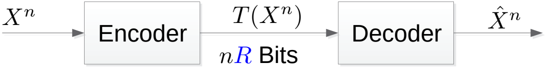
- **Result:**

$$C = \max_{P_X} I(X;Y).$$

Introduction to Information Theory - Source Coding



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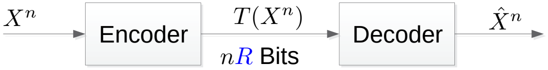

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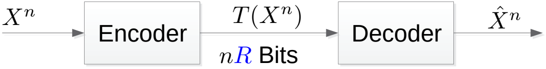

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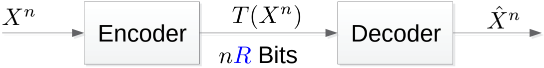

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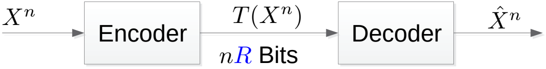

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Outline

- Motivation and past work
- AK problem with one-sided encoder cooperation
- Semi-deterministic BC with one-sided decoder cooperation
- Duality
- Summary

Motivation and Past Work

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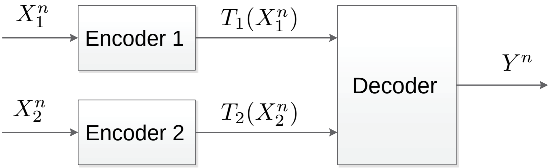

$$(X_1^n, X_2^n, Y^n) \in \mathcal{T}_\epsilon^{(n)}(P_{X_1,X_2} P^\star_{Y|X_1,X_2})$$

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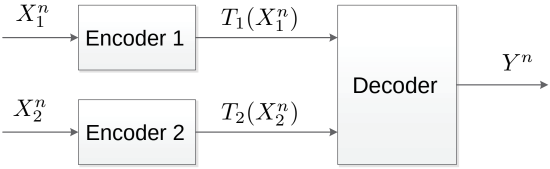

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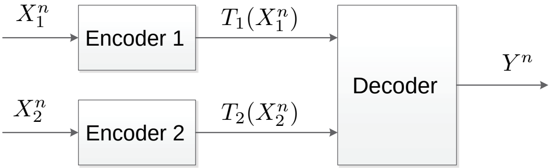

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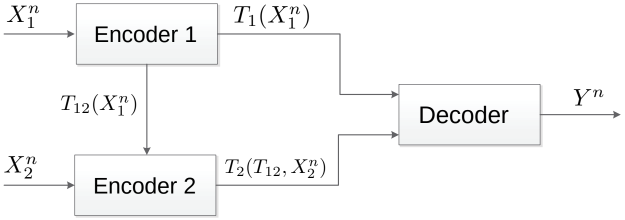
- Cooperation can dramatically boost performance of a network.

AK Problem with Cooperation - Definition



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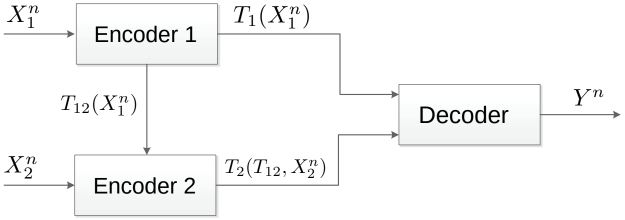

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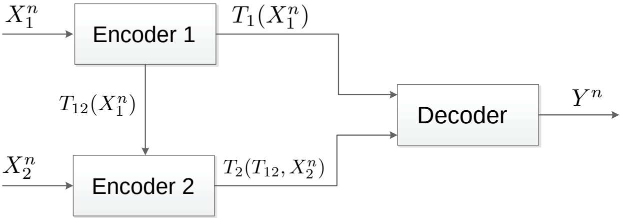

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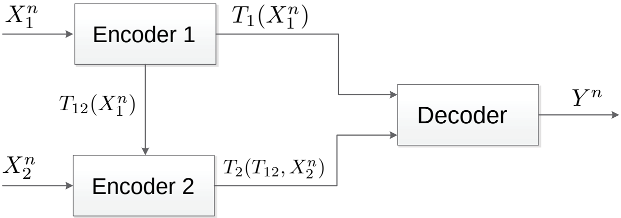

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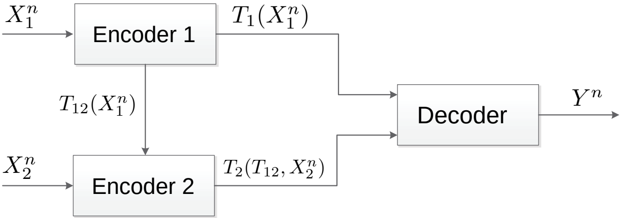

