Semantic Security versus Active Adversaries and Wiretap Channels with Random States

Ziv Goldfeld
Joint work with Paul Cuff and Haim Permuter

Ben Gurion University

Motivation

Information Theoretic Security over Noisy Channels

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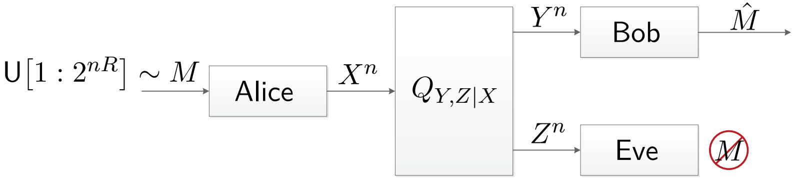
Our Goal: Stronger metric and remove "known channel" assumption.

Wiretap Channels - Security Metrics

Wiretap Channels and Security Metrics

Degraded [Wyner 1975], General [Csiszár-Körner 1978]



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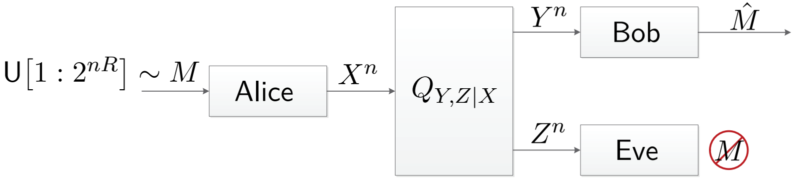

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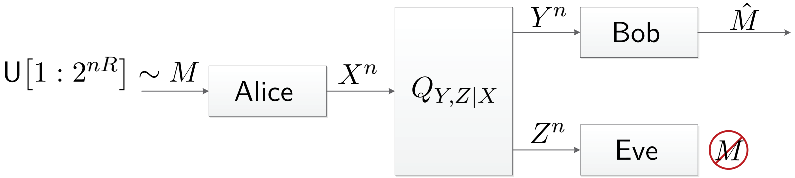

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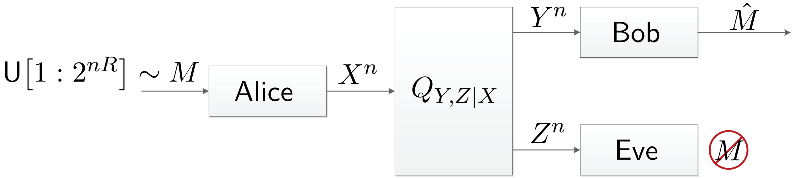

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- **Weak-Secrecy:** $\frac{1}{n}I_{\mathcal{C}_n}(M; Z^n) \xrightarrow[n\to\infty]{} 0.$ Only leakage **rate** vanishes

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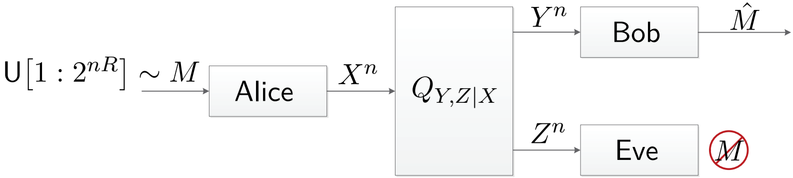

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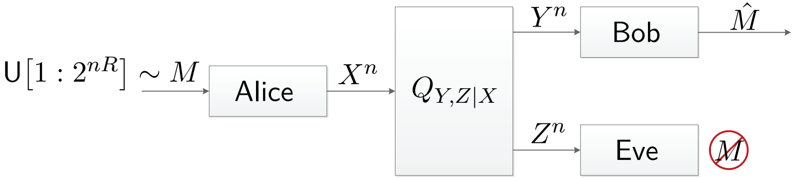

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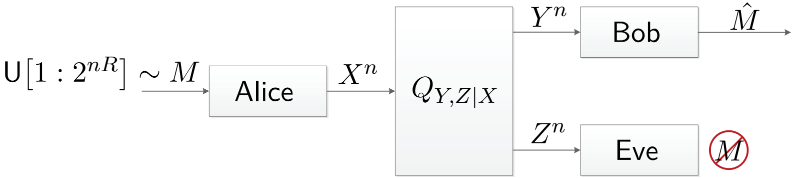

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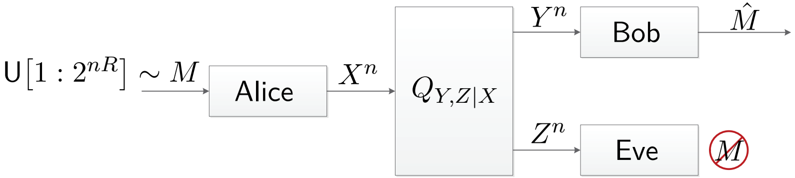
- **Strong-Secrecy:** $I_{\mathcal{C}_n}(M;Z^n) \xrightarrow[n\to\infty]{} 0$. Security only on **average**



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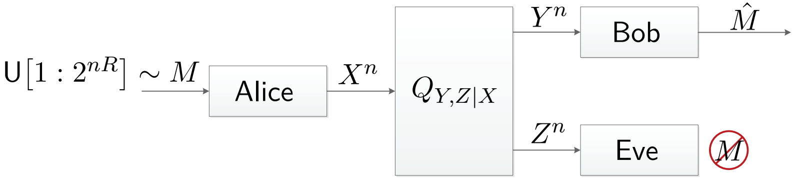

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★ A stronger secrecy metric is required for applications ★

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[Goldwasser-Micali 1982]

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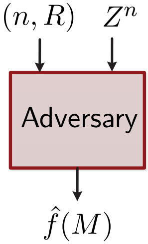

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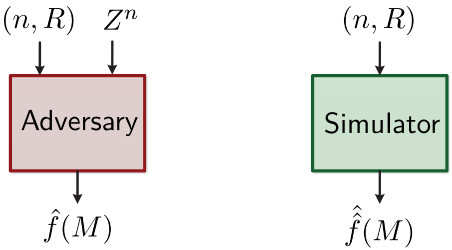

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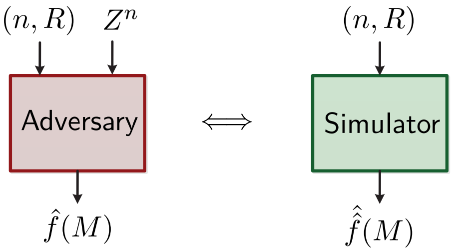

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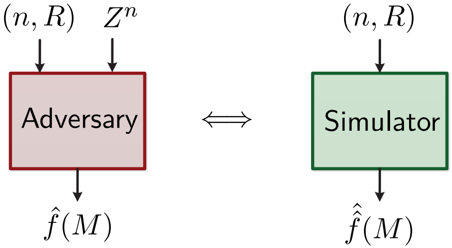

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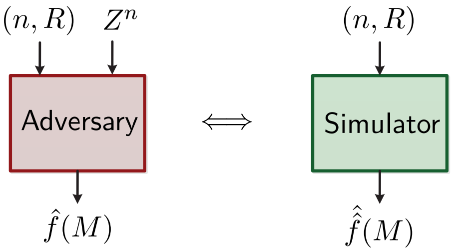

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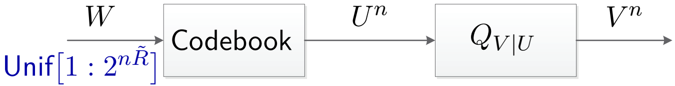
★ A single code must work well for all message PMFs ★

Strong Soft-Covering Lemma

Soft-Covering - Setup



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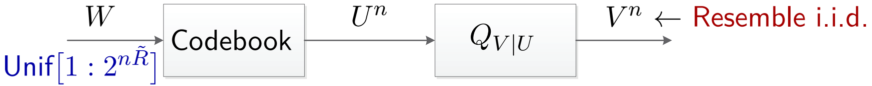

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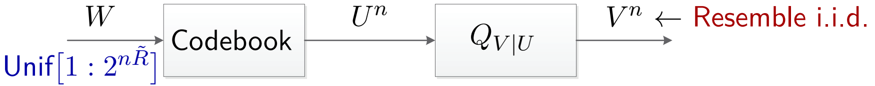

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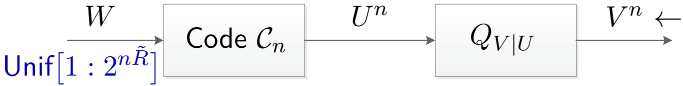

$\leftarrow$ Resemble i.i.d.

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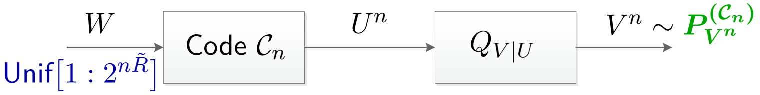

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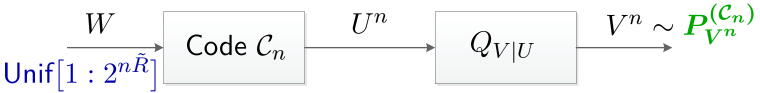

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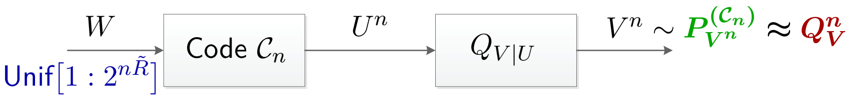

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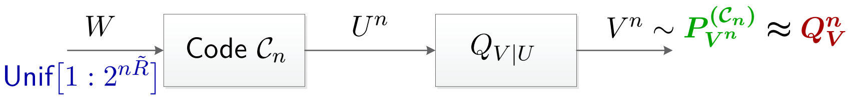

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⋆ **Goal:** Choose $\tilde{R}$ (codebook size) s.t. $P_{V^n}^{(\mathcal{C}_n)} \approx Q_V^n$ ⋆

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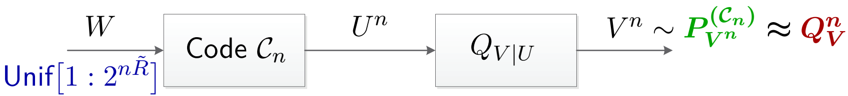

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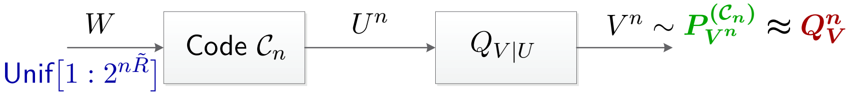

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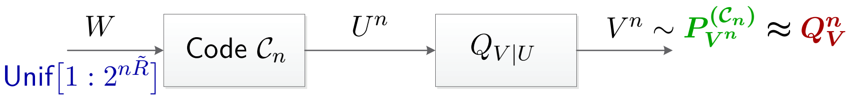

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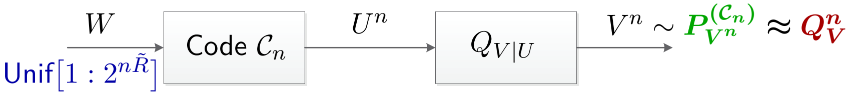

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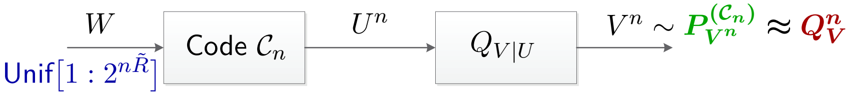

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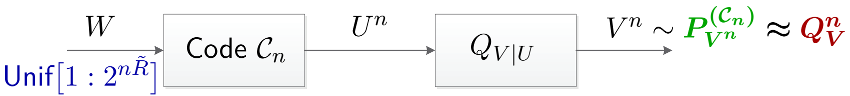

Lemma (ZG-Cuff-Permuter 2016)

If $\tilde{R} > I_Q(U;V)$*, then there exist* $\gamma_1, \gamma_2 > 0$ *s.t.*

$$\mathbb{P}_{\mathsf{C}_n}\left(D\left(P_{V^n}^{(\mathsf{C}_n)} \middle\| Q_V^n\right) > e^{-n\gamma_1}\right) \leq e^{-e^{n\gamma_2}}$$

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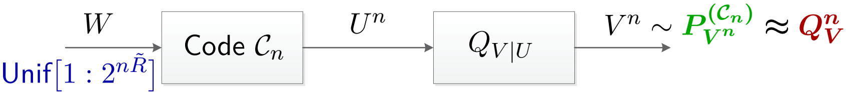

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- New proof via concentration of measure (McDiarmid Theorem).