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AK Problem with Cooperation - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination distribution $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *the coordination-capacity region is:*

$$\mathcal{C}_{AK} = \bigcup \left\{ \begin{array}{r} (R_{12}, R_1, R_2) \in \mathbb{R}_+^3 : R_{12} \geq I(V;X_1) - I(V;X_2) \\ R_1 \geq H(X_1|V,U) \\ R_2 \geq I(U;X_2|X_1,V) \\ R_1 + R_2 \geq H(X_1|V,U) + I(V,U;X_1,X_2) \end{array} \right\}$$

where the union is over all joint distributions $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

AK Problem with Cooperation - Proof Outline

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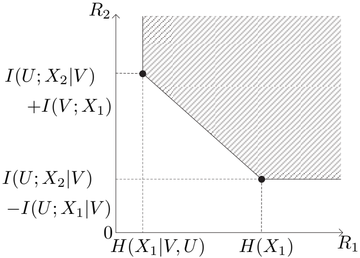

Figure : Region at $R_{12} = I(V;X_1) - I(V;X_2)$.

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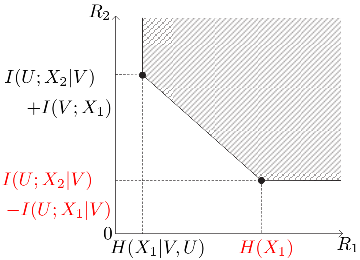

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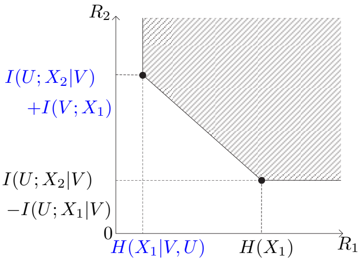

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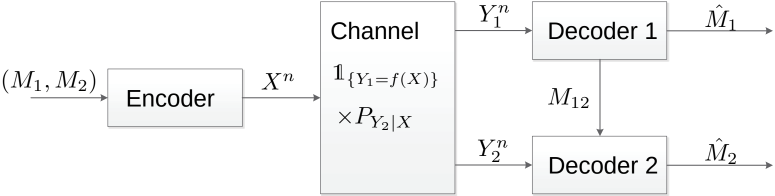
Semi-Deterministic BC with Cooperation - Definition



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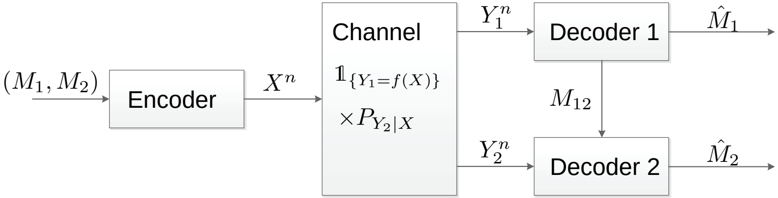

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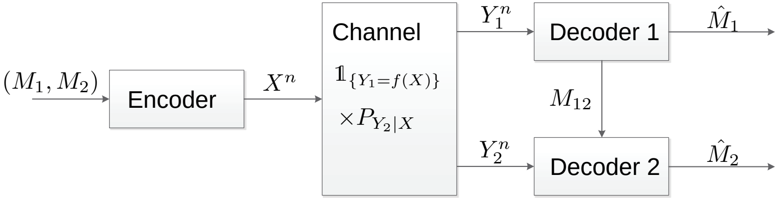

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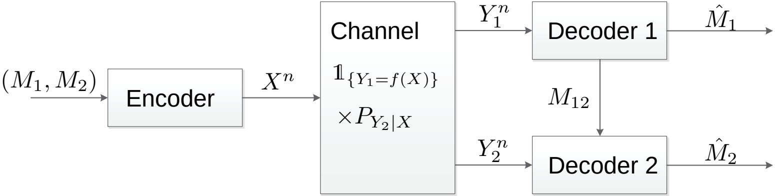

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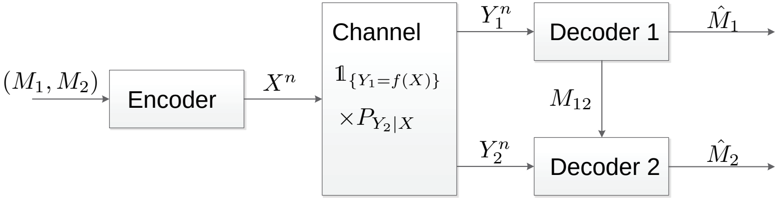

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- **Decoders' output:** $\hat{M}_1(Y_1^n)$ and $\hat{M}_2(M_{12}, Y_2^n)$;
  $P_e^{(n)} \triangleq \mathbb{P}\Big[(M_1, M_2) \neq (\hat{M}_1, \hat{M}_2)\Big] \to 0$ as $n \to \infty$.