Wiretap Channels of Type II

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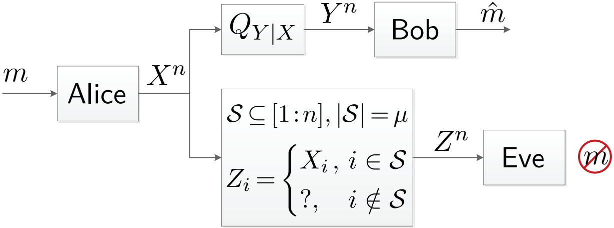
[Ozarow-Wyner 1984]



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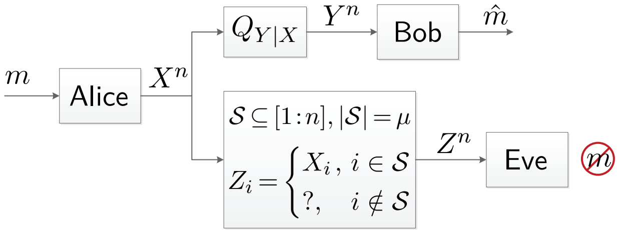

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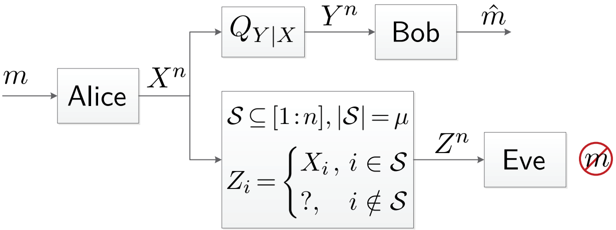

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- **Transmitted:**

0	0	1	0	1	1	1	0	1	0

$n = 10$ $\boldsymbol{\alpha = 0.6}$

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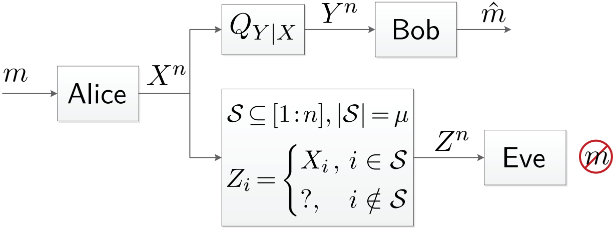

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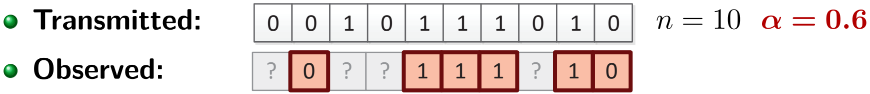

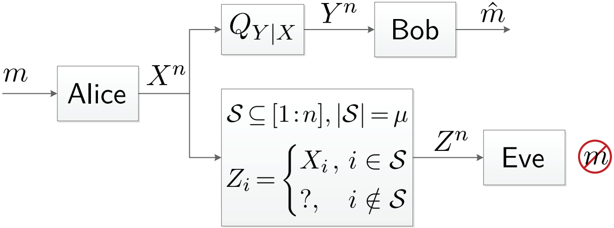

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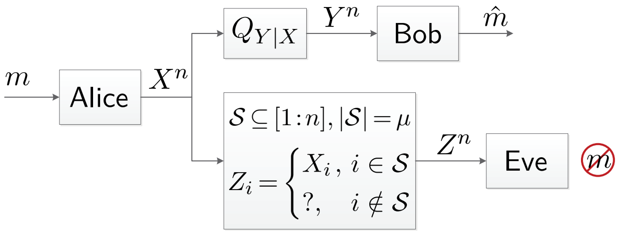
Transmitted:	0	0	1	0	1	1	1	0	1	0
Observed:	?	0	?	?	1	1	1	?	1	0

$n = 10$ $\boldsymbol{\alpha = 0.6}$

⋆ Ensure security versus all possible choices of $\boldsymbol{\mathcal{S}}$ ⋆

Wiretap Channels of Type II - Past Results

[Ozarow-Wyner 1984]



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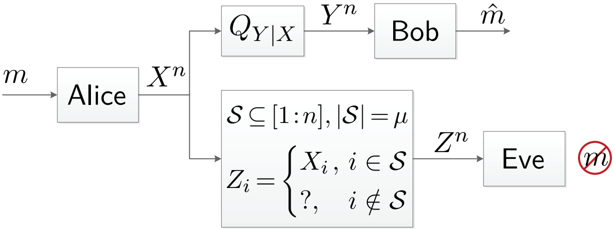

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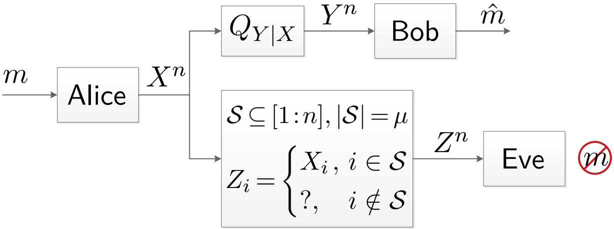

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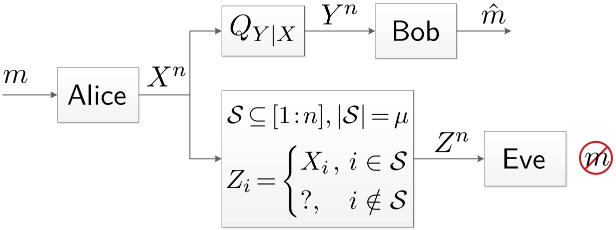

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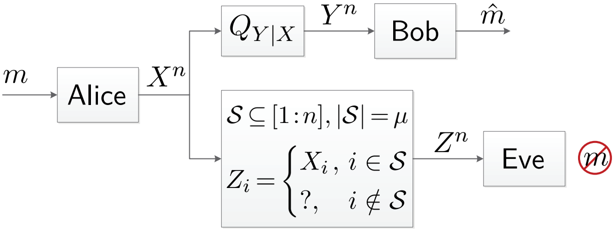

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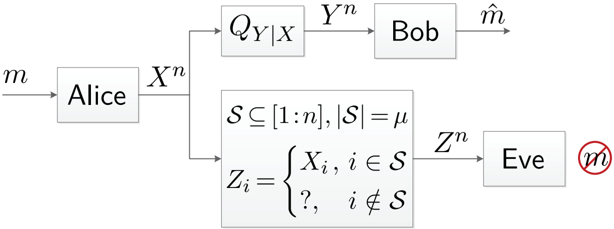

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Wiretap Channels of Type II - SS-Capacity

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- Standard (erasure) wiretap code & Stronger tools for analysis.

WTC II SS-Capacity - Achievability for U=X

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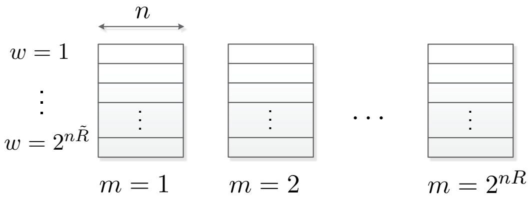

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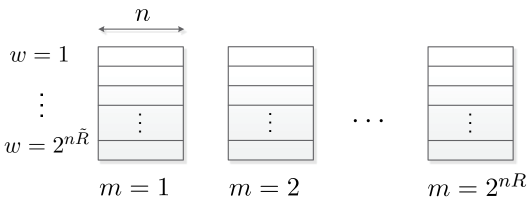

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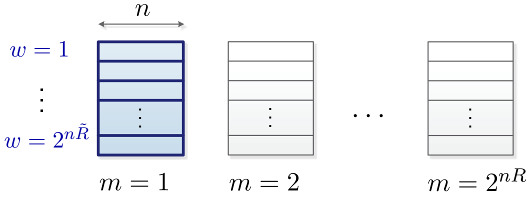

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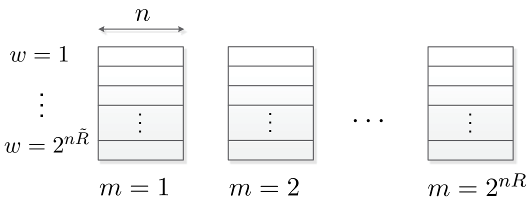

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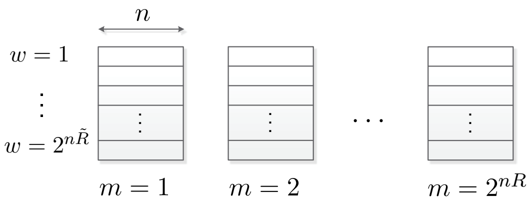

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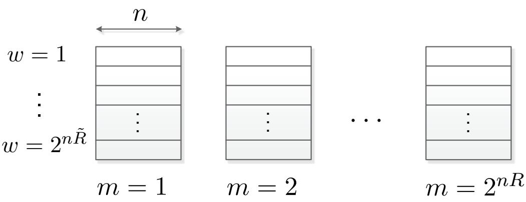

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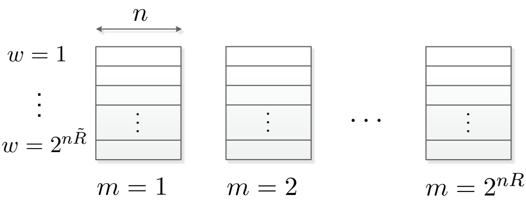

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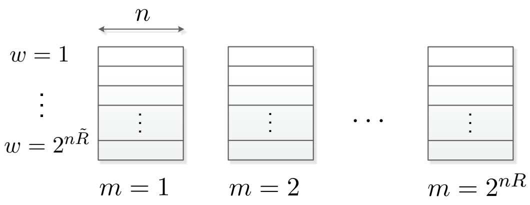

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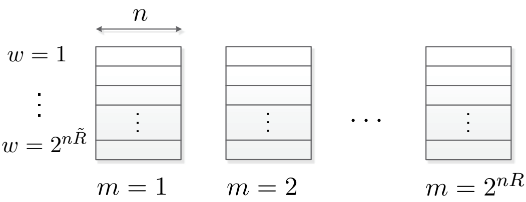

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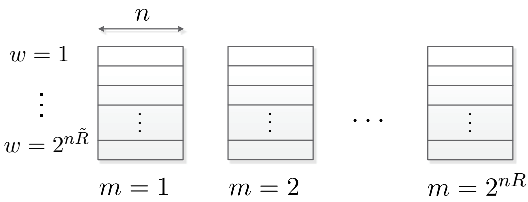

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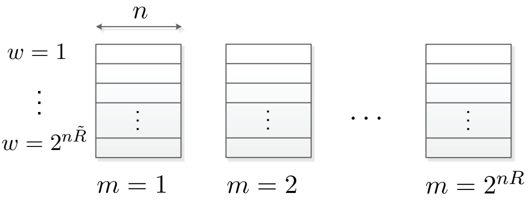

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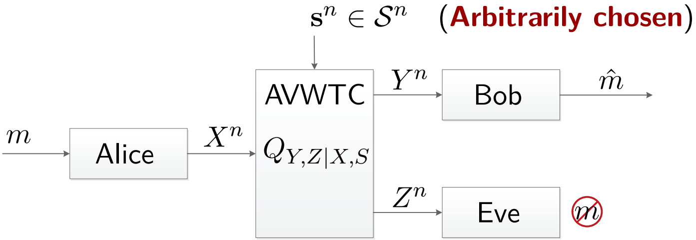

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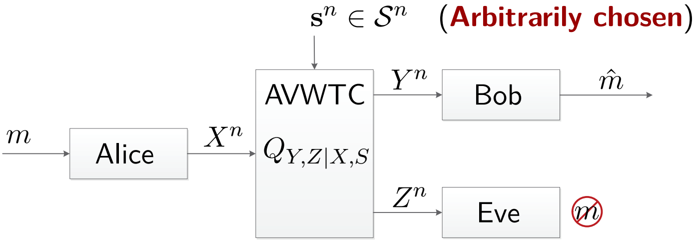

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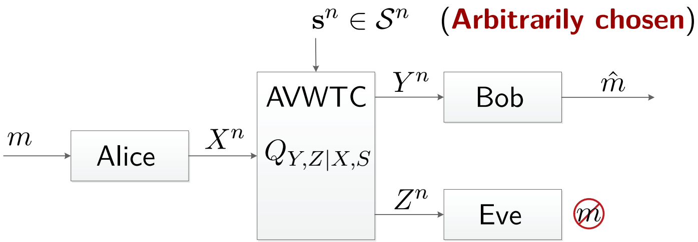

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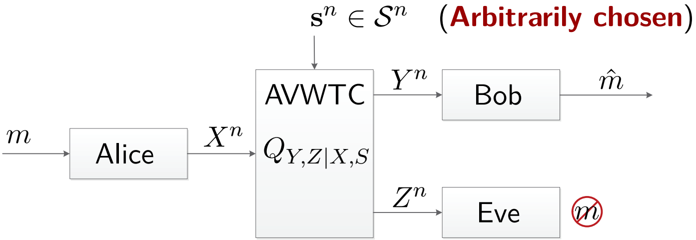