Semi-Deterministic BC with Cooperation - Solution

Theorem (Capacity Region)

The capacity region is:

$$\mathcal{C}_{BC} = \bigcup \left\{ \begin{array}{rl} (R_{12}, R_1, R_2) \in \mathbb{R}_+^3 : R_{12} & \geq I(V;Y_1) - I(V;Y_2) \\ R_1 & \leq H(Y_1) \\ R_2 & \leq I(V,U;Y_2) + R_{12} \\ R_1 + R_2 & \leq H(Y_1|V,U) + I(U;Y_2|V) \\ & \quad + I(V;Y_1) \end{array} \right\}$$

where the union is over all joint distributions $P_{V,U,Y_1} P_{X|V,U,Y_1} P_{Y_2|X}$ *with the property* $Y_1 = f(X)$.

Duality - Preface

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- Solving one problem provides valuable insight towards the solution of the other.

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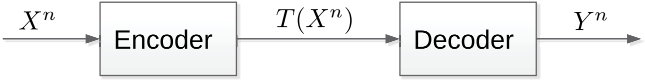

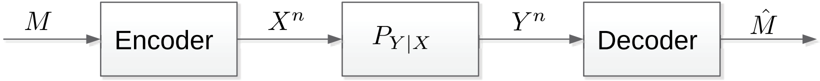

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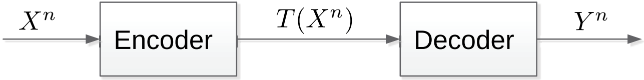

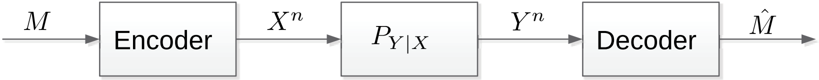

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Duality - Preface

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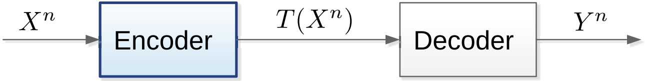

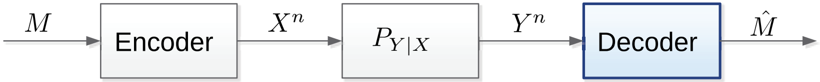

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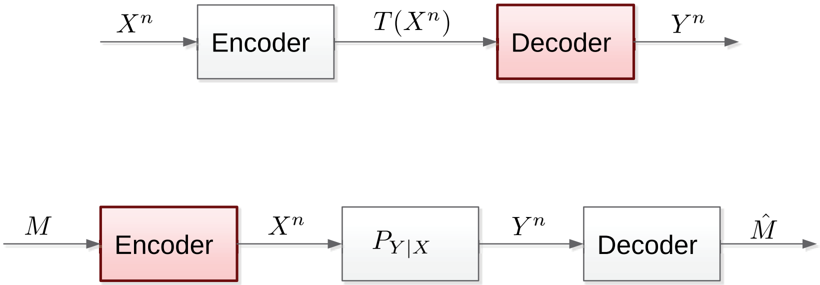
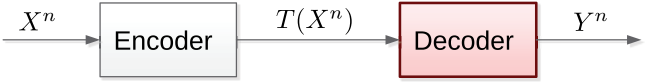
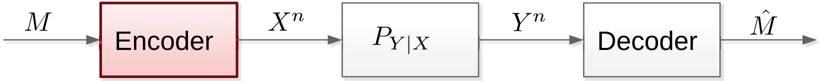