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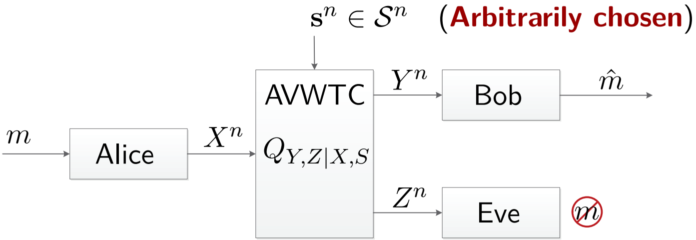

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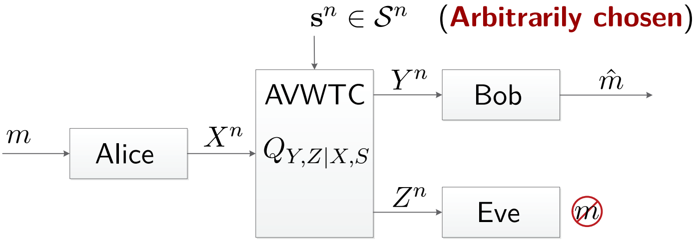

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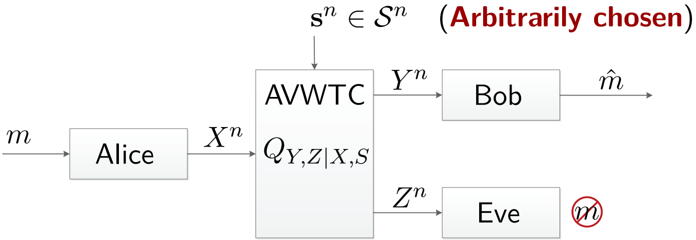

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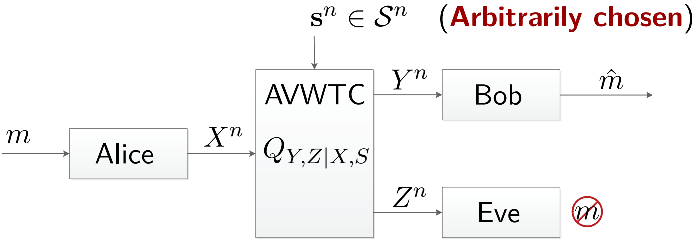

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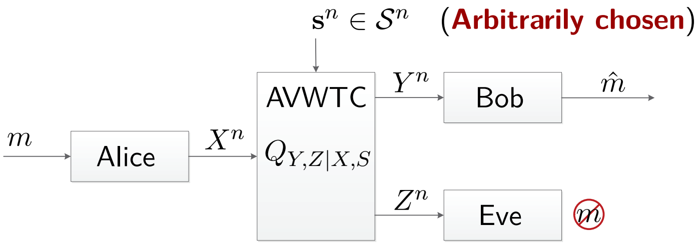

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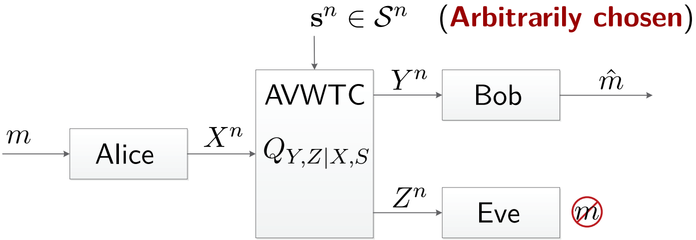

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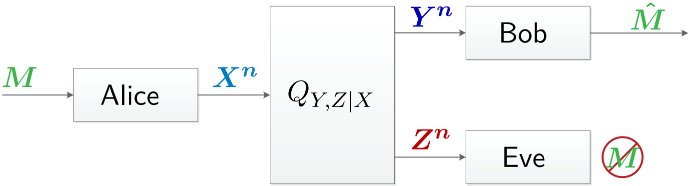
Wiretap Channels with Random States

The Wiretap Channel

Degraded [Wyner 1975], General [Csiszár-Körner 1978]



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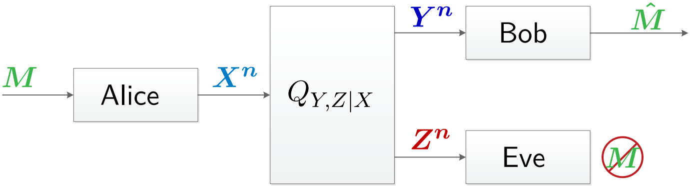

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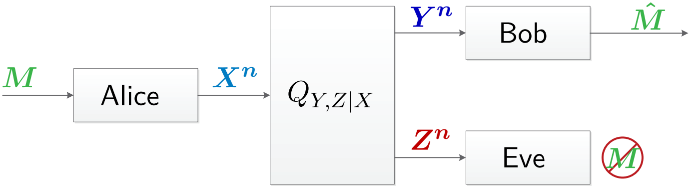

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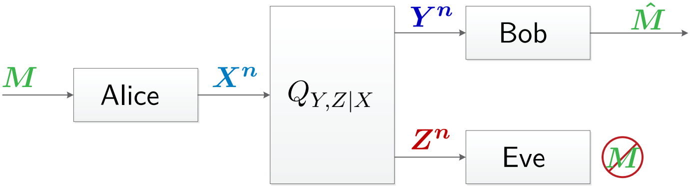

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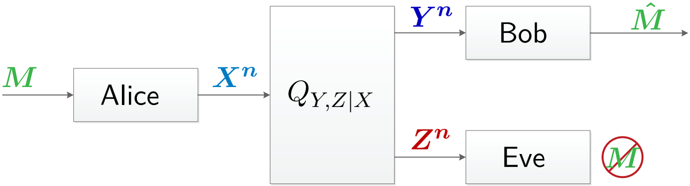

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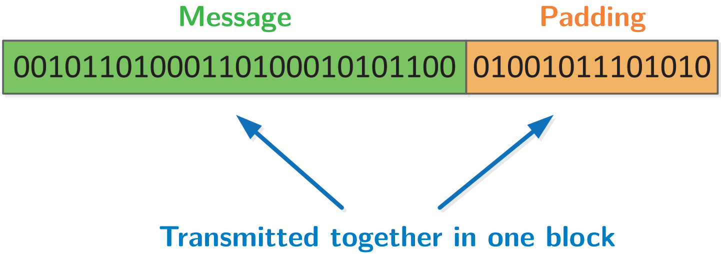

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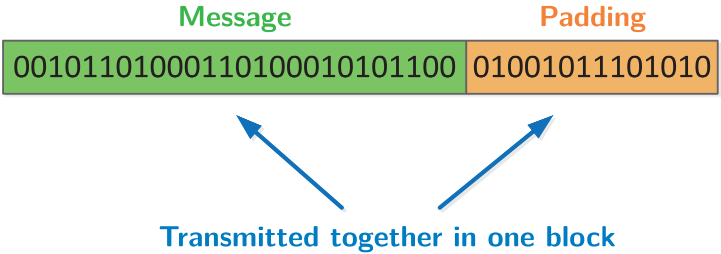

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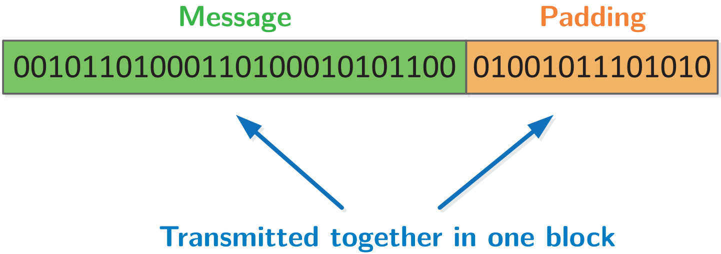

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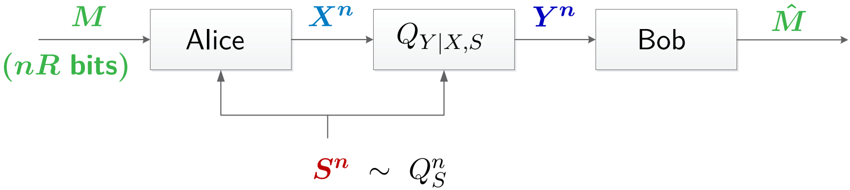
The Gelfand-Pinsker Channel

[Pelfand-Pinsker 1980]



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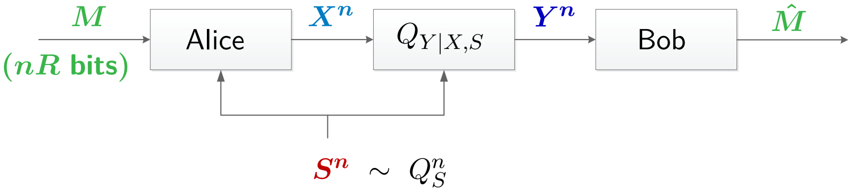

Capacity:

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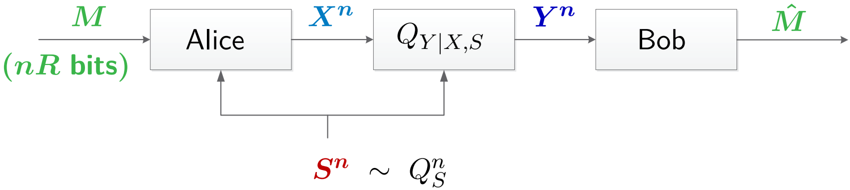

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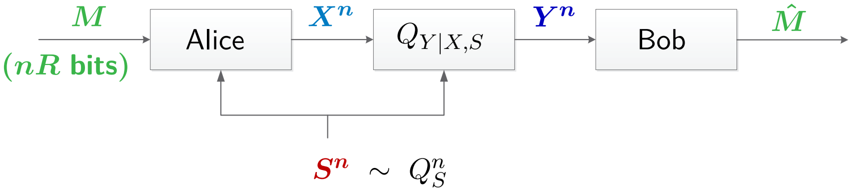

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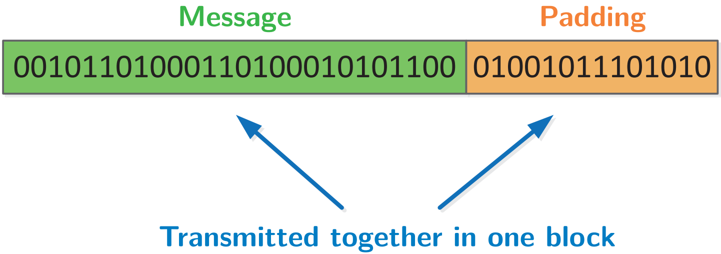

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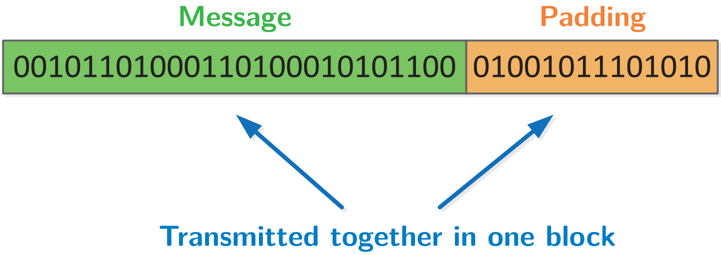

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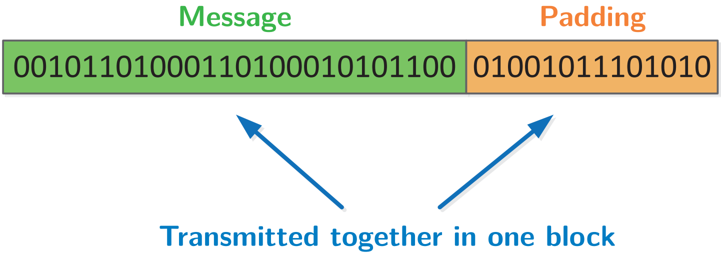

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Gelfand-Pinsker Channel vs. Wiretap Channel

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Gelfand-Pinsker Channel vs. Wiretap Channel

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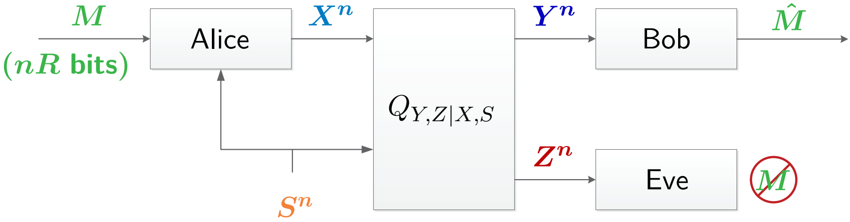