Duality - Preface

Point-to-Point Case:

$$R^\star = \min_{P_{Y|X}} I(X;Y)$$

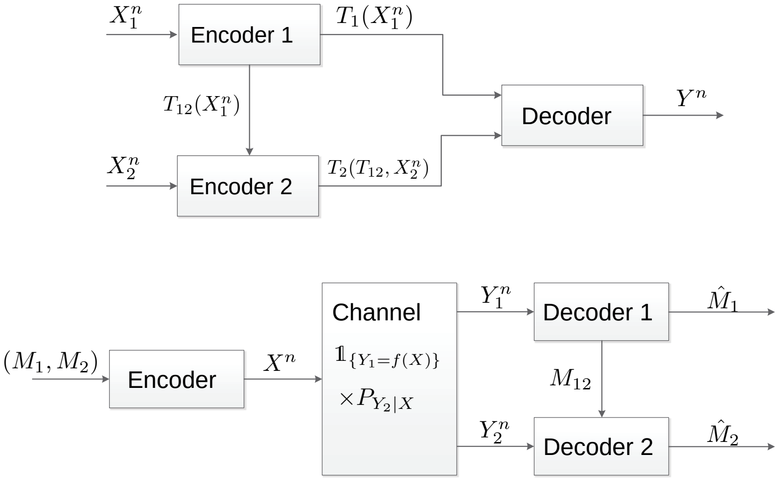




$$C = \max_{P_X} I(X;Y)$$

Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:



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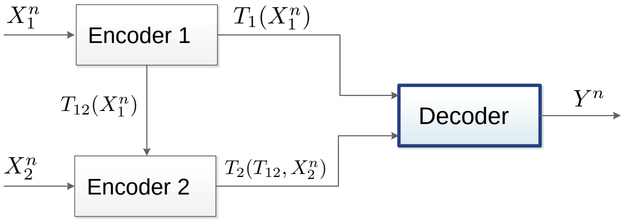

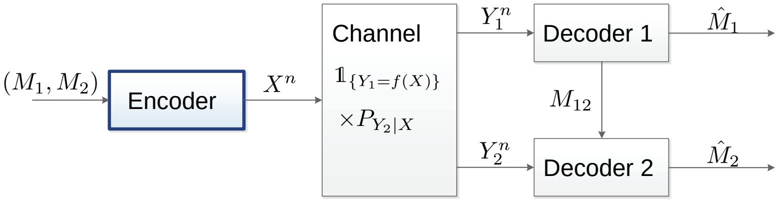

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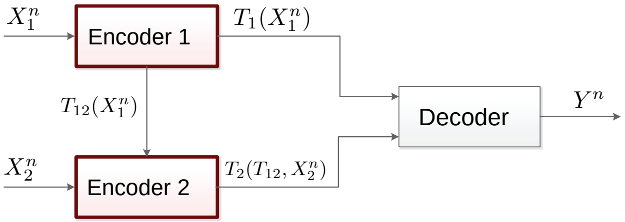
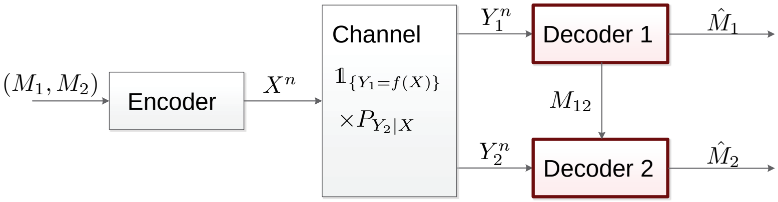




Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:

Probabilistic relations are preserved:

Duality - Multi-User Case

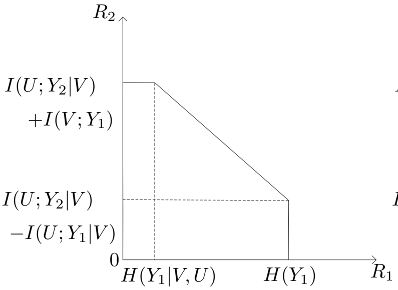
AK Problem vs. Semi-Deterministic BC:

Probabilistic relations are preserved:

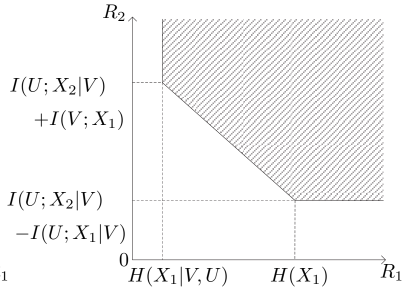
Semi-Deterministic BC		**AK Problem**
$(X^n, Y_1^n, Y_2^n) \in \mathcal{T}_\epsilon^{(n)}(X, f(X), Y_2)$	⟷	$(Y^n, X_1^n, X_2^n) \in \mathcal{T}_\epsilon^{(n)}(Y, f(Y), X_2)$
$(P_X^\star \mathbb{1}_{\{Y_1 = f(X)\}} P_{Y_2\|X})$		$(P_Y \mathbb{1}_{\{X_1 = f(Y)\}} P_{X_2\|Y}^\star)$