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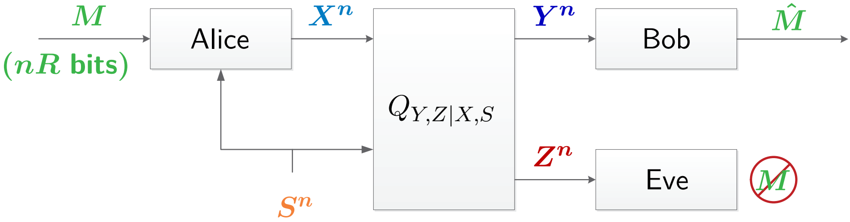

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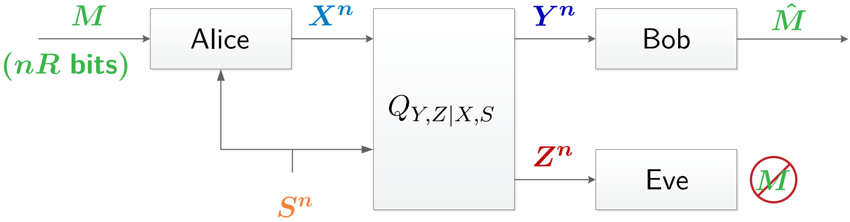

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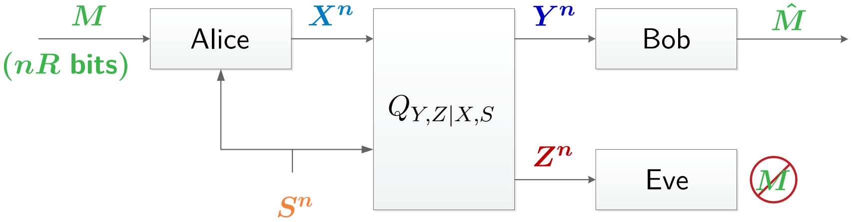

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The Gelfand-Pinsker Wiretap Channel

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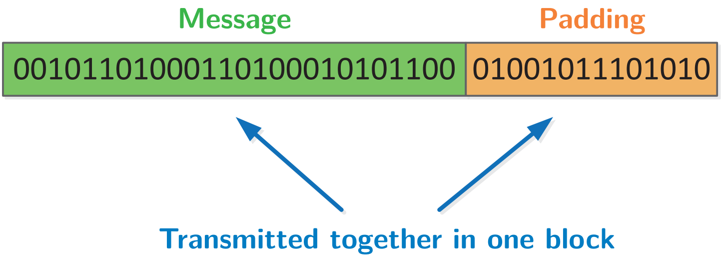

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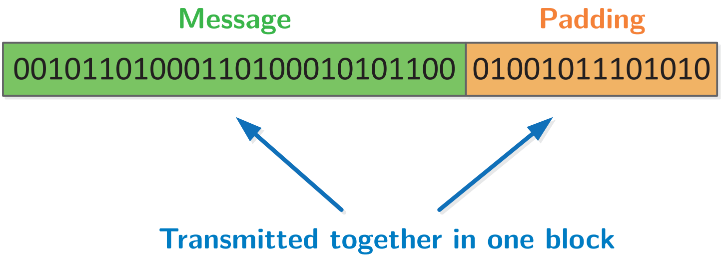

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Wiretap Channels with Encoder and Decoder CSI

Key Extraction Scheme [Chia-El Gamal 2012]

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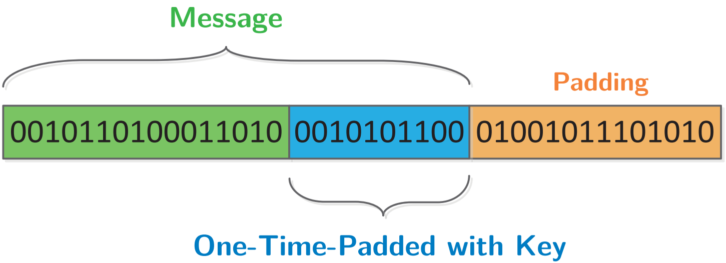

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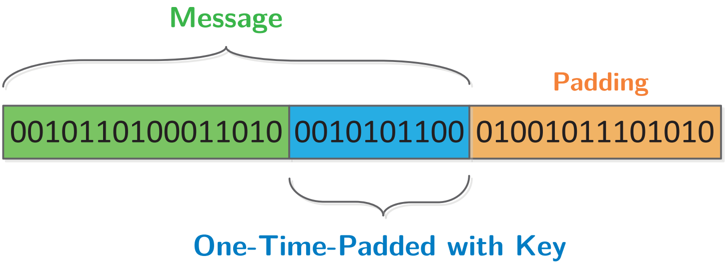

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The Gelfand-Pinsker Wiretap Channel - Our Scheme

Superposition Code:

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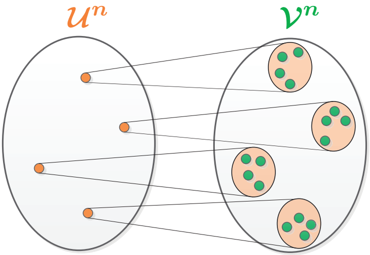

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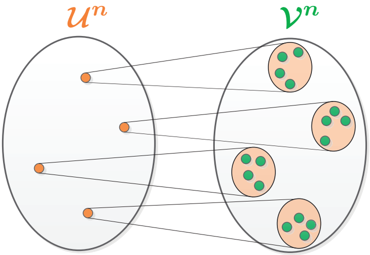

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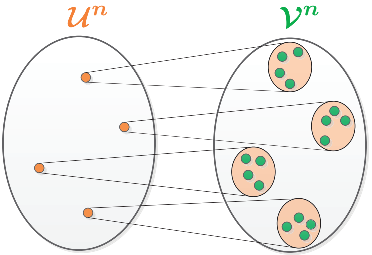

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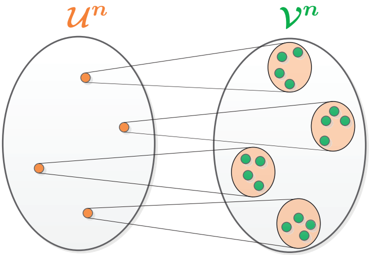

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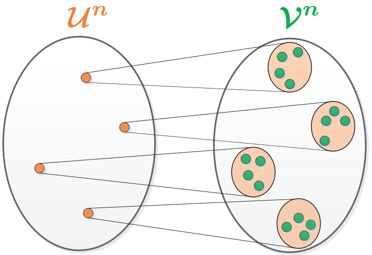

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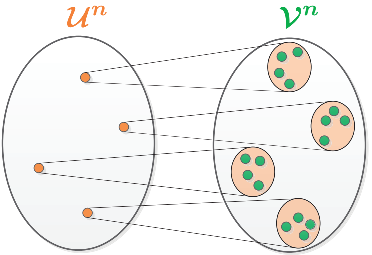

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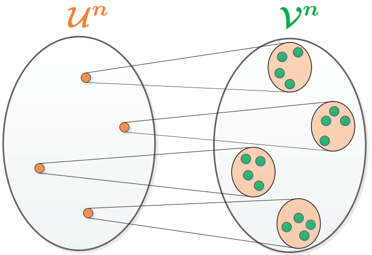

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⋆ **Analysis:** Likelihood Encoder & Superposition Strong SCL ⋆

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The Gelfand-Pinsker Wiretap Channel - Our Scheme

Theorem (ZG-Cuff-Permuter 2016)

$$\mathsf{C}_{\mathsf{GP-WTC}} \geq \max_{\substack{Q_{U,V,X|S}: \\ I(U;Y)-I(U;S)\geq 0}} \min \left\{ \begin{array}{c} I(V;Y|U) - I(V;Z|U), \\ I(U,V;Y) - I(U,V;S) \end{array} \right\}$$

Joint PMF: $Q_S Q_{U,V,X|S} Q_{Y,Z|X,S}$.

Relation to Previous Schemes:

- Upgrade from **weak-secrecy** to **semantic-security**.
- Recovers Chia-El Gamal's result when $Y = (Y,S)$.
- Beats previous regions even when S^n **not** known to Receiver.

Wiretap Channels with Random States - Recap

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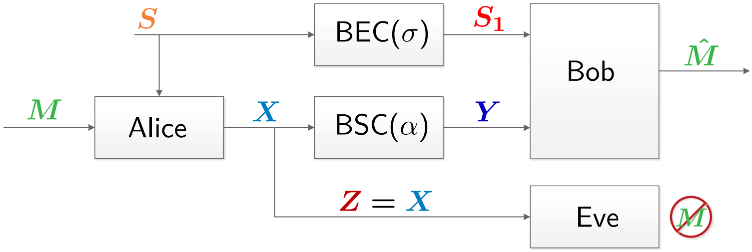
Thank you!

Outperforming Previous Schemes - An Example



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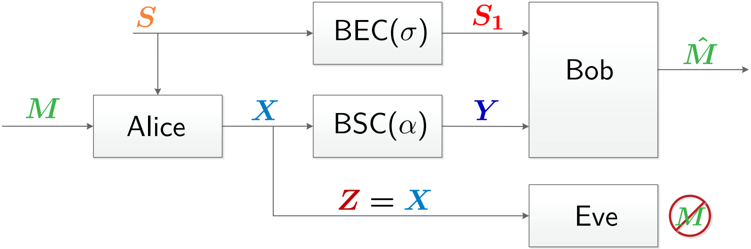

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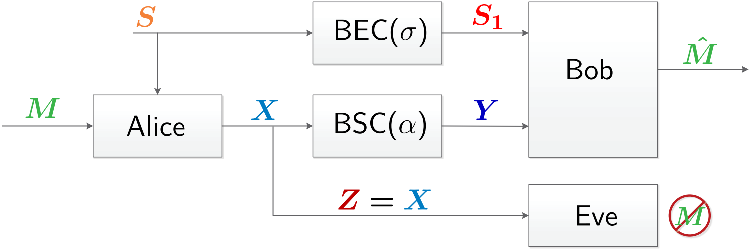

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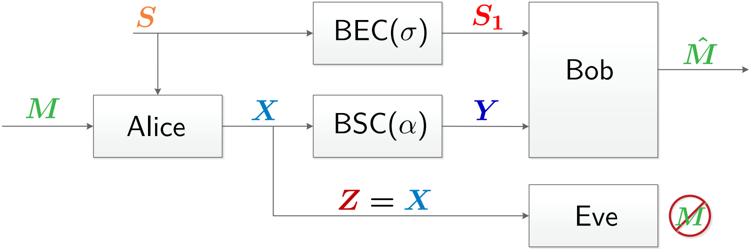

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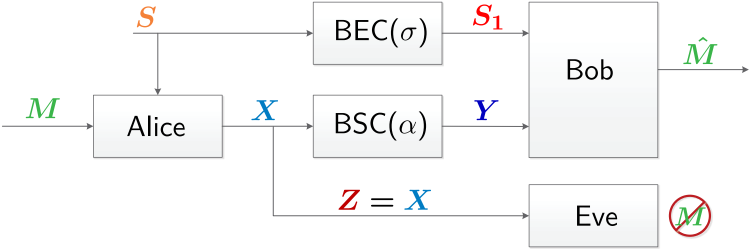

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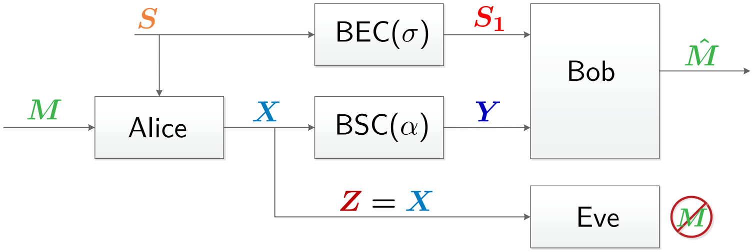

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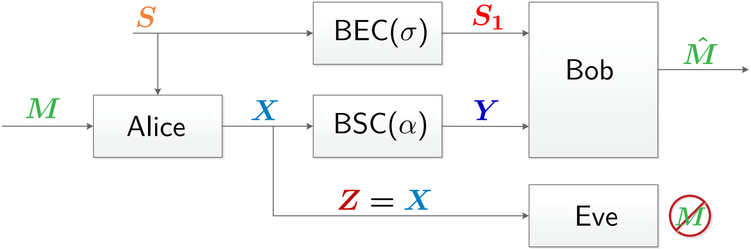

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- **Chen-Han Vinck scheme is suboptimal:**
  - Only one auxiliary - lacks flexibility to play both roles!

WTC II SS-Capacity - Converse

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- **Solution:** Sanov's theorem & Continuity of mutual information.