Duality - Corner Point Correspondence

For fixed joint distributions:



(a)



(b)

Duality - Corner Point Correspondence

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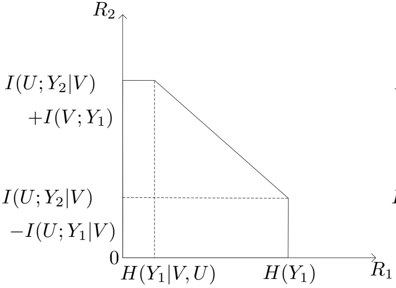

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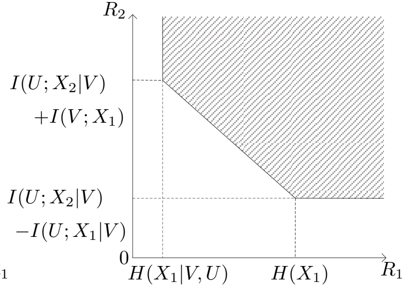

(b)

Semi-Deterministic BC with Cooperation	**Ahlswede-Körner Problem with Cooperation**
$R_{12} = I(V;Y_1) - I(V;Y_2)$	$R_{12} = I(V;X_1) - I(V;X_2)$
(R_1, R_2) at Lower Corner Point: $\big(H(Y_1)\ ,\ I(U;Y_2|V) - I(U;Y_1|V) \big)$	(R_1, R_2) at Lower Corner Point: $\big(H(X_1)\ ,\ I(U;X_2|V) - I(U;X_1|V) \big)$
(R_1, R_2) at Upper Corner Point: $\big(H(Y_1|V,U)\ ,\ I(U;Y_2|V) + I(V;Y_1) \big)$	(R_1, R_2) at Upper Corner Point: $\big(H(X_1|V,U)\ ,\ I(U;X_2|V) + I(V;X_1) \big)$

Duality - Corner Point Correspondence

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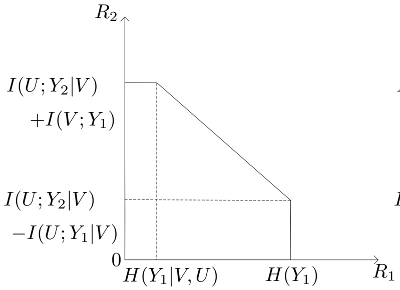

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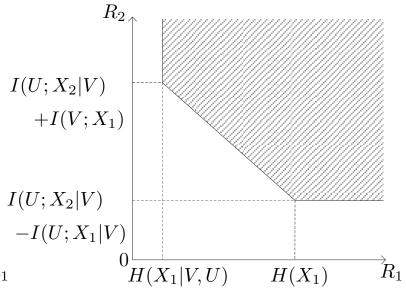

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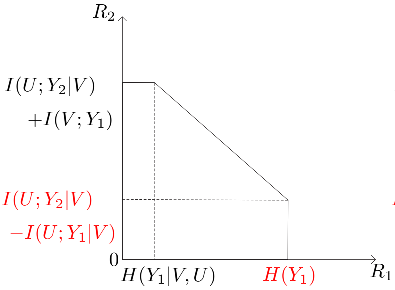

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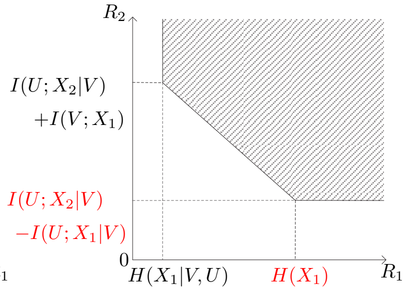

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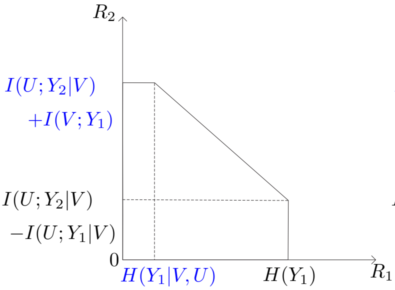

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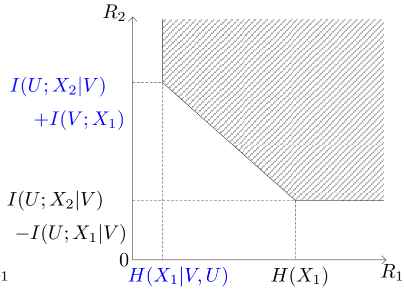

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Summary

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- Semi-deterministic BC with cooperation.
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Thank you!

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 1:

$$\Big(\,I(V;X_1|X_2)\;,\;H(X_1)\;,\;I(U;X_2|X_1,V)\,\Big).$$

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 1:

$$\Big(I(V;X_1|X_2)\ ,\ H(X_1)\ ,\ I(U;X_2|X_1,V) \Big).$$

- **Cooperation:** Wyner-Ziv coding to convey V^n from Encoder 1 to Encoder 2.

AK Problem with Cooperation - Achievability Outline

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- **Encoder 1 to Decoder:** Conveys X_1^n to the decoder in a lossless manner.
- **Encoder 2 to Decoder:** The decoder knows X_1^n and therefore V^n. Wyner-Ziv coding to convey U^n.

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

$$\Big(I(V;X_1|X_2)\ ,\ H(X_1|V,U)\ ,\ I(U;X_2|V)+I(V;X_1) \Big).$$

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

$$\big(\, I(V;X_1|X_2)\;,\; H(X_1|V,U)\;,\; I(U;X_2|V)+I(V;X_1)\,\big).$$

- **Cooperation:** Same.

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

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- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows V^n. Conveys the index of V^n and uses superposition coding to convey U^n.

AK Problem with Cooperation - Achievability Outline

Achieving Corner Point 2:

$$\big(I(V;X_1|X_2)\ ,\ H(X_1|V,U)\ ,\ I(U;X_2|V) + I(V;X_1) \big).$$

- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows V^n. Conveys the index of V^n and uses superposition coding to convey U^n.
- **Encoder 1 to Decoder:** The decoder knows (V^n, U^n). Binning scheme to convey X_1^n in a lossless manner.

AK Problem with Cooperation - Proof Outline

Converse:

AK Problem with Cooperation - Proof Outline

Converse:

- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\setminus i}, X_{2,i+1}^n),$$
$$U_i = T_2,$$

for every $1 \leq i \leq n$.

AK Problem with Cooperation - Proof Outline

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- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\backslash i}, X_{2,i+1}^n),$$
$$U_i = T_2,$$

  for every $1 \leq i \leq n$.
- Time mixing properties.

Semi-Deterministic BC with Cooperation - Proof Outline

Achievability: Split $M_i = (M_{i0}, M_{ii})$, $i = 1, 2$. Code construction:

Semi-Deterministic BC with Cooperation - Proof Outline

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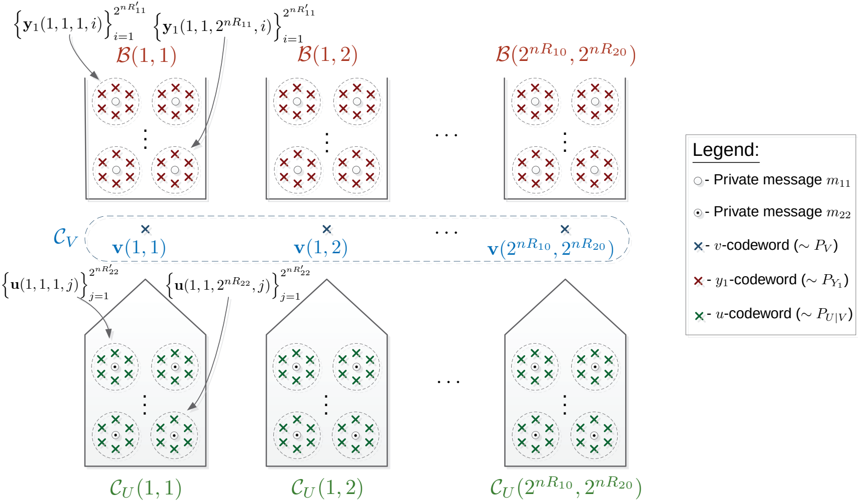

Semi-Deterministic BC with Cooperation - Proof Outline

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- Upper bound on the achievable region.

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- Upper bound on the achievable region.
- The auxiliaries are constructed in a probabilistic manner as a function of the joint distribution induced by each codebook.

Semi-Deterministic BC with Cooperation - Proof Outline

Converse: Via a novel approach - Probabilistic construction of auxiliary random variables:

- Upper bound on the achievable region.
- The auxiliaries are constructed in a probabilistic manner as a function of the joint distribution induced by each codebook.
- The upper bound is tightened to coincide with the achievable region.